Number and propagation of line outages in cascading events in electric power transmission systems

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Abstract— **Large blackouts typically involve the cascading outage of transmission lines. We estimate from observed utility data how transmission line outages propagate, and obtain parameters of a branching process model of the propagation. We show that the branching process model is consistent with the utility data by using it to estimate the distribution of the total number of lines outages and showing that this closely matches the distribution of the total number of line outages observed in the utility data. The branching process model and the measured propagation can then be applied to predict the distribution of total number of outages for a given number of initial failures. We study how the total number of lines outages depends on the amounts of propagation as the cascade progresses. The analysis gives a new way to quantify the effect of cascading failure from standard utility data about automatic line outages.**

I. INTRODUCTION

Cascading failure is a series of dependent failures that progressively weakens the system. Large electric power transmission systems occasionally have cascading failures that cause widespread blackouts, with up to tens of millions of people affected [20], [21], [22]. The cascades causing the larger blackouts have hundreds of dependent events, which include the outages of transmission lines. The multiple mechanisms involved these cascading outages are many and varied, and the power grid networks are heterogeneous and so large that methods relying on detailed enumeration of the full range of possibilities must fail. While methods that sample a subset of the possibilities can also be of use in maintaining power system reliability, in this paper we seek to capture and analyze the bulk statistical behavior of cascading transmission line outages from standard utility data that records the times of line outages.

The power system is carefully designed and operated so that most transmission line outages have only one or a few outages occurring together. Most of these short cascades do not cause blackouts (no load is shed), but the longer cascades can lead to blackouts. In this paper we statistically analyze the propagation of line outages in all the cascades of automatic line outages, regardless of whether load is shed.

II. PREVIOUS WORK

Branching processes have long been used to study cascading processes in many other subjects, including genealogy, cosmic rays, and epidemics [15], but their application to cascading failure is much more recent, first appearing in [9], [10].

Ren and Dobson [19] give a previous analysis of propagation in a transmission line outage data set. The data set of [19] is smaller and has a different source than the data set considered in this paper. The outages are simply grouped into stages and cascades according to their timing. [19] estimates an average value of propagation and shows how the distribution of the number of line outages can be predicted from the estimated propagation and the distribution of the initial line outages. One difference between this paper and [19] is that this paper accounts for varying propagation as the cascade progresses whereas [19] assumes a constant value of propagation throughout the cascade.

Chen and McCalley describe an accelerated propagation model for the number of transmission line failures in [8]. For parameters based on combined data for North American transmission line failures from [1], the accelerated propagation model applies to up to 7 failures. They examine the fit of the accelerated propagation model, a generalized Poisson distribution, and a negative binomial distribution to the North American transmission line failure data. Both the accelerated propagation model and the generalized Poisson distribution are consistent with the data. The generalized Poisson distribution is the distribution of the total number of outages produced by a Galton-Watson branching process with Poisson offspring distribution, whose mean number of offspring is constant except in the first stage.

More generally, there is some initial evidence that branching process models can represent probability distributions of blackout size Observed [6], [12] and simulated [4], [5], [7], [18], [12] blackout statistics show qualitative features such as probability distributions of blackout sizes with power law regions and criticality that are also shown by branching processes [9]. Moreover, branching processes have approximately reproduced the distribution of blackout sizes obtained from simulations of mechanisms of cascading failure in blackouts [14], [16]. Branching processes are used to analyze observed blackout data in [10], [19]. Branching processes also can approximate other high-level models of cascading failure [9], [17].

III. OUTAGE DATA

Transmission line outages are useful diagnostics in monitoring the progress and extent of blackouts. One common feature of large blackouts is the successive failure of transmission lines, and the number of transmission lines outaged

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is a measure of the blackout extent. The number of transmission lines outaged is not a measure directly impacting society as is energy unserved or customers disconnected, but it is a measure of blackout size internal to the power system that is useful to utilities, and for which data is available. The transmission line outages that do not lead to load shed can be regarded as precursor data for the transmission line outages that do lead to load shed and blackout.

