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## The Impact of Local Power Balance and Link Reliability on Blackout Risk in Heterogeneous Power Transmission Grids

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### Abstract

*Many critical infrastructures such as the power transmission grid are heterogeneous both in their basic structure and in some of their underlying characteristics. This heterogeneity can be good for system robustness if it reduces the spread of failures or bad if it adds risk or vulnerability to the system. In this paper we investigate the effect of heterogeneity in the strength of the links between parts of the system network structures, as well as the balance of local generation and demand, on the robustness of the power transmission grid using the OPA complex system model of the power transmission system. It is found that increasing or decreasing the reliability of the links between parts of the grid changes the likelihood of different size failures with neither being optimal for all sizes. Furthermore, imbalances between load and generation in the local regions further degrades the system reliability.*

### 1. Introduction

Many critical infrastructure networks including the power transmission network exist in a wide variety of configurations with various inhomogeneities, but overall share the characteristics that they are growing and becoming ever more interdependent and therefore more critical. Because of their importance to the functioning of modern society, it is essential that we understand their strengths and weakness so we can understand how to reduce their risks of failure and mitigate their vulnerabilities. Even within the same interconnection, different regions can have different characteristics and therefore it is important to understand how these regional characteristics and the

connections between the regions can affect the risk to the global network. Among the characteristics that may vary from region to region are the balance of sources and sinks (load and generation for the power grid) and heterogeneity of the “strength” of the links between regions of the network.

Since this work explores the network robustness as characterized by the long-term risk of large failures and temporal dynamics, we use the OPA (ORNL-PSerc-Alaska) model. The OPA model [1,2] was developed to study the long-term patterns of blackout of a power transmission system under the dynamics of an increasing power demand and the engineering responses to failure. In this model, the power demand is increased at a constant rate and is also modulated by random fluctuations. The generation capacity is automatically increased when the capacity margin is below a given critical level. The model is described more in the next section.

Using the OPA model we have been able to study and characterize the mechanisms behind the power tails in the distribution of the blackout size [1,2,3]. These algebraic tails obtained in the numerical calculations are consistent with those observed in the study of the blackouts for real power systems [4,5,6,7]. Most importantly, this model permits us to separate the underlying causes for cascading blackouts from the triggers that initiate them and therefore explore system characteristics that enhance or degrade resilience and reliability of the power transmission grid. One of these characteristics, the one investigated here, is the heterogeneity of the network.

Many real networks have an inhomogeneous structure with a series of relatively homogeneous regions more loosely coupled to each other like pearls on a string. The effect of this structure and size has previously been investigated with OPA [8]. However there are many other, perhaps more important, inhomogeneities in these networks. For example, the connections between the “pearls” will often have a different “strength” than the connections within the

“pearls”. Or, the sources and sinks may not be balanced within a region (“pearl”) leading to an underlying inhomogeneity in the flow of power. To understand the impact of this type of inhomogeneity, we investigate these types of configurations within a “pearls on a string” type of network structure using OPA. To do this we vary the inhomogeneities to determine the impact on the risk of failures of various sizes in networks with various number of regions and with various region sizes.

To evaluate the impact of these heterogeneities there must be a clear separation of scales (sizes) between the parts of the networks and the global size. This makes the problem very computationally challenging because of the long computation times to compute the complex system steady state statistics and dynamics of large networks.

The need for analysis and understanding of cascading in interconnected or coupled networks representing parts of the same or different infrastructures was recognized and described qualitatively in [9,10] in 2003 and 2004. The effect of the coupling in such systems has been studied with several different types of models, many of which were pioneered at Hawaii International Conference on System Sciences (HICSS). In 2005, [11] gave the first analysis with coupled probabilistic cascading interconnected networks and with coupled self-organizing complex system interconnected networks. The analysis and simulation showed how the coupling could affect the critical point behavior. In 2007, [12] modeled cascading failure based on forest fire type models to study coupled networks, and in particular how the coupling affected the critical point and the power law behavior of the coupled system. In 2014, [13] studied a tradeoff in blackout reliability that is related to network size.

In this paper, we describe the initial investigations and the impact on the long-term reliability of the system from the changes in system structure introduced in this model. Section 2 will briefly review the OPA model while section 3 will use OPA to analyze the impact of varying the reliability of the links connecting the regions. Section 4 investigates the propagation of the failures through the regions. Section 5 investigates the impact of load-generation imbalance within the regions while section 6 briefly discusses the case in which the power balance is added to the link reliability. Finally section 7 is a brief discussion and conclusion.

