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Quantifying transmission reliability margin

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Abstract

In bulk electric power transfer capability computations, the transmission reliability margin accounts for uncertainties related to the transmission system conditions, contingencies, and parameter values. We propose a formula which quantifies transmission reliability margin based on transfer capability sensitivities and a probabilistic characterization of the various uncertainties. The formula is verified by comparison with results from two systems small enough to permit accurate Monte Carlo simulations. The formula contributes to more accurate and defensible transfer capability calculations.

Keywords: Transfer capability; Uncertainty; Power transmission reliability

1. Introduction

Bulk power transfer capability computations have many uses in electric power system operation and planning. In the operation of bilateral markets, available transfer capability is used to allocate reservations of transmission rights [1–3]. In the operation of pooled markets, transfer capability combined with bid information can be used to help allocate financial transmission rights or transmission congestion contracts. In both planning and operations, transmission capability can be used to assess power system security when local power sources are replaced by imported power. Finally, transfer capability can be used to provide capacity data for simplified power system models suitable for locational price forecasting. All of these applications are reviewed in Ref. [4].

In many of these applications, it is desirable to quantify the uncertainty in the transfer capability computation as a safety margin so that if the computed transfer capability minus the safety margin is used, it is likely that the power system will remain secure despite the uncertainty. The transmission reliability margin (TRM) accounts for the uncertainties associated with the transmission system. Deregulation of power systems has increased the need for defensible calculations of transfer capability and related quantities such as the TRM.

This paper describes a straightforward method to estimate the TRM. The method exploits formulas for first order sensitivity of transfer capability [5–7]. These formulas can be quickly and easily computed when transfer capability is determined. The formulas determine a linear model for changes in transfer capability in terms of changes in any of the power system parameters. This paper supposes that the uncertainty of the parameters can be estimated or measured and shows how to estimate the corresponding uncertainty in the transfer capability. A formula for TRM is then developed based on the uncertainty in the transfer capability and the desired or agreed upon degree of safety.

2. Transfer capability and TRM

We summarize a generic transfer capability calculation [5,4] and discuss TRM.

The time horizon of the transfer capability calculation is established and a secure base case is chosen. A base case transfer including existing transmission commitments is

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chosen. Then a transfer limited case is determined. One method to determine the transfer limited case gradually increases the transfer starting at the base case until the first security violation is encountered. The real power transfer at the first security violation is the transfer capability. The calculation may be repeated for a short list of contingencies and the minimum of these transfer capabilities is used.

In our framework [5], the following limits are accounted for in the transfer capability computation:

- power flow or current limits (normal and emergency)
- voltage magnitude upper and lower limits (normal and emergency)
- voltage collapse limit

Our framework accounts directly only for limits which can be deduced from static model equations. Although oscillation and transient stability limits can be studied offline and approximated by surrogate power flow limits, the sensitivities of the surrogate power flow limits will not be the same as the sensitivities of the oscillation and transient stability limits. Thus, our methods do not extend to the estimation of uncertainties associated with oscillation and transient stability limits.

According to the North American Electric Reliability Council [2], 'The determination of ATC must accommodate reasonable uncertainties in system conditions and provide operating flexibility to ensure the secure operation of the interconnected network'. There are two margins defined to allow for this uncertainty: The TRM is defined in Ref. [2] as 'that amount of transmission capability necessary to ensure that the interconnected transmission network is secure under a reasonable range of uncertainties in system conditions'. The capacity benefit margin ensures access to generation from interconnected systems to meet generation requirements. The capacity benefit margin is calculated separately from the TRM.

Since uncertainty increases as conditions are predicted further into the future, the TRM will generally increase when it is calculated for times further into the future.

3. Quantifying TRM

3.1. Parameters and their uncertainty

The transfer capability is a function A of many parameters $p_1,p_2,\ldots,p_m$:

$$\text{transfer capability} = A(p_1, p_2, \ldots, p_m) \tag{1}$$

Uncertainty in the parameters p_i causes uncertainty in the transfer capability and it is assumed that this uncertainty in the transfer capability is the uncertainty to be quantified in the TRM. The uncertain parameters p_i can include factors such as generation dispatch, customer demand, system parameters and system topology. The parameters are assumed to satisfy the following conditions:

(1) Each parameter p_i is a random variable with known mean $\mu(p_i)$ and known variance $\sigma^2(p_i)$. These statistics are obtained from the historical record, statistical analysis and engineering judgment.
(2) The parameters are statistically independent. This assumption is a constraint that can be met in practice by careful selection of the parameters [8]. For example, if loads with the same weather have significant temperature dependence, then the temperature should be chosen as a random parameter and the loads should be computed as function of temperature.