Transmission owners in the USA are required to report transmission line outage data to NERC for the Transmission Availability Data System (TADS). The transmission line outage data used in this paper is 8864 outages in TADS data recorded by a North American utility over a period of ten years [3]. The TADS data for each transmission line outage includes the outage time (to the nearest minute) as well as other data. All the line outages are automatic trips. More than 99% of the outages are of lines rated 69 kV or above and more than 96% of the outages are of lines rated 115 kV or above. There are several types of line outages in the data and a variety of reasons for the outages. In processing the data, both voltage levels and all types of line outages are regarded as the same and the reasons for the line outages are neglected. For this initial bulk statistical analysis, neglecting these distinctions is a useful first step as we proceed. It is best to start new methods of analysis in the simplest way first.

IV. Grouping outages into cascades and stages

For our analysis it is necessary to group the line outages first into different cascades, and then into different stages within each cascade. Here we use a simple method based on outages' timing [19], [10]. Since operator actions are usually completed within one hour, we assume that successive outages separated in time by more than one hour belong to different cascades. Since fast transients or auto-recloser actions are completed within one minute, we assume that successive outages in a given cascade separated in time by more than one minute are in different stages within that cascade. The result of this grouping of the outages into cascades and stages is that there are 5227 cascades and the longest cascade has 110 stages. As discussed below, the results of the paper are not sensitive to the details of the time intervals that define the grouping of the outages into stages and cascades.

Table I is obtained by summing over all the 5227 cascades the number of outages in each of stages 0 to 10. That is, of the 8864 outages, 6254 are in stage 0 of a cascade, 1143 are in stage 1 of a cascade, and so on. The number of outages in each of stages 11 to 109 is shown in Fig. 1.

The probability distribution of the number of initial outages is shown by the circles in Fig. 2. The probability distribution of the total number of outages is shown by the squares in Fig. 2. Since there are 5227 cascades, an event that occurs for only one of these cascades has an estimated probability 1/5227=0.0002. Cascades with a specific large number of line outages occur only once and account for the events of probability 0.0002 shown in Fig. 2. The probability estimates for these rarer events are not reliable because of

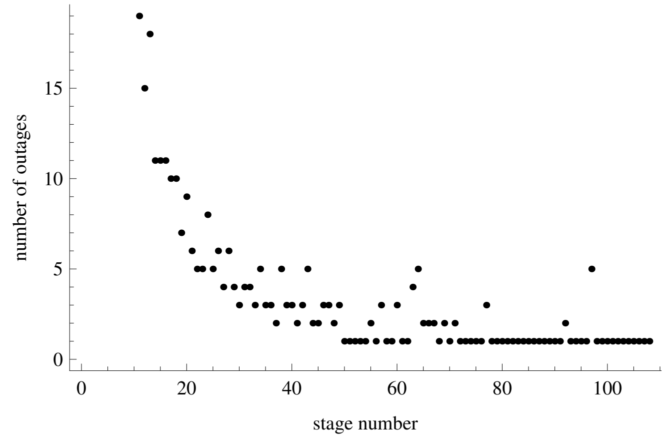

Fig. 1. Sum of line outages in each of stages 11 to 109 of all cascades.

their extremely large variance and it is better to bin the data for the larger number of outages as shown in Fig. 3. Fig. 3 shows that both distributions have a power law character up to the range of statistical validity of the data. The exponent of the power law distribution of the total number of failures is roughly -2.8.

TABLE I
Number of outages in initial stages summed over the cascades

stage number	0	1	2	3	4	5	6	7	8	9	10
number of outages	6254	1143	434	227	155	95	78	53	46	32	31

V. Propagation in the cascades

In our branching process model of cascading, each outage in each stage (a "parent" outage) independently produces a random number 0,1,2,3,... of outages ("child" outages) in the next stage according to an offspring distribution that is a Poisson distribution. The child outages then become parents to produce the next generation and so on. If the number of outages in a stage becomes zero, the cascade stops. The mean number of child outages for each parent (the average family size) is the parameter λ. λ quantifies the average tendency for the cascade to propagate.