## 2. The OPA model and the networks used

The OPA model for the dynamics of blackouts in power transmission systems [1, 2] shows how the slow opposing forces of load growth and network upgrades in response to blackouts could self organize the power system to dynamic equilibrium. Blackouts are

modeled by overloads and outages of lines determined in the context of Linear Programming (LP) dispatch of a DC load flow model. This model has been found to show complex dynamical behavior [1,2,3] consistent with that found in North American Electric Reliability Corporation (NERC) data [5]. Some of this behavior has the characteristic properties of a system near a critical transition point. That is, when the system is close to a critical point, the probability distribution function (PDF) of the blackout size (load shed, customers unserved, etc) has an algebraic tail and large temporal correlation lengths are possible. One consequence of this behavior is that at these critical points, both the system utilization is maximized and the risk for blackouts starts to increase sharply. Therefore, it may be natural for power transmission systems to operate close to this operating point.

In general, the operation of power transmission systems results from a complex dynamical process in which a variety of opposing forces regulate both the maximum capacity of the system components and the loadings at which they operate. These forces interact in a highly nonlinear manner and may cause a self-organization process to be ultimately responsible for the regulation of the system. This view of a power transmission system considers not only the engineering and physical aspects of the power system, but also the engineering, economic, regulatory and political responses to blackouts and increases in load power demand. A detailed, comprehensive inclusion of all these aspects of the dynamics into a single model would be extremely complicated if not intractable due to the intrinsic human interactions involved. However, it is useful to consider simplified models with some approximate overall representation of the opposing forces in order to gain some understanding of the complex dynamics in such a framework and the consequences for power system planning and operation. This is the basis for OPA.

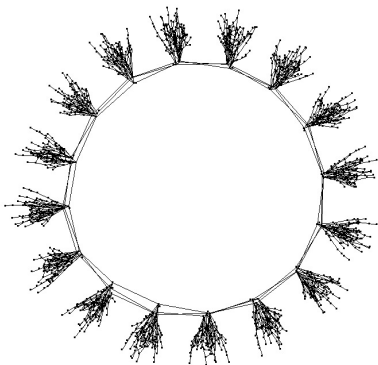
In the OPA model the dynamics involves two intrinsic time scales. There is a slow time scale, of the order of days to years, over which load power demand slowly increases and the network is upgraded in engineering responses to blackouts. These slow opposing forces of load increase and network upgrade self organize the system to a dynamic equilibrium. There is also a fast time scale, of the order of minutes to hours, over which cascading overloads or outages may lead to blackout. On the fast time scale we start from a solved base case and blackouts are initiated by random line outages with a probability  $p_0$ . Whenever a line is outaged, the generation and load is re-dispatched using standard linear programming methods. This is because there is more generation power than the load requires and one must choose how to select and optimize the generation that is used to exactly balance the load. The cost function is weighted to ensure that load shedding is avoided where possible. If any lines

were overloaded during the optimization, then these lines are outaged with probability  $p_l$ . The process of re-dispatch and testing for outages is iterated until there are no more outages. The total load shed is, then, the power lost in the blackout. The  $p_l$  parameter is the parameter that controls the reliability or strength of a line and will be one of the important control parameters for our study.

The OPA model computes the long-term reliability taking into account the complex systems dynamics and feedbacks; that is, OPA is run until it converges to a complex systems steady state with stationary statistics and long time correlations. Because of the time correlations intrinsic to such a system, these simulations are different from the more common Monte Carlo method for generating statistics. In the case of OPA, we generally run the simulation for longer times to generate better statistics, thereby sampling more of the allowed system states with the probabilities of sampling a given state being generated by the system itself.

The main purpose of the OPA model is to study the complex behavior of the dynamics and statistics of series of blackouts in various scenarios. This allows us to easily investigate the impact of different levels of inhomogeneity on the risk and dynamics as well as other network characteristics. Despite its simplifications, OPA has been validated against real data[6] making it ideal for this type of study. For the rest of the paper, OPA results are used for the computational analysis.

Let us consider a series of networks build by linking several small networks. An example a network made by linking 16 100-node networks is shown in Fig. 1. Each of the smaller subnetworks, which we will refer to as zones or regions, is connected to each of its neighbors with three lines. We refer to this system as 16x100.



**Fig. 1. 16 linked 100-node networks.**

Each of the networks is an artificial power network with realistic parameters constructed by following the algorithms of [14,15]. It is worth noting that Fig. 1 should not be taken as a geographical representation and the length of the lines connecting the zones (“pearls”) is really a normal length line whose characteristics we will change in the next section to investigate the effect of heterogeneity.