3.2. Transfer capability sensitivity

We assume that the nominal transfer capacity has been calculated when all the parameters are at their mean values The uncertainty U in the transfer capability due to the uncertainty in all the parameters is:

$$U = A(p_1, p_2, \ldots, p_m) - A(\mu(p_1), \mu(p_2), \ldots, \mu(p_m)) \tag{2}$$

The mean value of the uncertainty is zero:

$$\mu(U) = 0 \tag{3}$$

Approximating the changes in transfer capability linearly in Eq. (2) gives:

$$U = \sum_{i=1}^{m} \frac{\partial A}{\partial p_i}(p_i - \mu(p_i)) \tag{4}$$

$(\partial A/\partial p_i)$ is the small signal sensitivity of the transfer capability to the parameter p_i evaluated at the nominal transfer capability.

When the transfer capability is limited by voltage magnitude or thermal limits, the sensitivity of the transfer capability to parameters can be computed using the formulas of Refs. [5,6].

When the transfer capability is limited by voltage collapse, the sensitivity of the transfer capability to parameters can be computed using the formulas of Ref. [9]. (Topology changes can also be accommodated with limited accuracy using the fast contingency ranking techniques in Ref. [10].)

In each case a static, nonlinear power system model is used to evaluate the sensitivities. The computation of $(\partial A/\partial p_i)$ is very fast and the additional computational effort to compute $(\partial A/\partial p_i)$ for many parameters p_i is very small [5,6,9]. For example, the sensitivity of the transfer capability to all the line admittances can be calculated in less time than one load flow in large power system models [5,9].

3.3. Approximate normality of U

Since the parameters are assumed to be independent

$$\sigma^2(U) = \sum_{i=1}^{m} \sigma^2\left(\frac{\partial A}{\partial p_i}(p_i - \mu(p_i))\right) \tag{5}$$

$$\sigma^2(U) = \sum_{i=1}^{m} \left(\frac{\partial A}{\partial p_i}\right)^2 \sigma^2(p_i) \tag{6}$$

and the standard deviation of U is:

$$\sigma(U) = \sqrt{\sum_{i=1}^{m} \left(\frac{\partial A}{\partial p_i}\right)^2 \sigma^2(p_i)} \tag{7}$$

The central limit theorem asserts that, under suitable conditions which are discussed in Appendix A, the sum of n independent random variables has an approximately normal distribution when n is large. Ref. [11] states: ‘in practical cases, more often than not, $n=10$ is a reasonably large number, while $n=25$ is effectively infinite.’ Hence for practical power system problems with many parameters, we expect that the uncertainty U is approximately a normal random variable with mean zero and standard deviation given by Eq. (7). This approximation gives a basis on which to calculate the TRM. The conditions described in Appendix A are mild and require little knowledge of the distribution of the parameters.

There are cases in which the central limit theorem approximation does not work well: As stated in Ref. [11], ‘the separate random variables comprising the sum should not have too disparate variances: for example, in terms of variance none of them should be comparable with the sum of the rest.’ This can occur in Eq. (4) when there are a few parameters which heavily influence the transfer capability (large$(\partial A/\partial p_i)$) and the other parameters have little influence on the transfer capability (small$(\partial A/\partial p_i)$) and are insufficiently numerous. In these cases, accurate answers can be obtained by using the central limit theorem to estimate the combined effect of the numerous parameters of little influence as a normal random variable and then finding the distribution of U with the few influential parameters by Monte Carlo or other means (cf. [12] in the context of probabilistic transfer capacity). This partial use of the central limit approximation dramatically reduces the dimension of the problem and the computational expense of solving it.

In all cases the central limit theorem approximation improves as the number of similar parameters increases and thus the approximation generally improves as the power system models become larger and more practical.