There are two main ways to estimate the propagation. The first way is to count all the outages in the cascade that are children and divide this by all the outages that are parents. This gives the propagation λ averaged over the number of stages [19]. The second way is to look at how many children are produced by each parent at each stage. This gives an estimate λ_k for each stage $k = 1, 2, 3, ...$ which is computed by dividing the number of outages in stage k by the number of outages in stage $k-1$. For example, stage 0 has 6254 outages and these parents produced 1143 child outages in stage 1. Therefore the average number of children in stage 1 per parent in stage 0 is $\lambda_1 = 1143/6254 = 0.18$. Stage 1 has 1143 outages and these outages, considered as parents, produced 434 child outages in stage 2. Therefore the average number of children in stage 2 per parent in stage 1 is $\lambda_2 =$

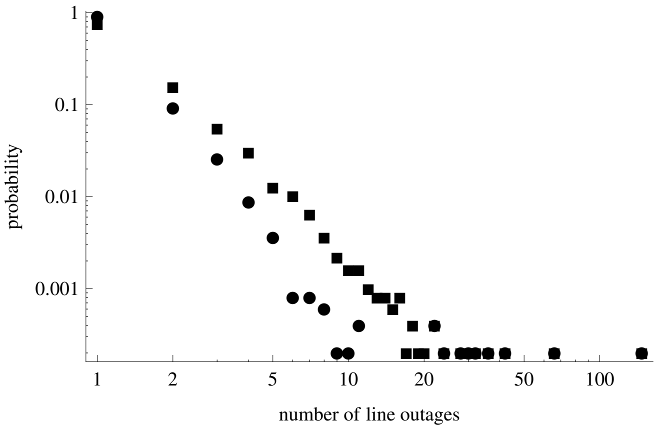

Fig. 2. Probability distribution of initial (circles) and and total (squares) line outages. Raw data shown with no binning.

$434/1143 = 0.38$. The results of computing λ_k for stages $k = 1, 2, 3, ..., 19$ are shown in Fig. 4 and Table II. As the cascade progresses, λ_k increases from 0.18 and appears to level off at approximately 0.75. The higher stages have too few outages to accurately estimate λ_k and the results for higher stages become noisy.

TABLE II
ESTIMATED STAGE PROPAGATIONS λ_k

k	1	2	3	4	5	6	7	8	9	10	11	12
λ_k	0.18	0.38	0.52	0.68	0.61	0.82	0.68	0.87	0.70	0.97	0.61	0.79

If we compute the propagation λ averaged over the number of stages using the method of [19] by dividing the total number of children in all the cascades by the total number of parents in all the cascades, we get $\lambda = 0.29$. This value averaged over the stages is dominated by the early stages that have the majority of the outages. It seems unsatisfactory to be using methods that essentially assume λ to be roughly constant when it is increasing significantly, so in this paper we use a new method that accounts for the increase.

We have tried doubling the time intervals that were assumed to define the stages and cascades and we found that the stage propagation estimates are insensitive to these assumptions. (We are unable to try decreasing the interval of one minute that separates different stages because the line outage timings are specified in minutes.)

VI. Results of predicting total outage distribution with a branching process

We predict the distribution of the total number of outages using a branching process model from the distribution of initial outages and the propagation. The new aspect different than [19] is that we account for the change in propagation as the cascade proceeds. In particular we assume that the stage propagation is given by Table III, which is obtained from the estimated propagations for the first 4 stages in Table III, followed by an assumption of $\lambda_k = 0.75$ for $k \geq 5$. $\lambda_k = 0.75$ is a rough estimate of the asymptotic propagation based on the noisy data in Fig. 4. The branching process is

