

## Risk Metrics for Dynamic Complex Infrastructure Systems such as the Power Transmission Grid

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### Abstract

*Complex critical infrastructures such as power transmission networks, communication systems, transportation network and others display many of the characteristic properties of complex systems. They exhibit infrequent large cascading failures that often obey a characteristic power law distribution in their probability versus size. This power law behavior suggests that conventional risk analysis does not apply to these systems. They also often display correlations between events suggesting that the system memory is important in its evolution. It is thought that much of this behavior comes from the dynamical evolution of the system as it is upgraded, ages, is repaired, and as the operational rules evolve. Metrics for the “system state”, i.e. quantifying how likely and risky large failures are is discussed in the context of the power transmission grid. How these metrics change, implying changed risk, with different upgrade and operational strategies is initially explored.*

### 1. Introduction

The goal of this paper is to identify possible metrics that give a measure of risk of extreme events in an infrastructure system such as the power transmission system. A first step in identifying such a metric is determining the parameter of the system that plays a critical role in leading to such events.

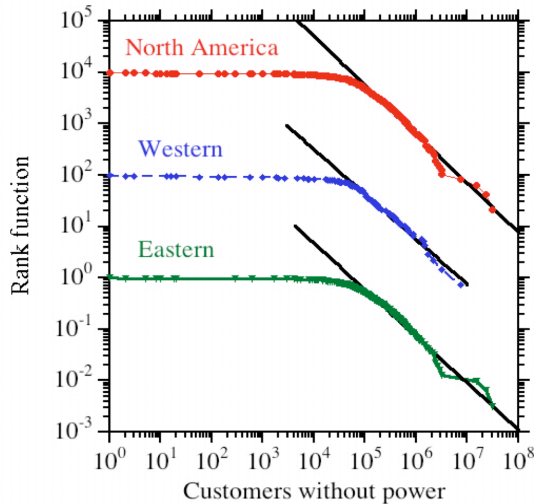
In a complex system, extreme events may be triggered by a random event. However, the much higher than Gaussian probability of extreme events (the heavy tail) [1] is a consequence of the correlations induced by operating near the operational limits of the

system and has nothing to do with the triggering events. The result is that the tail of the extreme event distribution is independent of the triggering events. Therefore, trying to control the triggering events does not lead to a change of the form of the power tail distribution. A careful reduction of triggering events may reduce the frequency of blackouts but will not change the functional form of the distribution. The process of trying to plan for and impact the triggering events can in fact lead to a false sense of security since one might think one is having an effect on risk by doing so when in reality, the unexpected triggers which will certainly occur will lead to a similar distribution of blackout sizes.

These types of “heavy tail” distributions are seen in many infrastructure systems. Figure 1 shows the cumulative distribution function for customers unserved during blackouts for the US power transmission system. The figure shows that the distribution of sizes in the blackout has a power law (“heavy”) tail with a slope of  $\sim -0.98$ . It also is apparent that there is a very similar functional distribution for the Western region and Eastern region separately, despite differences in the network grid characteristics. It is these tails, which contain the rare yet very important extreme events, which dominate the overall risk [2].

In these complex systems, an initiating event cannot be identified by just the random trigger event, but by the combination of the triggering event and the state of the system. This “state of the system” can be characterized by different measurements of the elements of the system. In our case for example, the system state includes the distribution and amount of loads and power flows in the network. A model like OPA [2-6] is continually, self-consistently, changing the network loading and power flows. This, importantly, gives a large sample of initiating events and system states. The statistics of the results therefore reflect these many combinations of initial events and system states. It is some measure of the system state

that we would like to find which can be used as a risk metric since by their nature the random triggers are not predictable.



**Fig.1. The cumulative distribution functions for a measure of the size of blackouts (Customers without power), for the total US grid and for two sub-regions, the eastern and western regions. Power law tails can be clearly seen in all three.**

The dynamics of these complex systems is driven by the constant push toward the breaking point coming from aging and or increased demand while the pull away from that critical point is driven by upgrade, repair, replacement and sometimes regulation [2,7]. This dynamic push-pull coupled with the cascading failures initiated by the random triggers leads to the heavy tails which characterize the increased risk of “extreme events”. It is important to note that while often these systems are modeled by increasing the demand, aging can be equally important in driving the system toward the critical point as it also often leads to an increased risk of component failure for a given load.