3.4. Formula for TRM

We want to define the TRM large enough so that it accounts for the uncertainty in U with rare exceptions. More precisely, we want

$$\text{probability}\{-U \le \text{TRM}\} = P \tag{8}$$

where P is a given high probability. This can be achieved by choosing the TRM to be a certain number K of standard deviations of U:

$$\text{TRM} = K\sigma(U) \tag{9}$$

K is chosen so that the probability that the normal random variable of mean zero and standard deviation 1 is less than K is P. (i.e. $1/\sqrt{2\pi}\int_{-\infty}^{k} e^{-t^2/2}\,dt = P$.) It is straightforward to calculate K from P by consulting tables of the cumulative distribution function of a normal random variable [13]. For example, if it is decided that the TRM should exceed the uncertainty $-U$ with probability $P=95\%$, then $K=1.65$. (i.e. a normal random variable is less than 1.65 standard deviations greater than the mean 95% of the time.) If it is decided that the TRM should exceed the uncertainty $-U$ with probability $P=99\%$, then $K=2.33$.

Combining Eqs. (7) and (9) yields a formula for TRM:

$$\text{TRM} = K\sqrt{\sum_{i=1}^{m} \left(\frac{\partial A}{\partial p_i}\right)^2 \sigma^2(p_i)} \tag{10}$$

In order to use formula (10) we need:

- A choice of uncertainty parameters $p_1, p_2,\ldots,p_m$ satisfying the two conditions above.
- The variance $\sigma^2(p_i)$ of each parameter.
- Calculation of the sensitivity $(\partial A/\partial p_i)$ of the transfer capability to each parameter p_i.

4. Uncertainty parameters

The transfer capability is computed from a base case constructed from system information available at a given time. There is some uncertainty or inaccuracy in this computation. There is additional uncertainty for future transfer capabilities because the transfer capability computed at the base case does not reflect evolving system conditions or operating actions. These two classes of uncertainty are detailed in Sections 4.1 and 4.2.

4.1. Uncertainties in the base case transfer capability

These uncertainties are:

- inaccurate or incorrect network parameters
- effects neglected in the data (e.g. the effect of ambient temperature on line loading limits)
- approximations in transfer capability computation

4.2. Uncertainties due to evolving conditions

These uncertainties are:

- ambient temperature and humidity (contributes to loading) and weather
- load changes not caused by temperature
- changes in network parameters
- change in dispatch

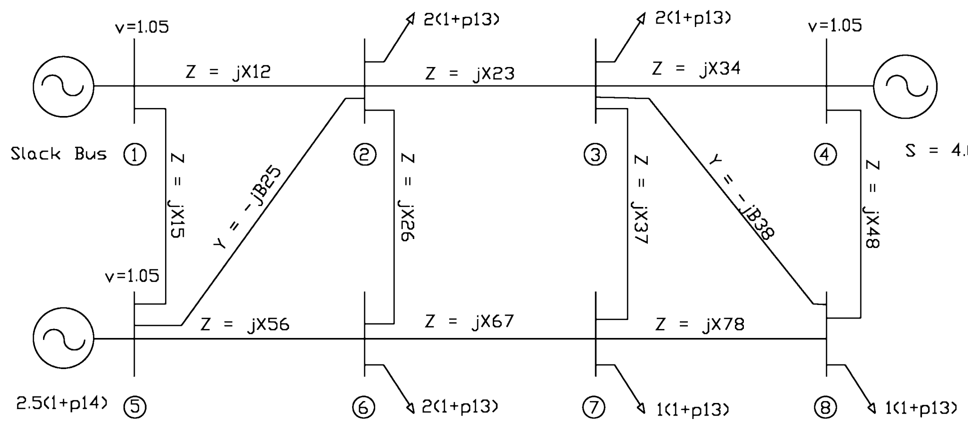

Fig. 1. Eight-bus test system.

- topology changes. This is often referred to as 'con-contingencies.' The probabilities of these contingencies can be estimated.
- changes in scheduled transactions

These uncertainties generally increase when longer time frames are considered. While some of these uncertainties may be quite hard to characterize a priori, it is important to note that it would be practical to collect empirical data on the changes in base cases as time progresses. Then variances of the uncertain parameters corresponding to various time frames could be estimated.