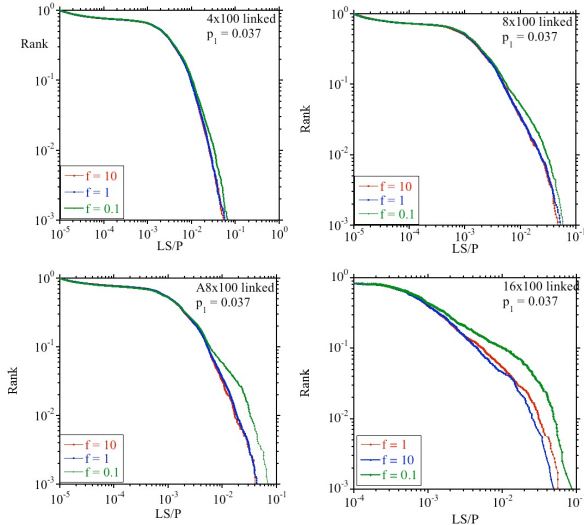
### 3. Reliability of links

There are various schemes for joining the regions shown in Fig. 1, two of which we will call the intelligently built network and the simply built network. The difference between them is the mean connection distance between nodes. In the intelligently built network the connection lines are chosen to minimize the connection distance in the overall system. This is a smaller world system than the simply built network in which random edge nodes are connected. For this work, either minimizing the number of hops or the electrical resistance gave the same results.

Here we consider cases (in both the simple and intelligent types of networks) in which the probability of a line outage in case of an overload of the linking lines,  $p_2$ , is different from the probability  $p_l$  for the lines in the networks. We use the following notation:

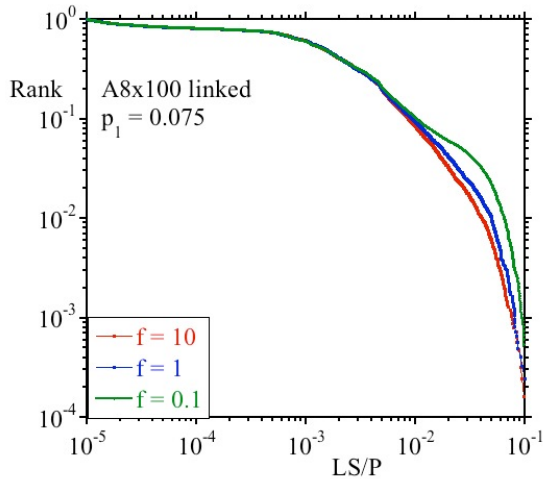
$$p_2 = \frac{p_l}{f} \quad (1)$$

For a given case the same  $f$  is applied to all the linking lines. For  $f > 1$  the linking lines are more reliable than the standard lines since the probability of failure when overloaded is lower; conversely, for  $f < 1$  the lines are less reliable as the probability of failure is higher. There is a fairly systematic behavior of the rank function of the normalized load shed (LS/P) of the blackouts: when  $f > 1$  and the links are more reliable, the rank function, which is 1-CDF (Cumulative Distribution Function), is very close to the case with  $f = 1$ . When  $f < 1$ , less reliable links, the size of the blackouts are larger. This can be seen in Fig. 2. The effect seems to increase with the number of component networks. Note that more intelligently built network, case A8x100, shows a more pronounced effect than the simply built network, case 8x100 perhaps due to the smaller world nature of the system giving higher importance to each of the connecting lines.



**Fig. 2. The rank function (1-CDF) for four different linked systems, each with normal linking line failure probability ( $f = 1$ ), higher failure probability ( $f = 0.1$ ), and lower failure probability ( $f = 10$ ). The system A8x100 is an “intelligently” linked system.**

This effect is not specific to the base value of  $p_1$  chosen; for other values of  $p_1$ , similar behavior is seen as shown in Fig. 3.



**Fig. 3. The rank function for an A8x100 network, an “intelligently” linked system, with a higher base probability ( $p_1 = 0.075$ ) of failing when overloaded.**

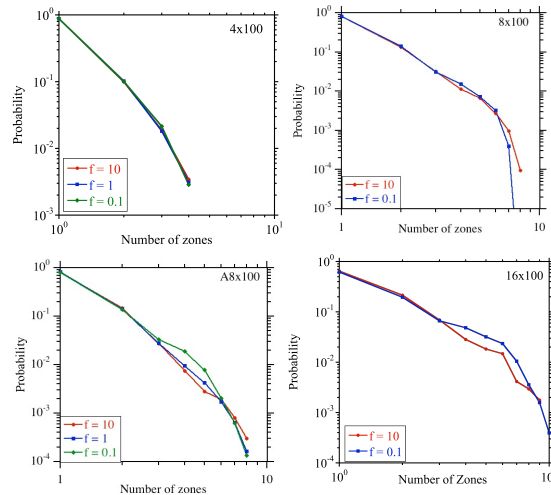
These results were somewhat surprising because we expected that in both cases, increasing and reducing reliability of the connecting lines could have a negative

effect on the overall system performance. The actual dynamics are more complex, and details that are hard to see in the rank function plots are important. We know that increased reliability helps in allowing transfer of power among the components of the network, but it also allows the propagation of large cascades. Decreasing the reliability of the connecting lines can have the opposite effects. The issue is how these different effects contribute to the overall performance.