The rest of the paper is organized as follows. In Section II, we introduce a risk measure that is going to be used for the different potential metrics discussed in this paper. For different values of these potential metrics we evaluate the probability of a blackout in Section III and the average size of the blackout in Section IV. The risk as a function of these quantities is evaluated in Section V. This risk measure is then tested in Section VI for the occurrence of extreme events. Finally the conclusions of the paper are given in Section VII.

## 2. Risk Metrics in a Complex System

The determination of the risk of an event requires the determination of the probability of the event and a measure of the consequences (or cost) of the event. Here, for simplicity, we will quantify those consequences simply by the size of the event. Estimating the real cost of an event as a function of its size is difficult, and we have discussed elsewhere several possible cost measures for blackouts for which the cost of a blackout scales as a power greater than 1 as a function of the the blackout power shed [2]. However in order to discuss the basic ideas of the metrics proposed, we will simply use a cost that is proportional to the size measured by the power shed normalized to the power demand.

In determining the risk, the first step is the identification of the parameters that play a critical role in the occurrence of the extreme events (the tail of the PDF or CDF). From this identification, we can select which parameters are directly measurable and this then would allow us to implement such a metric. An alternative approach is to find ways to measure the proximity to critical point. This type of metric is in general an a posteriori measure, which is useful in order to learn how to operate the system in a safer manner.

For the second step, we need to determine how these parameters correlate with the size of the event. Because of the intrinsic dynamics of a complex system, we can only give a correlation between critical parameter and the probability of certain event sizes.

Let  $Q$  be one of the parameters identified. Then the range of  $Q$  values can be divided into  $N$  bins of width  $\Delta Q$ . If Probability ( $Q_j$ ) is the probability of a blackout when  $Q$  is in the range  $(Q_j, Q_j + \Delta Q)$  and  $\langle \text{Size}(Q_j) \rangle$  is the averaged size of the blackout, we can define the risk associated with the blackouts in which  $Q$  is in the range  $(Q_j, Q_j + \Delta Q)$  as:

$$\text{Risk}(Q_j) = \text{Probability}(Q_j) \langle \text{Size}(Q_j) \rangle \quad (1)$$

In this work, for power transmission systems, we use the OPA model to investigate these possible relationships. The OPA model provides guidance as to which parameters may play a critical role in leading to extreme events. We can then build testable metrics for the OPA model based on these parameters. Clearly however, once the parameters have been identified with OPA it will be necessary to validate them with measurements in a real power transmission system in order to calibrate the metric.

## 3. The OPA Model

The OPA model [2-6] for a fixed network configuration represents transmission lines, loads and generators with the usual DC load flow approximation

using linearized real power flows with no losses and uniform voltage magnitudes.

There are two basic timescales modeled in OPA. For the slow, long time scale part, the OPA blackout model represents the essentials of slow load growth, cascading line outages, and the increases in system capacity coming from the engineering responses to blackouts. The short timescale part captures the cascading line outages leading to a blackout, which are regarded as fast dynamics and are modeled as follows. Starting from a solved base case, blackouts are initiated by random line outages. Whenever a line is outaged, the generation and load is re-dispatched using standard linear programming methods. This is because there is more generation power than the load requires and one must choose how to select and optimize the generation that is used to exactly balance the load. The cost function is weighted to ensure that load shedding is avoided where possible. If any lines were overloaded during the optimization, then these lines are outaged with probability  $p_l$ . The process of re-dispatch and testing for outages is iterated until there are no more outages. The total load shed is, then, the power lost in the blackout.

The slow dynamics model the growth of the load demand and the engineering response to the blackout by upgrades to the grid transmission capability. The slow dynamics is carried out by the following small changes applied each time a potential cascading failure is simulated: All loads are multiplied by a fixed parameter that represents the rate of increase in electricity demand. If a blackout occurs, then the lines involved in the blackout have their line flow limits increased slightly. The grid topology remains fixed in the upgrade of the lines for model simplicity. In upgrading a grid it is important to maintain coordination between the upgrade of generation and transmission. The generation is increased at randomly selected generators subject to coordination with the limits of nearby lines when the generator capacity margin falls below a threshold.

#### 4. Critical Parameters for the Occurrence of Blackouts

To demonstrate the identification of critical parameters, we use the OPA code with the WECC model networks. These are fairly large (over 1000 bus reduced representations of the Western region. Input parameters for these models have been determined [8] by matching the blackout data from NERC, historical performance data [9,10,11], as well as data on cascading events [12].