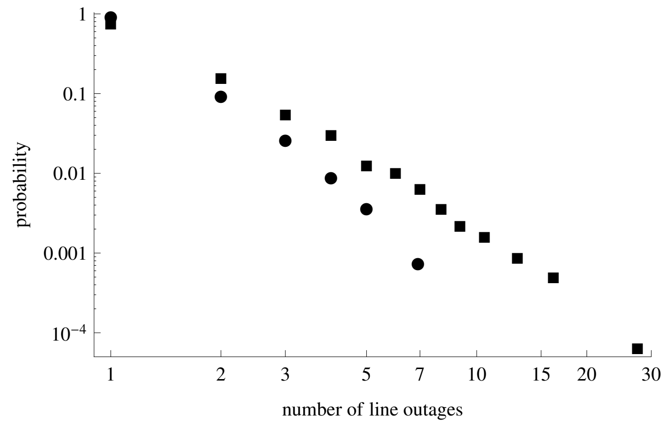

Fig. 3. Probability distribution of initial (circles) and and total (squares) line outages. Data is binned to have at least 10 outages per bin.

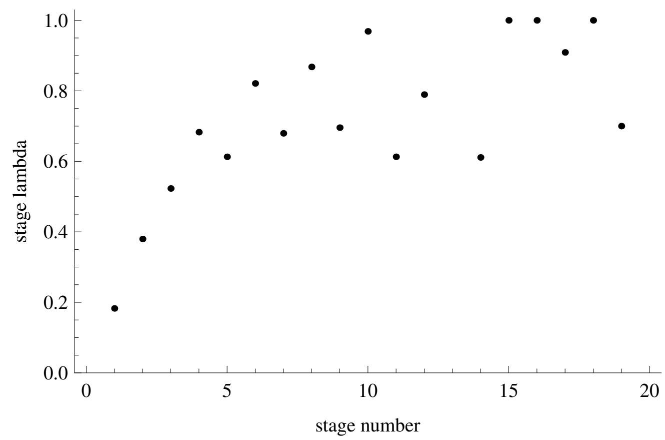

Fig. 4. λ_k estimated from the outage data at stages $k = 1, 2, ..., 19$.

TABLE III
STAGE PROPAGATIONS λ_k FOR PREDICTING TOTAL OUTAGE DISTRIBUTION

k	1	2	3	4	≥ 5
λ_k	0.18	0.38	0.52	0.68	0.75

assumed to have the initial distribution of outages given by the data as shown by the circles in Fig. 2 and propagation at each stage with a Poisson distribution with mean given by the stage propagations in Table III. The general reasons for assuming a Poisson distribution are explained in [19]. The details of this new computation are given in the next section. The predicted distribution of total number of outages is shown by the line in Figure 5 and it can be seen that the match with the empirical distribution of total number of outages is good.

VII. Calculation of total outages using branching process

This section assumes some familiarity with branching processes [15], [2]. Let the generating function of the offspring distribution producing stage k from stage $k-1$ be $f_k(s)$. Then $f_0(s)$ is the generating function of the initial distribution of failures. $f_0(s)$ is computed from the empirical

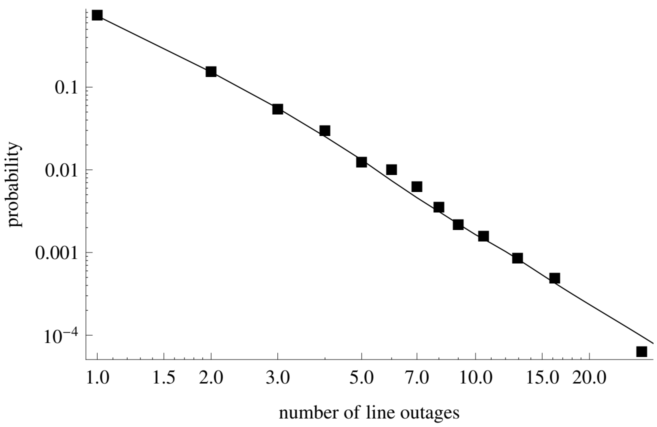

Fig. 5. Distribution of total number of outages from data (squares) and estimated using branching process (line).