One more detailed way of looking at this is by separating the blackouts in zones, the smaller component networks, and looking at how many zones are affected by the blackouts. Results are plotted in Fig. 4.

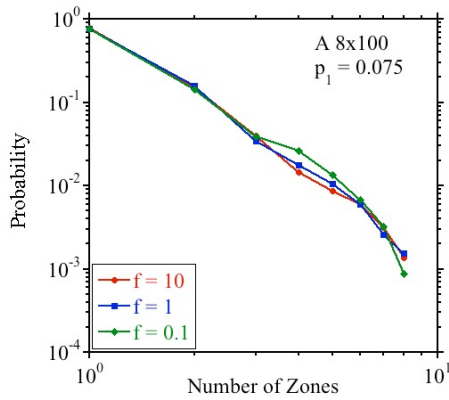
From these graphs we see:

- 1) When less than 4 zones are involved, the change in reliability has little effect
- 2) When 5 or 6 zones are involved, high reliability seems better than low reliability
- 3) When more than 6 zones are involved, low reliability seems better than high reliability.



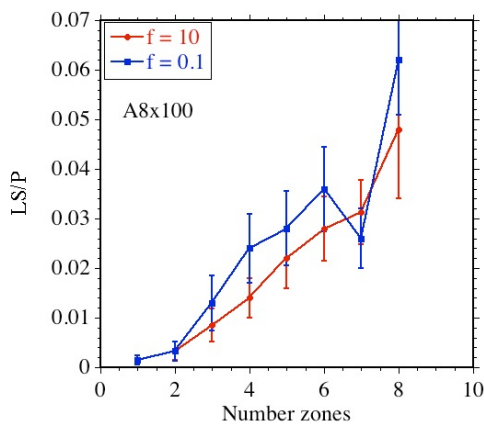
**Fig. 4. The probability of a blackout occurring in a given number of zones for the four  $p_1 = 0.037$  cases.**

This suggests that the increased propagation across zones in the more reliably linked networks propagation is detrimental for large failures while the less reliably linked networks are more susceptible to mid range number of failures. At the small number of zone end the high reliability networks once again seem worse which could be due to the fact that the higher reliability networks might be letting the individual zones get closer to their critical point. When  $p_1$  is changed once again we obtain similar a result, see Fig. 5.



**Fig. 5. The probability of a blackout occurring in a given number of zones for the A8x100 network, an “intelligently” linked system, with a higher base probability ( $p_1 = 0.075$ ) of failing when overloaded.**

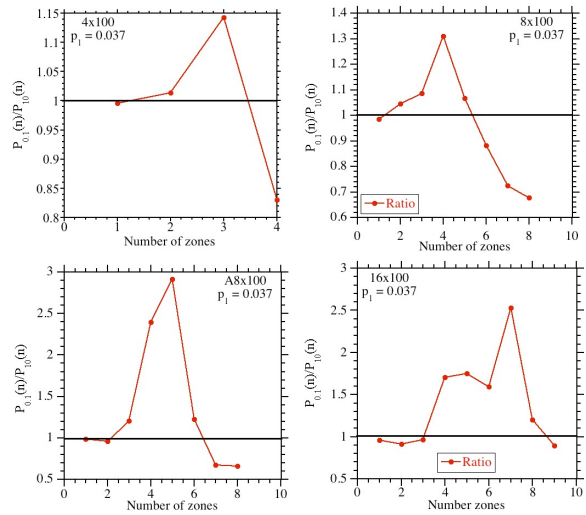
Note that the largest load shed blackouts do not always correspond to the blackouts involving all zones. If we look at the load shed as a function of the number of zones included in a blackout, we see that for 3 to 6 zones the blackout are systematically larger for  $f = 0.1$ , however, that is not the case for higher number of zones. For 8 zones, it is difficult to say anything because the number of events is too small. These results are shown in Fig. 6.



**Fig. 6. The normalized load shed in a blackout affecting a given number of zones for an “intelligently” linked system.**

An easier way to visualize the impact of the reliability is to look at the ratio of the probability of

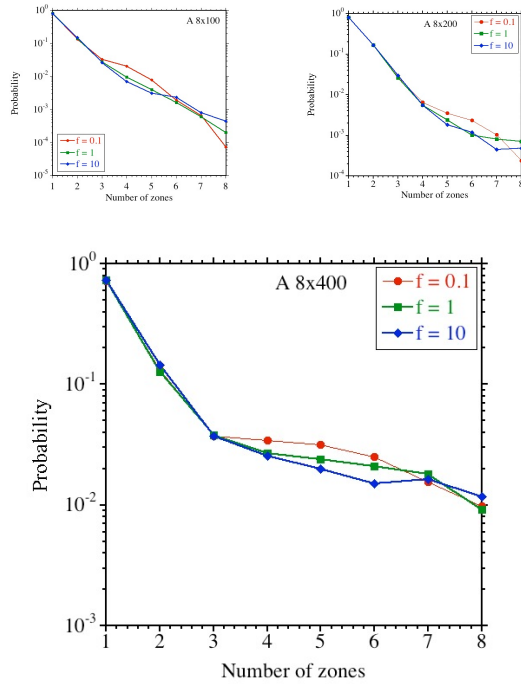
spreading to the various number of regions for the low reliability case to the high reliability case (Fig. 7).