Some variables of the system that we measure every “simulation day” are:

- $\langle M \rangle$ , the averaged value of  $M_i$  (the fractional line loading) over all lines every day. Here,

$$\langle M \rangle = \frac{1}{N_{Lines}} \sum_{i=0}^{N_{Lines}} M_i = \frac{1}{N_{Lines}} \sum_{i=0}^{N_{Lines}} \frac{F_i}{F_i^{\max}} \quad (2)$$

where  $F_i$  is the power flow in line  $i$ ,  $F_i^{\max}$  is the maximum power flow allowed in this line, and  $N_{Lines}$  is the number of lines in the network.

- The variance of  $M$ ,  $\langle (M - \langle M \rangle)^2 \rangle$ , every day

• The distribution of values of  $M_i$  for each line. The OPA code calculates a distribution of  $M_i$  values with ten bins of width 0.1. We often consider the number of lines with  $M_i > M_0$  and we will be using  $M_0 = 0.9$ .

- Load shed and load shed normalized to the power demand. This is used as our blackout size measure.

Let  $Q(t)$  be one of the parameters measured every day during one of the simulations of the power transmission system evolution. This same quantity, now called  $Q_b(i)$ , is also measured every time a blackout occurs. The range of  $Q$  values is divided into  $N$  bins of width  $\Delta Q$ . If there are  $n_j$  values of  $Q(t)$  in the bin  $j$ , and  $m_j$  values of  $Q_b(i)$  in the same bin, the probability of a blackout when  $Q$  is in the range  $(Q_j, Q_j + \Delta Q)$  is,

$$p_j = \frac{m_j}{n_j} \quad (3)$$

Using this approach, we can evaluate the probability of a blackout for a given value of each of the network parameters listed in the previous section. In this way, we can see if the measurement of these quantities can tell us if a blackout is likely. For example, if, in a simulation, there are 50 instances (days) in which the average value for  $M$  is between 0.6 and 0.65 ( $\Delta Q$  being 0.05 and  $Q_j$  being 0.6 in this example), and there are 30 blackouts on those 50 days, then  $n_j = 50$  and  $m_j = 30$  giving a probability of a blackout  $p_j = 0.6$  for  $\langle M \rangle$  between 0.6 and 0.65.

The results of this diagnostic measure, for the main four quantities listed in the previous section calculated for both the WECC 1553 and 2507 bus networks are plotted in Fig. 2. From this figure the following conclusions can be reached:

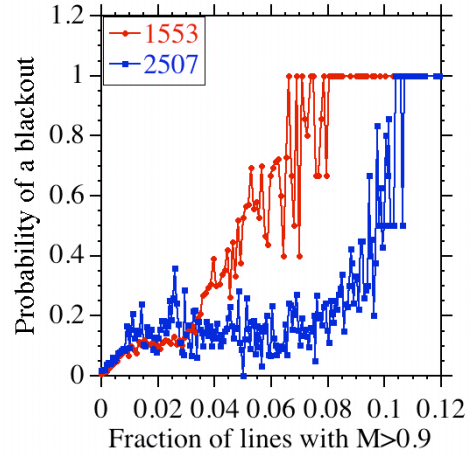
- The averaged value of  $M$  ( $\langle M \rangle$ ): The two networks show that the probability of a blackout increases with  $\langle M \rangle$ . The increase becomes sharper with a threshold type of effect that displays a size scaling behavior.
- The variance of  $M$  ( $\langle (M - \langle M \rangle)^2 \rangle$ ): The functional form of this plot is very similar to that

of  $\langle M \rangle$ . This also shows a threshold type effect for both networks.