It is important to satisfy the statistical independence assumption when modelling parameter uncertainty. For example, if the uncertainty of different loads has a common temperature component, then this temperature component should be a single parameter and the load variations should be modelled as a function of temperature.

5. Simulation test results

This section tests the TRM formula by comparing it with Monte Carlo simulations in two examples. The examples are chosen to be small enough that comprehensive validation against the formula is practical. However, the formula is applicable to extremely large systems and situations with numerous parameters. In these larger examples, validation against extensive Monte Carlo analysis is impractical.

The first example uses the 8 bus system shown in Fig. 1. The transfer capability from area 1 (buses 1, 2, 5 and 6) to area 2 (buses 3, 4, 7 and 8) is limited by the power flow limit on the line joining buses 2 and 3. Fig. 1 also defines the parameters; for example, B25 is the susceptance of the line joining buses 2 and 5 and $p13$ is the change in real power of all the loads. The parameters are listed in Table 1.

Several simple types of models for the parameter uncertainty are illustrated in the first example. If a transmission line is assumed to be usually in service with a nominal susceptance and occasionally out of service with zero susceptance, then the probability distribution of its susceptance is binary with a high probability for the nominal value. If a transmission line has variable impedance due to temperature and the maximum and minimum impedances are known, then a conservative assumption for computing the variance of the impedance is that its probability distribution is uniform between the maximum and minimum impedances. The uncertainty in changes in system loading, generation, and line flow limits can take many forms, but here we assume that these parameters are normally distributed. The important points are that the TRM calculation allows any modelling choice for the parameter uncertainty as long as the variance of each parameter can be computed and that the TRM calculation depends on the parameter uncertainty only via these variances. Thus, the TRM calculation is insensitive to much of the detail of the modeling assumptions for parameter uncertainty. It also

Table 1
Parameter distributions

Parameter	Distribution
Line susceptance B25	Binary; prob{B25 = 5.0} = 0.95, prob{B25 = 0} = 0.05
Line susceptance B38	Binary; prob{B38 = 2.5} = 0.95, prob{B38 = 0} = 0.05
Line impedance X12	Uniform; $\mu = 0.1$, $\sigma = 0.0029$
Line impedance X23	Uniform; $\mu = 0.2$, $\sigma = 0.0058$
Line impedance X34	Uniform; $\mu = 0.1$, $\sigma = 0.0029$
Line impedance X15	Uniform; $\mu = 0.1$, $\sigma = 0.0029$
Line impedance X26	Uniform; $\mu = 0.1$, $\sigma = 0.0029$
Line impedance X37	Uniform; $\mu = 0.1$, $\sigma = 0.0029$
Line impedance X48	Uniform; $\mu = 0.1$, $\sigma = 0.0029$
Line impedance X56	Uniform; $\mu = 0.1$, $\sigma = 0.0029$
Line impedance X67	Uniform; $\mu = 0.2$, $\sigma = 0.0058$
Line impedance X78	Uniform; $\mu = 0.1$, $\sigma = 0.0029$
System loading p13	Normal; $\mu = 0.0$, $\sigma = 0.1$
Bus 5 generation p14	Normal; $\mu = 0.0$, $\sigma = 0.1$
Line 2–4 flow limit	Normal; $\mu = 1.5$, $\sigma = 0.1$
Line 6–7 flow limit	Normal; $\mu = 1.5$, $\sigma = 0.1$

Parameters are defined in Fig. 1. p13 and p14 are *changes* in load and generation.

Table 2
TRM for 8-bus system (p.u.)

P	90%	95%	99%	99.5%
TRM formula	0.6012	0.7750	1.0944	1.2118
Monte Carlo	0.6027	0.7846	1.1083	1.2171

would be straightforward to estimate the variance directly from historical records of the parameter.

The base case of the system assumes all parameters at their mean values. At the base system, the transfer capability is 2.8253 p.u. (with no contingency). Sensitivity of the transfer capability to these parameters can be calculated with no difficulty. Given a desired high, probability P, the TRM defined in Eq. (8) is calculated using formula (10). Table 2 lists TRMs with respect to different probabilities P. Ten thousand samples are used in the Monte Carlo simulation.