**Fig. 7. The ratio of probability of affecting a given number of zones for a low reliability to high reliability linked systems for the 4 cases.**

It can once again be seen that the probability of spreading to an intermediate number of zones is higher in the low link reliability ( $f = 0.1$ ) cases, but for the highest and lowest number of zones risk probability is highest in the high link reliability ( $f = 10$ ) cases.

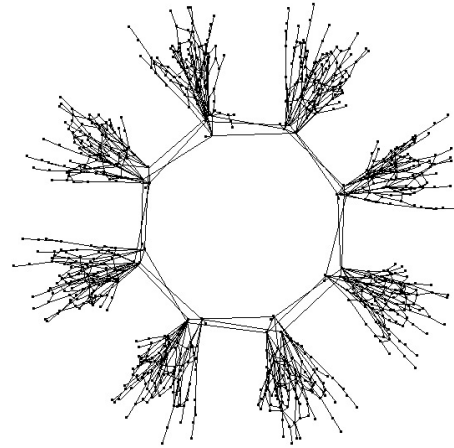
The effect of a higher risk of larger and smaller number of zones impacted by a blackout with the higher reliability links is independent of the size of the zones as seen in Fig. 8. In fact, the enhancement of the risk in the intermediate region (between 4 and 6 zones) is clearest in the 8x400 node cases. However somewhat counterintuitively, as the size of the individual zones gets bigger, the probability of more zones being involved increases. This can be seen in the progressive flattening at the larger number of zones of the probability curves for all the cases as the zone size increases. A possible explanation for this is that as the zones get larger a major failure in one zone is a bigger perturbation to the system and is therefore more likely to spread, and once it spreads it again is a larger perturbation so the spreading is more likely to continue.



**Fig. 8. The probability of a blackout occurring in a given number of zones for 3 “intelligently” linked systems, all with 8 zones of sizes 100, 200 and 400. The effect is seen in all but there is a noticeable flattening of the probability in the larger zone systems**

#### 4. Propagation of failure

To further investigate the dynamics of the blackouts in the heterogeneous networks, we can look at the propagation of the cascades in a failure. Instead of looking at the fine structured detail of the cascades, we will take a coarser view and look at the cascade across the various regions since that is an important characteristic of large failures. For these cases we use an intelligent linked network again with 8 identical 100 node networks. As before this is called the A8x100 network and again the linking between the regions is done with three lines as shown in Fig. 9. The linking busses may as before have different reliability than the busses in the 8 zones. For these cases we use the parameters:  $p_0 = 0.00025$  and  $p_1 = 0.037$ . We have done three sets with the linking busses having the  $p_2$  values of 0.37 (low reliability links), 0.037 (normal), and 0.0037 (high reliability links) respectively.



**Fig. 9. The A8x100 network.**

First we explore the high reliability set. To look at the propagation, we choose two representative specific times with cascades that have the maximum number of zones involved in the blackout.

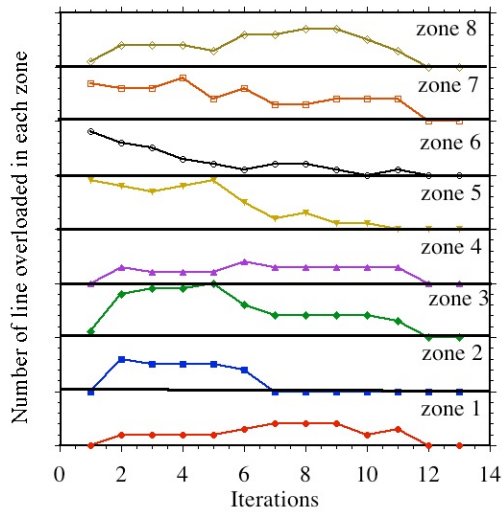
*Case I:  $p_2 = 0.0037$  for linking lines at time 33444 in the evolution*

In this case, all 8 zones of the network are involved. The load shed normalized to the power demand in each of the 8 zones is higher than 0.00001, our criterion for declaring a blackout. This is shown in Table I.