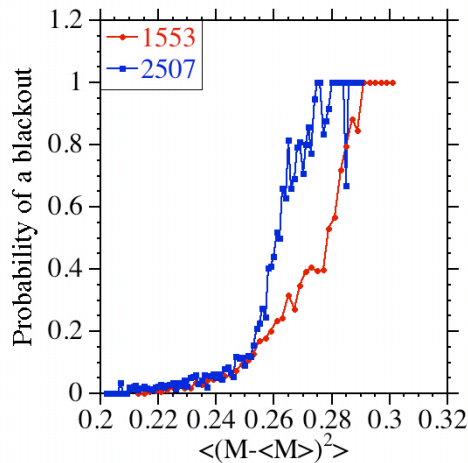
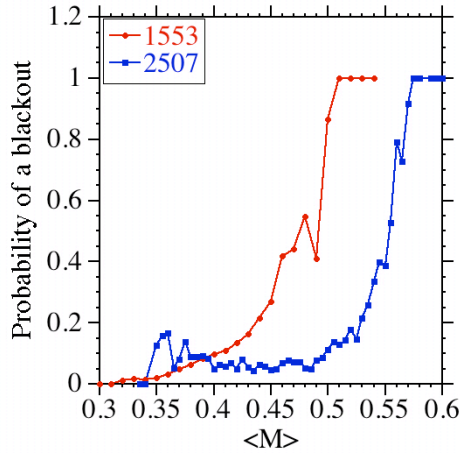
- The number of lines with  $M > 0.9$ : The probability of a blackout grows strongly with the number of lines with  $M > 0.9$ . This threshold depends on the size of the network and it is not directly correlated neither with the number of lines overloaded nor with the fraction of lines overloaded. Here by fraction of lines overloaded we refer to the ratio of overloaded lines to the total number of lines of the network.

All three of these metrics saturate at a probability of 1 for large enough values.

Based on these results, there appear to be several useful quantities to monitor the system and calculate the probability of a blackout occurring. However, this information alone does not provide guidance as to the size of the blackout to expect.



**Fig.2. Probability of a blackout as a function of three network measures. These results are for the WECC 1553 bus and 2507 bus networks**



## 5. Critical Parameters and the Size of Large Blackouts

After determining parameters that correlate with blackouts, we will look for correlations between the values of the critical parameters and the size of the blackout. Let us consider one of the parameters,  $Q$ . For each blackout  $j$ , we have a value of this parameter  $Q_j$  and a value  $S_j$  for the load shed normalized to the power demand. A number of useful quantities can be calculated:

- 1) Binning the parameter  $Q$  in bins of size  $\Delta Q$ , the average value of  $S$  in the bin  $k$ ,  $\langle S \rangle_k$  can be calculated permitting  $\langle S \rangle_k$  to be plotted as a function of  $Q_k$ . When the standard deviation of  $S$  in the bin  $k$  is calculated, it is used as an error bar.
- 2) Consider a bin  $k$ . In this bin, there are  $n_k$  values of  $S$ . Of them, there are  $m_k$  values of  $S > S_0$ . Therefore, as before, the probability of having a blackout of size  $S > S_0$  when  $Q_k < Q < Q_k + \Delta Q$  is  $P_k = m_k/n_k$ . Then,  $P_k$  can be plotted as a function of  $Q_k$ .

Finally, in each of the bins, the minimum and maximum size of the blackouts can be determined and tabulated.

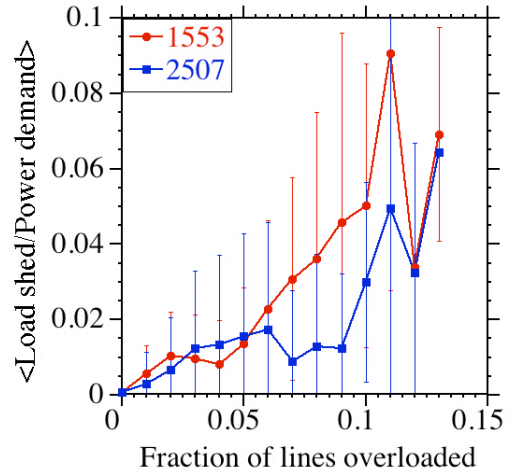
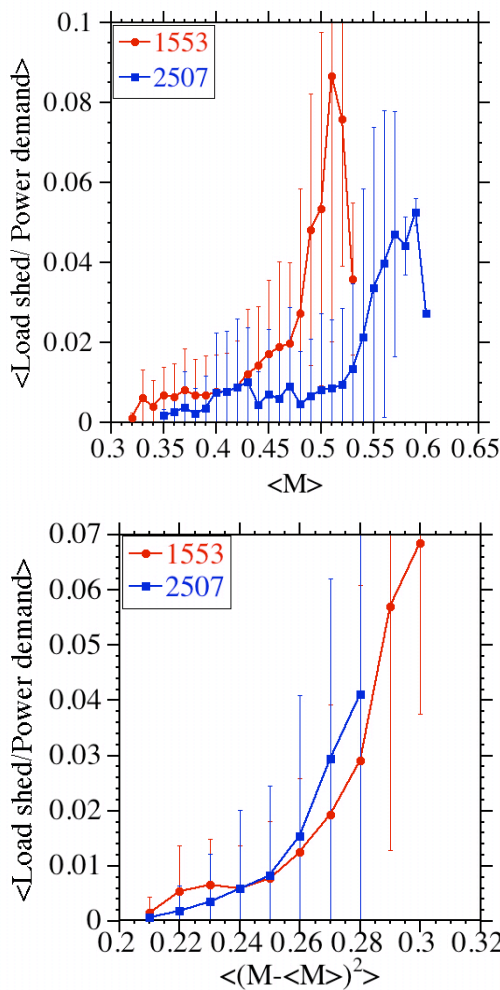
Using this method, the correlation of  $\langle M \rangle$  and the size of the blackout is analyzed. Binning the variable  $\langle M \rangle$  and proceeding as above, the average blackout size as a function of  $\langle M \rangle$  can be plotted. In general, as