The second example uses the IEEE 118 bus system. There are 186 lines and the real power flow limit was assumed to be 1.0 p.u. at all lines except that the real power flow limit for line 54 was assumed to be 3.0 p.u. We consider a point to point power transfer from bus 6 to bus 45. The uncertain parameters are the power injections to all buses. The power injections are assumed to have a uniform distribution between 95% and 105% of their nominal values.

An AC power flow model was used. At the base case, the transfer capability is 1.8821 p.u. Given a desired probability P, TRM defined in Eq. (8) is calculated using formula (10). Table 3 lists TRMs with respect to different probabilities P. Ten thousand samples were used in the Monte Carlo simulation.

In both the 8 and 118 bus examples, the Monte Carlo results confirm the TRM estimates from formula (10).

6. Probabilistic transfer capacity

Our approach is not limited to the determination of TRM. Since our approach yields an approximately normal distribution of transfer capability uncertainty U and an estimate (7) of the standard deviation of U, this is an alternative way to find the probabilistic transfer capacity as presented in Refs. [8,12,14–16]. The probabilistic transfer capacity can be used for system planning, system analysis, contract design and market analysis. Ref. [14] suggests promising applications of probabilistic transfer capacity in the new market environment.

Table 3
TRM for 118-bus system (p.u.)

P	90%	95%	99%	99.5%
TRM formula	0.0803	0.1036	0.1462	0.1619
Monte Carlo	0.0795	0.1027	0.1427	0.1585

7. Conclusion

This paper presents a way to estimate TRM from parameter uncertainties with a formula. The formula requires estimates of the variances in independent parameters, the evaluation of transfer capability sensitivities, and specification of the degree of safety. The transfer capability sensitivities with respect to many parameters are easy and quick to evaluate once the transfer capability is determined [5,9]. This ability to quickly obtain sensitivities with respect to many parameters makes it practical to account for the effects of many uncertain parameters in large power system models. The validity of the formula has been confirmed by comparison with Monte Carlo runs on 8 and 118 bus systems. However, the central limit theorem approximation used to derive the formula improves as the number of parameters increases so that the formula is most applicable to larger power system models for which Monte Carlo comparisons are impractical. No cost information is used in the formula.

The approach includes estimating the statistics of the uncertainty in the transfer capability and thus gives an alternative way to obtain a probabilistic transfer capacity with a more formal way of accounting for uncertainty than is ordinarily used in such calculations. The formula provides a defensible and transparent way to estimate TRM; in particular, the degree of safety assumed and the sources of uncertainty are apparent in the calculation. The improved estimate of TRM will improve the accuracy of transfer capabilities and could be helpful in resolving the tradeoff between security and maximizing transfer capability. The sensitivities used in the calculation highlight which uncertain parameters are important. Indeed, the calculation provides one way to put a value on reducing parameter uncertainty because a given reduction in uncertainty yields a calculable reduction in TRM and this can be related to the monetary benefits that accrue from an increased transfer.

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Appendix A

Let $X_1,X_2,\ldots,X_m$ be independent, zero mean random variables and write $s_m^2=\sum_{k=1}^m \sigma^2(X_k)$ for the variance of $\sum_{k=1}^m X_k$. The approximate normality of $\sum_{k=1}^m X_k$ requires a central limit theorem. (Note that the most straightforward version of the central limit theorem does not apply because we do not assume that $X_1,X_2,\ldots,X_m$ are identically distributed.) A special case of the Lindeberg theorem [17] states that if

$$\lim_{m\to\infty}\sum_{k=1}^{m}\frac{1}{s_m^2}\int_{|X_k|>\in s_m} X_k^2\,\mathrm{d}F = 0 \tag{11}$$

hold for all positive $\in$ then $\frac{1}{s_m}\sum_{k=1}^{m} X_k$ converges in distribution to a normal random variable of mean zero and variance unity.

One useful class of random variables satisfying the Lindeberg condition (11) is random variables which are both uniformly bounded and whose variance uniformly exceeds some positive constant. It is also possible to augment the random variables in this class with some normal random variables.

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