Table I. Load shed normalized to the power demand in each of the 8 zones

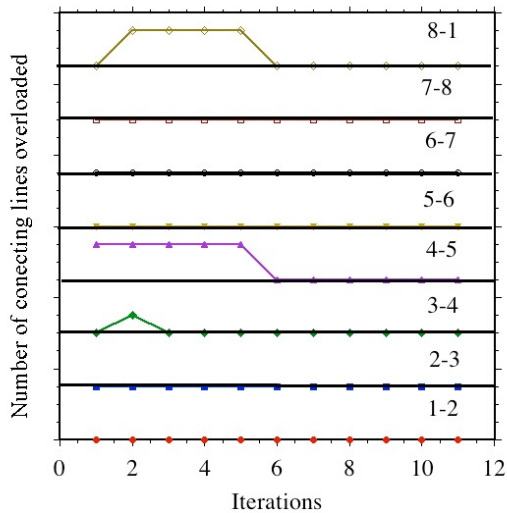
Zone	Zone	Zone	Zone	Zone	Zone	Zone	Zone
1	2	3	4	5	6	7	8
0.0243	0.00523	0.00712	0.00437	0.232	0.129	0.0475	0.0694

This blackout goes through 11 iterations and one way to see the propagation is by looking at the number of overloaded lines in each zone at each iteration. Fig. 10 shows these results. We can see that the initiation is in zones 5, 6 and 7. Note that in this blackout there were 4 simultaneous failures due to  $p_0$  as initiation event. Then, we can see that it very rapidly propagates across all other zones. As can be seen in table I, there was the largest load-shed in the initiating zones 5 and 6, but there was also a significant load shed in the other zones where the blackout propagated, such as zones 1 and 8.



**Fig. 10. Number of overloaded lines vs iteration for each zone during a blackout showing the propagation across the zones.**

If we look now at the linking lines, the main overloads are on the lines linking zones 4 to 5, and 8 to 1. These are the links between the region where the failure was initiated and the rest of the network. This is shown in Fig. 11.



**Fig. 11. Number of connecting lines overloaded vs iteration for each zone during a blackout showing the propagation across the zones.**

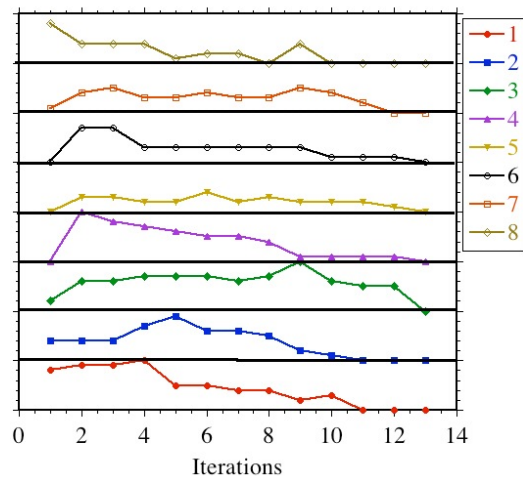
*Case II:  $p_2 = 0.0037$  for linking lines at time 81395 in the evolution*

This case is interesting because even though the initiating events were only in two of the zones, zone 1 and zone 8, it still propagated to the other six zones of the network. The load sheds for each of the zones are shown in Table II.

Table II. Load shed normalized to the power demand in each of the 8 zones

Zone	Zone	Zone	Zone	Zone	Zone	Zone	Zone
1	2	3	4	5	6	7	8
0.1165	0.0801	0.1343	0.0286	0.0506	0.0259	0.0941	0.111

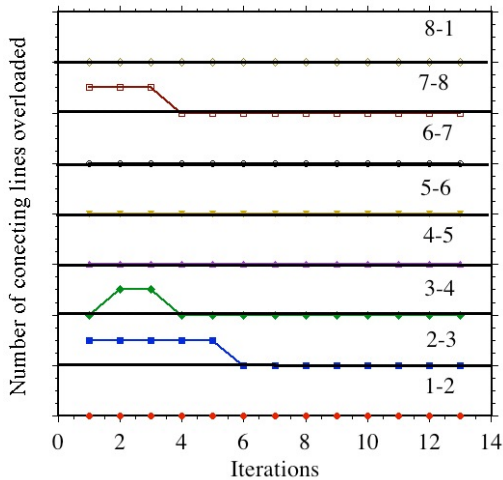
The propagation is again seen through the change of the number of overloaded lines in each zone as shown in Fig. 12.



**Fig. 12. Number of overloaded lines vs iteration for each zone during a blackout showing the propagation across the zones.**

We can see that the event initiates in zones 1 and 8 with some impact already apparent in the first iteration in zone 7. This then propagates through the system with the largest load shed in this case being in zone 3, where the blackout lasted longer. All zones were seriously affected in this case also. Lines linking zones 2 and 3 and zones 7 and 8 were the most affected by this blackout. Again these are the main lines

connecting the initiating region to the rest. This is shown in Fig. 13.