$\langle M \rangle$  increases the blackout size increases. Interestingly, this rolls off at the highest values of  $\langle M \rangle$ . This is shown in Fig. 3 for the same two WECC network models. There is a clear network size dependence in this relationship.

Also in Fig. 3, the calculation of the blackout size as a function of variance of  $M$  is shown. The function has very similar behavior for both networks. In this case the curves overlap, exhibiting no network size dependence.

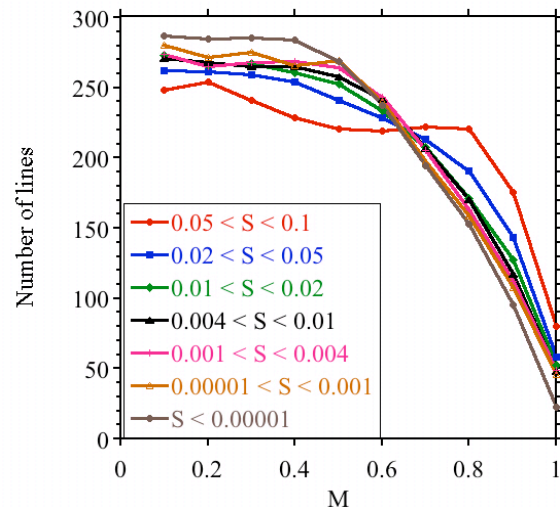
Finally, the fraction of lines with  $M > 0.9$  (the overload value) is considered and the same method is applied. In Fig. 3, the averaged size of the blackout as a function of the fraction of number of lines with  $M > 0.9$  is once again plotted for the two WECC network models.

The dependence of the probability of a blackout and its size on the several measures of the  $M$  distribution reflects the structure of this distribution associated with the different blackout sizes. Plotting this distribution for different ranges of  $S$ , it becomes apparent how the probability distribution function (PDF) of  $M$  changes.



**Fig. 3. Averaged size of the blackout as a function of  $\langle M \rangle$  (top), the variance of  $M$  (middle), and the fraction of lines with  $M > 0.9$  (bottom) in the initiating event for two WECC network models. Error bars represent the standard deviation**

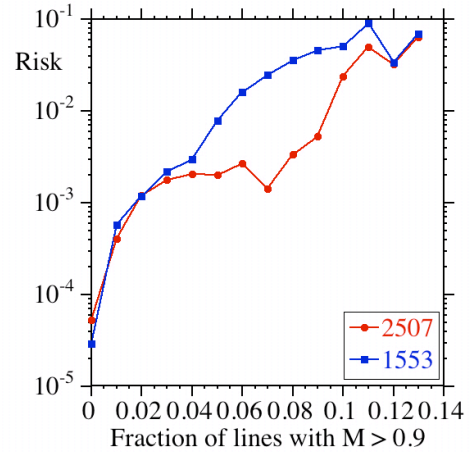
In Fig. 4 the plot of the PDF for the different ranges of  $S$  and for the WECC 1553 bus network is shown. It can be seen that as the blackout size gets larger, the distribution is shifted towards larger values of  $M$ , or perhaps more properly, as the distribution is shifted towards larger values of  $M$ , the blackout size gets larger.