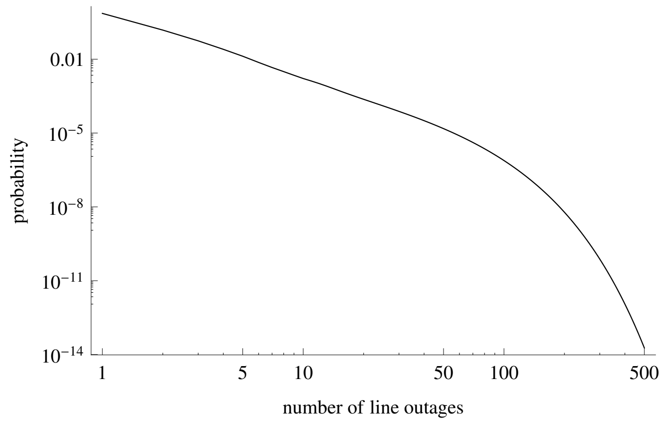

Fig. 6. Distribution of total number of outages predicted using branching process up to 500 outages.

initial distribution of outages. For $k \geq 1$,

$$f_k(s) = e^{\lambda_k(s-1)} \quad (1)$$

is the generating function of the Poisson offspring distribution with mean λ_k.

Consider a single line outage that occurs in stage k and let the total number of outages that are descendants of this outage in any subsequent stage (children plus grandchildren plus great grandchildren and so on) be Y_k. Let the generating function of Y_k be $F_k(s) = Es^{Y_k}$. The number of descendants of the single line outage plus the single line outage itself is $Y_k + 1$ and $Y_k + 1$ has generating function $sF_k(s)$. Then the basic recursion for computing all the descendants of an outage at a given stage is

$$F_{k-1}(s) = f_k(sF_k(s)) \quad (2)$$

Since $\lambda_k = \lambda_{5+}$ for $k \geq 5$, the total number of outages that are descendants of an outage in stage 4 plus the outage itself is given by a Borel distribution with parameter λ_{5+}. We write $f_B(s)$ for the generating function of the Borel distribution with parameter λ_{5+}:

$$f_B(s) = \sum_{r=0}^{\infty} (r\lambda_{5+})^{r-1} \frac{e^{-r\lambda_{5+}}}{r!} s^r \quad (3)$$

We write $F(s)$ for the generating function of the total number of outages. We wish to compute $F(s)$ to obtain the distribution of the total number of outages. Then applying the recursion (2) successively, we get

$$\begin{aligned} sF_4(s) &= f_B(s) \\ F_3(s) &= f_4(sF_4(s)) = f_4(f_B(s)) \\ F_2(s) &= f_3(sf_4(f_B(s))) \\ F_1(s) &= f_2(sf_3(sf_4(f_B(s)))) \\ F_0(s) &= f_1(sf_2(sf_3(sf_4(f_B(s))))) \quad (4) \\ F(s) &= f_0(sf_1(sf_2(sf_3(sf_4(f_B(s)))))) \quad (5) \end{aligned}$$

Equation (5) shows that $F(s)$ is a complicated power series in s, but it can be evaluated by computer algebra [24] for as many terms as needed. For example, computing 500 terms of $F(s)$ predicts the total number of outages as shown in Fig. 6. Note the power law character of the distribution up to about 100 outages.

The recursive structure of formula (5) mirrors the stages of the cascade. One way to show this for part of the formula (5) is by writing in full the formula (4) for $F_0(s)$:

$$F_0(s) = e^{-\lambda_1 + \lambda_1 s e^{-\lambda_2 + \lambda_2 s e^{-\lambda_3 + \lambda_3 s e^{-\lambda_4 + \lambda_4 f_B(s)}}}}$$

VIII. Predicting total outages from initial outages

Once the branching process model with the estimated propagation is validated as accurately enough matching the distribution of the total number of line outages from empirical data, we can use it to predict the distribution of the total number of line outages from other assumptions of initial line outages. A particular number or distribution of initial line outages is assumed to specify f_0 and then (5) is applied with the stage propagation of table III to predict the distribution $F(s)$. For example, if there are 5 initial line outages, then $f_0(s) = s^5$ and the distribution of total number of outages obtained by computing $F(s)$ with (5) is shown in Figure 7. This new capability is significant because traditional risk analysis analytic and observational methods can give good estimates of initial failures, and the method presented here can, based on observed data, quantify the number of additional failures caused by cascading. Note that total number of failures is predicted, but there is no information about which failures occur.