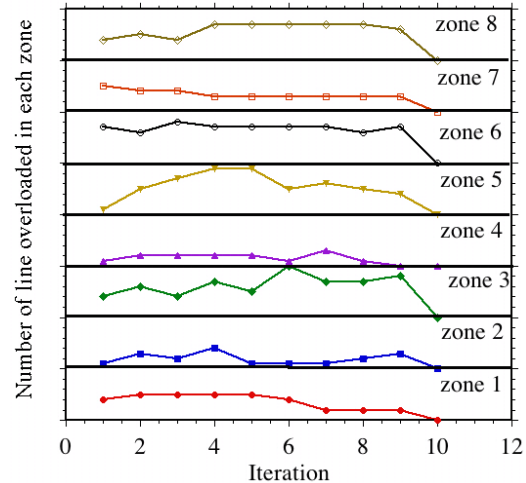
**Fig. 13. Number of connecting lines overloaded vs iteration for each zone.**

Next we briefly look at the low reliability set. In this case we will look at one specific cascade, again with the maximum number of zones involved in the blackout. Because of this it looks very similar to the high reliability cases and is included mainly for completeness.

*Case I:  $p_2 = 0.37$  for linking lines at time 41260 in the evolution*

This case has initiating events in two of the zones, zone 6 and zone 7, and it too propagated to the other six zones of the network in 10 iterations.

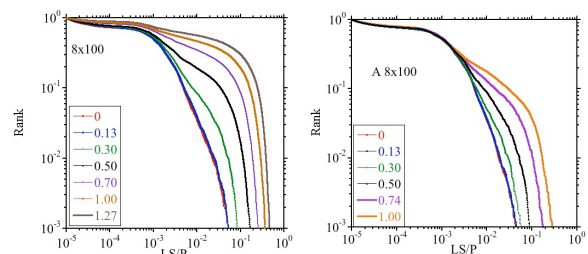
The propagation is seen through the change of the number of overloaded lines in each zone as shown in Fig. 14. One interesting point to note in this cascade is that the maximum impact was in zone 3 but much later in the cascade at iteration 6 and after having peaked in zone 5 earlier. It is also worth noting that because of the nonlocal nature of the electrical power transmission, the propagation does not need to be through nearest neighbors or even through neighboring zones.



**Fig. 14. Number of overloaded lines vs iteration for each zone during a blackout showing the propagation across the zones.**

## 5. Load - Generation balance

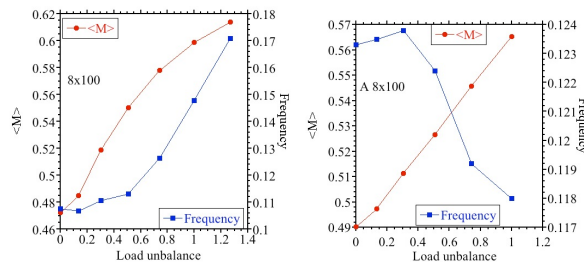
We next consider the inhomogeneity of an imbalance of power within the zones. In these investigations we will again use the 8x100 networks. We will use both the simple 8x100 and the intelligent A8x100 linked network. The imbalance initially investigated is an excess load (demand) in zone 4 and an excess generation (supply) in zone 6. Figure 15 shows the rank functions for the two networks (simple on the left and intelligent on the right) as the imbalance is varied. The simple network loses the self-similar nature of the rank function for values of imbalance of  $\sim 0.5$  as the large events grow rapidly. Impact on the intelligently linked network is also significant but the self-similarity remains although the tail gets much heavier.



**Fig. 15. Rank functions for the 8x100 and A8x100 networks as the power imbalance in the networks is varied.**

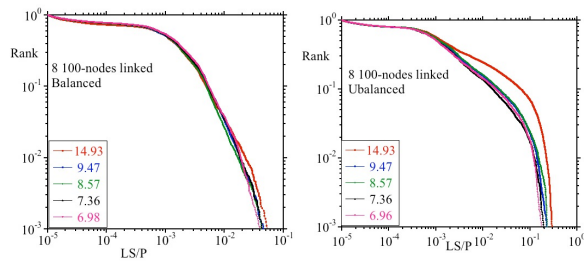


In Fig. 16 we see the statistical quantities,  $\langle M \rangle$  and the frequency of blackouts.  $\langle M \rangle$  is the average fractional loading of the line, where 1 would be lines at their limit and 0 would be a line with no load. For both types of network there is a significant change in both measures. However the changes are very different for the two networks. For the simple network, the frequency increases by more than 60% and  $\langle M \rangle$  increases by  $\sim 30\%$ . This, combined with the rank functions suggests that for the largest imbalance the simple network is continuously collapsing. In contrast, the intelligently linked network has a small decrease in frequency and a more modest increase in  $\langle M \rangle$ . This, combined with the rank functions suggests that with the imbalance, the intelligently linked network is negatively impacted. However, the intelligently linked network is much more resilient to the imbalance.