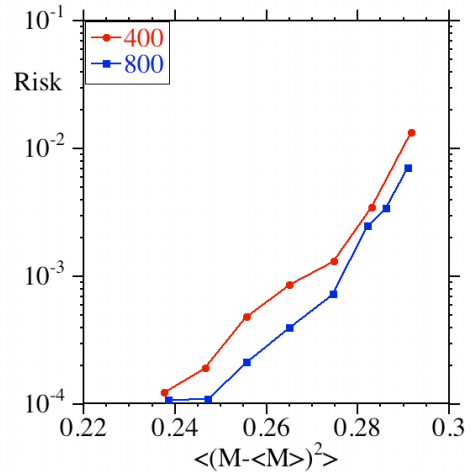
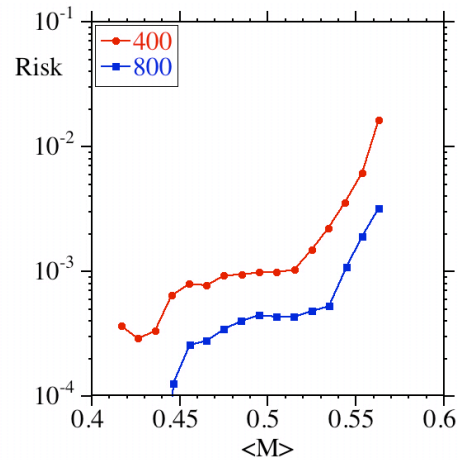
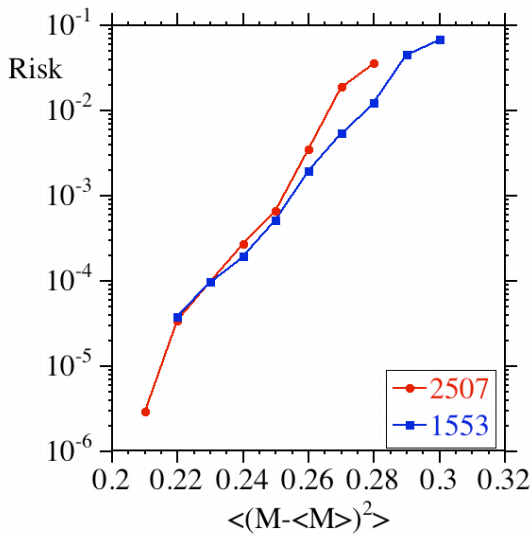
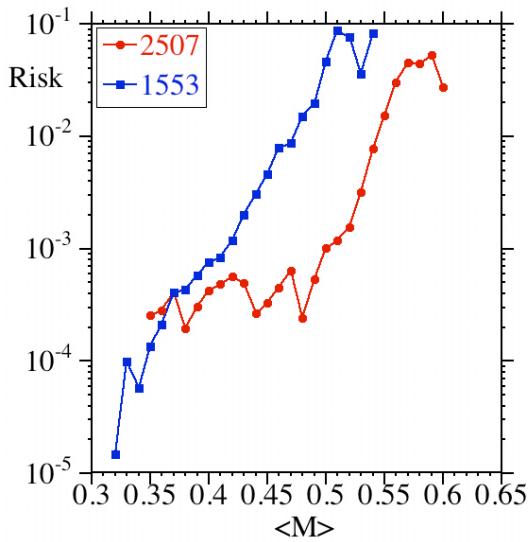
**Fig. 4. PDF of the values of  $M$  associated with the lines of the WECC 1553 bus network for different ranges of the blackout size.**

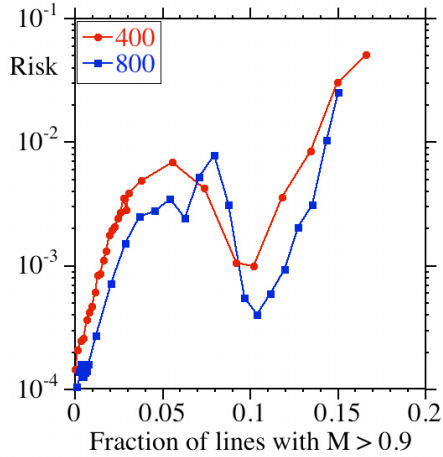
## 6. Risk Metrics for Power Transmission Systems

Having evaluated the probability of an event and the averaged size of the event as a function of the measurable parameters, we can calculate the risk using Eq. (1). In Fig. 5, we show the risk as a function of the  $\langle M \rangle$ , the variance of  $M$  and the fraction of lines with  $M > 0.9$  for the WECC 1553 and 2507 bus network models. Similar plots are shown in Fig. 6 for artificial 400 and 800 bus network models. For these smaller network models, we have considerably better statistics, which allow us to check some of the features of the risk function obtained for the WECC network models.