IX. Sensitivity to stage propagation

We can compute the sensitivity of the distribution of the total number of failures to the stage propagation λ_k. To do this, it is convenient to first rewrite (5) using functional composition notation

$$F = f_0 \circ S \circ f_1 \circ S \circ f_2 \circ S \circ f_3 \circ S \circ f_4 \circ f_B \quad (6)$$

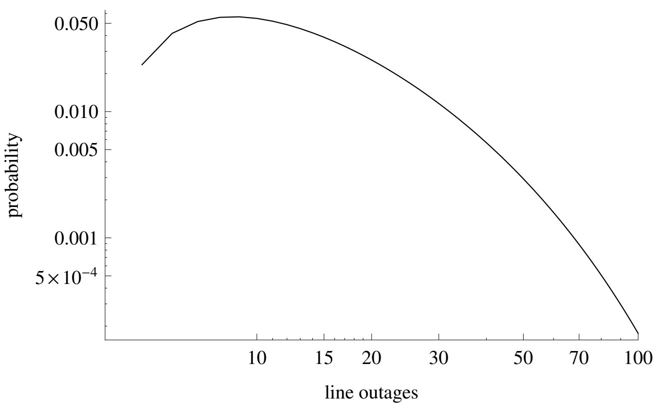

Fig. 7. Probability distribution of total line outages assuming 5 initial line outages predicted by the branching process model with the estimated propagation in Table III.

Here the function S is multiplication by s. Also note that

$$Df_k(s) = \lambda_k f_k(s), \qquad k \geq 1 \tag{7}$$

$$D_{\lambda_k} f_k(s) = (s-1) f_k(s), \qquad k \geq 1 \tag{8}$$

Then (6) can be differentiated. For example, differentiating (6) with respect to λ_3 gives

$$\begin{aligned} D_{\lambda_3}F &= Df_0(sF_0)sDf_1(sF_1)sDf_2(sF_2)sD_{\lambda_3}f_3(sF_3) \\ &= s^3 Df_0(sF_0)Df_1(sF_1)Df_2(sF_2)D_{\lambda_3}f_3(sF_3) \\ &= s^3 Df_0(sF_0)\lambda_1 F_0 \lambda_2 F_1 (sF_3 - 1)f_3(sF_3) \\ &= s^3 \lambda_1 \lambda_2 Df_0(sF_0)F_0F_1F_2(sF_3 - 1) \end{aligned}$$

The results for differentiating with respect to λ_k for $k = 1, 2, 3, 4$ are similarly obtained:

$$\begin{aligned} D_{\lambda_1}F &= sDf_0(sF_0)F_0(sF_1 - 1) \\ D_{\lambda_2}F &= s^2\lambda_1 Df_0(sF_0)F_0F_1(sF_2 - 1) \\ D_{\lambda_3}F &= s^3\lambda_1\lambda_2 Df_0(sF_0)F_0F_1F_2(sF_3 - 1) \\ D_{\lambda_4}F &= s^4\lambda_1\lambda_2\lambda_3 Df_0(sF_0)F_0F_1F_2F_3(sF_4 - 1) \end{aligned} \tag{9}$$

For the data set of the paper, formulas (9) are evaluated in Table IV. Consider DF_{λ_3}. Increasing λ_3 has no effect on the probability of 1 or 2 lines outaged, decreases the probability of 3 lines outaged (since it is then more likely that 3 lines outaged increases to more lines outaged), and increases the probability of 5 or more lines outaged. These general effects are expected, but they are quantified in Table IV. The positive values of DF_{λ_k} for the larger line numbers implies that large cascades can be mitigated by reducing λ_k, but it can be seen that the effectiveness of this mitigation reduces as k increases. Thus, if measures can be taken to reduce propagation λ_k at a single stage k, then it is more effective to do this for the early stages. However, it can be expected that measures to reduce propagation will often affect many stages of propagation, and then the effect on the distribution of the total number of lines outaged can be estimated by suitably combining the DF_{λ_k} with the changes in λ_k. The positive and negative signs in the entries of Table IV suggest that mitigation measures could involve tradeoffs between shorter and longer cascades.