**Fig. 16. Frequency and average line loading as a function of the power imbalance for the two networks showing large changes but differing for the two networks.**

There are different ways of doing intelligent linking, all of which reduce the averaged distance between nodes. Figure 17 shows that the different degrees of intelligent linking do not affect the rank function of the balanced case; however it is important in the unbalanced cases.



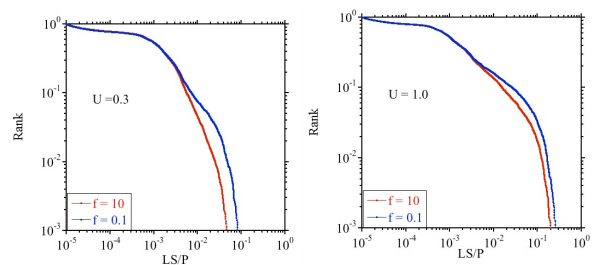
**Fig. 17. Rank functions for balanced and unbalanced cases as the average nodal distance is changed. All the intelligently links cases look the same but in the unbalanced case the intelligent linking makes a significant difference to the tail.**

For all the cases presented, the two imbalanced zones were zones 4 and 6. A reasonable question asks

what would be the effect of changing the separation between the imbalanced zones. Figure 18 shows that for the rank function, the effect of changing the separation is small. L4 G4 is the balanced case, L4 G6 is the case we have been looking at, L4 G5 are closer together and L4 G8 is the maximum separation. We see that the rank function does not change much with the separation between the enhanced load and the enhanced generation though they all are significantly changed from the balanced cases.

## 6. Connection reliability and generation imbalance

Preliminary work combining the reliability of the links and the load-generation imbalance shows that as the unbalance increases the effect of the reliability of the zones connecting lines is less important (Fig. 19). This is likely because the impact of the load imbalance dominates.



**Fig. 19. Rank functions for the unbalanced and inhomogeneous link reliability cases. With increased imbalance (right panel) the effect of the reliability is less visible.**

## 6. Conclusion

Because of the efficiency gains of having larger systems it is natural for multiple smaller regions to be connected together. Though this gives the benefit of being able to share power across the boundaries, it also allows problems (blackouts) to be shared (to propagate) across these boundaries. The way these smaller regions are connected can incorporate heterogeneities into the system which can impact the dynamics of the failures. Using a complex systems power transmission grid model (OPA) to investigate the impact of grid inhomogeneity on the system, our analysis suggests that adding heterogeneity in the form of changed reliability of the links between zones can have a significant impact on the risk of failures of different sizes. Interestingly, increasing the reliability of the links decreases the risk of mid range failures but increases the risk of large (and small) failures. Conversely, decreasing the reliability of the linking

lines decreases the risk of large failures but increases the risk of mid size failures. A likely mechanism for this is that the failure size and the reliability of the links both affect the propagation of the failures between the different zones. This overall effect is found to be insensitive to size of the individual parts, but the probability of propagation through the system does increase as the individual zones get larger. It should also be noted that the rules for power dispatch are system-wide in our present implementation and if dispatch were weighted toward the local zone, that too could have an impact on the results. It should be mentioned that the apparent heterogeneity in Fig. 1 is just an artifact of how the system is illustrated. If the linking lines are constructed to minimize distance, the overall system is very close to a homogeneous system of the same size. It is not until the linking lines are changed (through length or reliability changes) that we start to see a real impact of heterogeneity.

Additional inhomogeneities in the form of imbalances in the power in the different zones in the system significantly added to the risk in the system but the construction of the system greatly changed that impact. The power balance issues include a real load/generation imbalance causing systematic long-distance power transfers such that one region (zone) is not locally in balance and imports power from other regions in the base case operation. This has a large impact on the risk of large failures.

Because the impacts are varied, this suggests that the natural islanding between zone that the linking lines may cause do not on their own improve the reliability of the overall system. Therefore a natural next step will be to investigate policies or smart agents controlling the islanding schemes to see if we can get the best of both worlds by cutting the zones apart when appropriate to halt the spread of the failures.

It is clear that inhomogeneities can have a number of impacts on the system robustness and reliability coming from both the inhomogeneity in the network structure and in the power flows across the inhomogeneous network. These seem to work together to move the system closer to or perhaps even past its critical point leading to a heavy tail or even a bump on the tail blackout (failure) size distribution with the increased large blackout risk inherent in those distributions.

While the work presented here is specifically for a single type of infrastructure (the power transmission grid as modeled by OPA) that has an inhomogeneous structure, similar results likely would hold for coupled infrastructures where the inhomogeneity arises from the coupling.

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