**Fig. 5. Risk of the blackout as a function of  $\langle M \rangle$  (top), the variance of  $M$  (middle), and the fraction of lines with  $M > 0.9$  (bottom) for the WECC 1553 and 2507 bus network models.**





**Fig. 6. Risk of the blackout as a function of  $\langle M \rangle$  (top), the variance of  $M$  (middle), and the fraction of lines with  $M > 0.9$  (bottom) for the 400 and 800 bus network models.**

For the 2507 bus WECC network and the artificial networks, as a function of  $\langle M \rangle$ , the risk is at moderate level for  $\langle M \rangle < 0.5$ . However, and perhaps very importantly, for higher values the risk increases exponentially. Regarding the variance of  $M$ , risk seems to be an exponentially increasing function of  $\langle (M - \langle M \rangle)^2 \rangle$ . Finally, for the fraction of lines with  $M > 0.9$ , the risk function has strange form with a peak in some cases for low values of the fraction. This deserves further examination; therefore in the next section we discuss the reason for this behavior.

## 7. Risk Metrics and Extreme Events

In calculating the risk function we have used the average event size over a group of events. Since the blackout probability distribution function is not Gaussian and has a heavy tail, this averaging is not necessary a good description of the event size associated with the value of the parameter considered. It is important to submit these potential metrics to a further test.

To address the latter point, when we determine the probability of an event using a sample with  $n_i$  points, the minimum value of the probability that we can measure within a factor of 2 with a 95% confidence level is given by  $p_{min} = 11/n_i$  [13].

If the probability that we want to measure is lower than this minimum value, we have only two solutions. One is run this case for a longer time to accumulate more statistics. The other is to increase the size of the bin  $\Delta \langle M \rangle$ . Naturally, we can weaken the accuracy of the determination, but that is not very satisfactory. In what follows, we will keep this accuracy criterion and

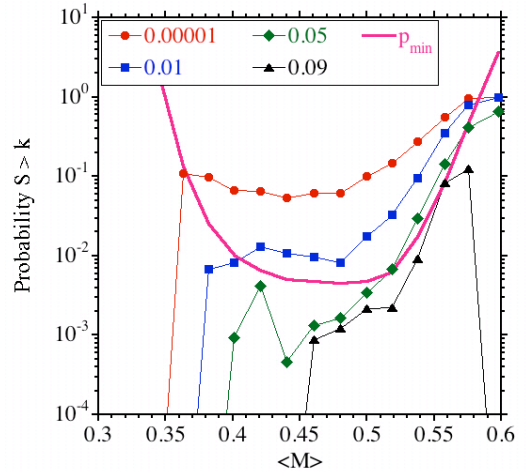
do a first estimate of what we can do with our usual OPA code.

To find a metric for large events, in the following, we consider only events larger than a given size. To evaluate the probability of a blackout with size  $S$  larger than a value  $k$ , we determine the number  $m_i$  of blackouts with  $S > k$  in the bin  $i$ , then the probability of a blackout when  $\langle M \rangle$  is in the range  $M_i - \Delta \langle M \rangle \leq \langle M \rangle \leq M_i + \Delta \langle M \rangle$  is

$$p_i = \frac{m_i}{n_i} \quad (3)$$

For this value to be correct within a factor of 2 with a 95% confidence level we need to have  $p_i \geq p_{min}$ . Here  $S = \text{load shed}/\text{Power demand}$  during a blackout.

We can now calculate the probability of blackouts with size  $S \geq k$ , for  $k = 0.00001$  (that is, for all blackouts according to our definition),  $k = 0.01$ ,  $k = 0.05$  and  $k = 0.09$ . The calculated probabilities for the 2507 bus network are plotted in Fig. 7.

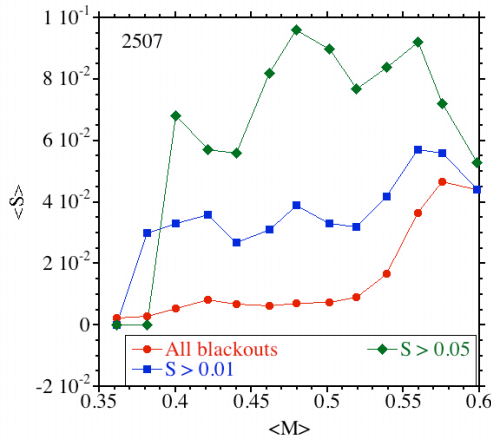


**Fig. 7. Probability of blackouts with size  $S \geq k$ , for the WECC 2507 bus network for four values of  $k$ . In the figure,  $p_{min}$  is also plotted.**

We can see that the probability function has a similar structure for all three values of  $k$ . It is relatively flat for  $\langle M \rangle \leq 0.48$ , then increases sharply. The rate of increase increases with  $k$ . By looking at the larger blackouts, we have confirmed the results obtained previously. Similar results are obtained for other networks.