The derivatives DF_{λ_k} will also be useful in estimating how errors in estimating in λ_k affect the distribution of the total number of lines outaged.

X. Conclusions

We analyze ten years of transmission line outage data recorded by a North American utility. The key information used in this first analysis is the timing of each outage, and this is included in the standard Transmission Availability Data System (TADS) data that must be reported to NERC by American transmission owners.

For this North American utility data set we conclude that:

1) The probability distributions of the initial and total number of line outages have an approximate power law character, at least until the number of line outages is large. This conclusion is consistent with the data for the total number of line outages aggregated from North America in [1], [8].
2) Propagation of line outages increases as the cascade progresses and then appears to level out.
3) To mitigate long cascades it appears to be more effective to reduce the amount of propagation at the early stages of cascading.
4) The distribution of the total number of outages predicted with a probabilistic branching process model matches well the empirical distribution of the total number of outages. This validates the branching process model for predicting the distribution of the total number of outages in the sense that it is consistent with this data set. That is, a branching process model that accounts for the varying propagation as the cascade progresses can give a good prediction of the distribution of the total number of line outages from the initial number of line outages.

The significance of the fourth conclusion is that conventional risk analysis can give the distribution of the initial number of line outages. And the propagation can be estimated from standard recorded data. Then the branching process model can be used to estimate the distribution of the total number of line outages. This is a new method to predict the effect of cascading failure acting on known or assumed initial line outages. This new method of cascading failure analysis appears to be practical and the computations are easy to implement with computer algebra. A similar method was developed on a shorter data set that assumed a constant amount of propagation throughout the cascade in [19]. This paper uses a larger and different data set than [19] and discovers and accounts for the variable propagation throughout the cascade.

In this paper we report an initial analysis and there are several directions for improvements:

- The statistical accuracy should be analyzed. Similarly to [19], it is expected that much less than ten years of data is needed for an accurate prediction of the stage propagation and the distribution of the total number of failures, but this has not yet been quantified.

- In this initial analysis, all the line outages are regarded as the same, and only the timing of line outages is considered. Also we statistically analyze the propagation of line outages in all the cascades, regardless of whether load is shed or whether the cascade is short or long. Analyzing the statistics of all the cascades regardless of these distinctions is a good initial assumption because it is simple, but the assumption that all the outages and cascades are sufficiently similar should be examined to the extent feasible.
- There is parallel work applying branching processes to estimate the probability distribution of the number of line outages or load shed from short data sets produced by simulations of cascading failure [14], [16]. Perhaps further developments of these methods could be adapted from simulated data to observed data.
- More data sets should be obtained and analyzed to find out the general characteristics of propagation of cascading line outages.

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TABLE IV

DERIVATIVE DF_{λ_k} OF DISTRIBUTION OF TOTAL NUMBER OF FAILURES WITH RESPECT TO λ_k

	number of lines outaged										
	1	2	3	4	5	6	7	8	9	10	11
DF_{λ_1}	-0.73	0.28	0.16	0.090	0.054	0.036	0.023	0.016	0.012	0.0087	0.0064
DF_{λ_2}	0	-0.091	0.0067	0.016	0.014	0.011	0.0081	0.0061	0.0046	0.0036	0.0029
DF_{λ_3}	0	0	-0.02	-0.0057	0.00027	0.0022	0.0026	0.0026	0.0023	0.0019	0.0017
DF_{λ_4}	0	0	0	-0.0054	-0.0034	-0.0015	-0.00032	0.00028	0.00058	0.00070	0.00072