The next step is the determination of the average size of the blackout for the different thresholds considered. We use as a measure of blackout size the ratio of the load shed to the power demand. The calculated average size of the blackouts as a function

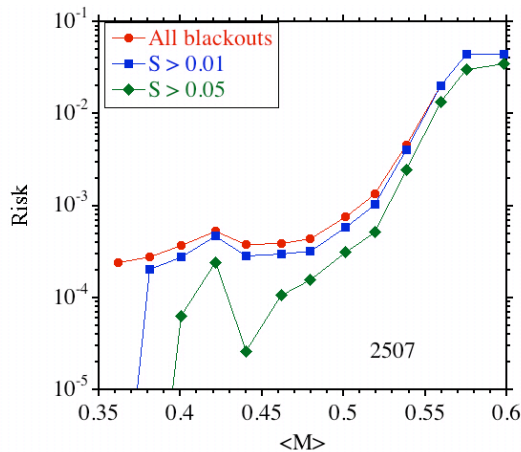
of  $\langle M \rangle$  under different conditions and for the four networks are shown in Fig. 8.



**Fig.8. Average size of the blackouts as a function of  $\langle M \rangle$  under different conditions for the 2507 bus network.**

Now, having evaluated both the probability of an event and the average size of the event as a function of the measurable parameters, we can calculate the risk using Eq. (1). In Fig. 9, we have plotted the calculated risk for the 2507 bus network.

Similar results have been obtained for all networks. These all show that at low values of  $\langle M \rangle$  the risk is practically independent of  $\langle M \rangle$ , but for higher  $\langle M \rangle$  the risk increases rapidly as a function of  $\langle M \rangle$ . The form of the risk function is the same for all the cases considered with  $S \geq k$ . This verifies the result suggesting that to minimize risk in this system, one must operate the system with  $\langle M \rangle < 0.5$ .



**Fig.9. Risk of the blackouts as a function of  $\langle M \rangle$  under different conditions and for the 2407 bus network.**

## 8. Conclusions

Because the heavy tails found in size distributions of failures in many complex infrastructure systems dominate the risk and therefore the societal cost/impact, understanding these risks and finding metrics to quantify them is essential. The dynamics which cause the heavy tails are often driven by a combination of the aging of the system components and the increased load placed on those components and the counteracting forces of upgrade, repair, replacement, economics and changes in operation. Approaches like those shown here, with simplified models used to determine the important parameters and important values of the risk metrics, are very promising and allow the investigation of the real impact of changing upgrade, repair [14] and replacement policies and different types of aging profiles.

Some initial conclusions can be drawn from these results:

1) The probability of a blackout occurring correlates well with  $\langle M \rangle$  every day, variance of  $M$ ,  $\langle (M - \langle M \rangle)^2 \rangle$ , every day, and the number of lines with  $M > 0.9$

2) When a blackout occurs, the size of a blackout also correlates with  $\langle M \rangle$  every day, variance of  $M$ ,  $\langle (M - \langle M \rangle)^2 \rangle$ , every day, and the number of lines with  $M > 0.9$ . In fact it appears that the risk of the largest events starts to rapidly grow when  $\langle M \rangle$  crosses a threshold of  $\sim 0.5$ . This type of criterion could give guidance to operators or designers of the power grid.

3) One could construct an even more meaningful composite metric for the risk of large blackouts using two of the three quantities. Both the  $\langle M \rangle$  and  $\langle (M - \langle M \rangle)^2 \rangle$  seem to be very good metrics, they show a clear separation between an operational region of moderate risk and a high risk region.

## ACKNOWLEDGMENTS

We gratefully acknowledge support in part from NSF grants SES-0623985 and SES-0624361 and in part by the California Energy Commission, Public Interest Energy Research Program. This paper does not necessarily represent the views of the Energy Commission, its employees or the State of California. It has not been approved or disapproved by the Energy Commission nor has the Energy Commission passed upon the accuracy or adequacy of the information. One of us (BAC) thanks the financial support of Universidad Carlos III and Banco Santander through a Càtedra de Excelencia.

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