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An approach to statistical estimation of cascading failure propagation in blackouts

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***Abstract*— Load power is progressively shed as large, cascading blackouts of electric power transmission systems evolve. We propose a statistical estimator to measure the extent to which the load shedding is propagated. The estimator uses data from a series of simulated blackouts. The estimator is derived from a continuous state branching process that is a high level probabilistic model of the cascading process. The estimator is tested on failure data generated by a power system model of cascading line outages. The estimates for propagation of load shed are consistent with estimates for the propagation of line outages. Estimating the initial load shed and the propagation of load shed leads to estimates of the probability distribution of blackout size. This work opens up possibilities of monitoring infrastructure failures to quantify the vulnerability to cascading and the overall risk of large cascading failures.**

I. Introduction

Blackouts in electric power transmission systems become widespread by a cascading process in which power system components are progressively disconnected and customer load power is progressively shed (disconnected). For example, the August 2003 blackout spread to a sizable region of Northeastern America by cascading [22].

We think of a cascading blackout as occuring in stages, with an initial amount of load shed in the first stage and further amounts of load shed in subsequent stages. We are particularly interested in estimating from simulated blackout data a quantity λ that describes the extent to which load shedding propagates in the subsequent stages. If the initial load shed is not very large and the load shedding propagates weakly (λ small), then it is likely that the blackout will be small. On the other hand, if load shedding propagates strongly ($\lambda > 1$), then it is likely that the blackout can become large. Moreover, we show how to estimate the probability distribution of blackout size from λ and other estimated quantities.

The ability to efficiently compute the probability distribution of blackout size from data is significant. Since risk is probability multiplied by cost, the blackout risk as a function of blackout size can be obtained from the probability distribution of blackout size and knowledge of the blackout cost. Efficient estimation of overall risk of blackouts of all sizes from simulated or real blackout data is an overall goal and this paper advances towards this goal.

We illustrate and begin to test the estimators using blackout data from the OPA model [3] of cascading failure blackouts. The estimators are tested by comparing the probability distribution of blackout size predicted by the estimators with the empirical probability distribution of blackout size produced by exhaustively running the OPA model.

In related previous work [12], we proposed an efficient estimator for propagation of transmission line outages in blackouts and predicted the distribution of the total number of lines outaged from this estimator and an estimate of the initial number of line outages. For the case of line outages, the cascading process was modeled by a Galton-Watson branching process [16], [2]. In this paper, we similarly apply a branching process model to the load power shed. The difference is that numbers of line outages are nonnegative integers whereas load power shed is a continuously varying nonnegative number. Therefore for load power shed we use a continuous state branching process model. The theory for continuous state branching processes [17], [21], [14] closely parallels the theory for Galton-Watson branching process, but the computation of the probability distribution of blackout size is more complicated.

We discuss reasons for using branching processes as high-level models for cascading blackouts. Branching processes are an obvious stochastic model to capture the gross features of cascading blackouts because they have been applied to other cascading processes such as genealogy, epidemics and cosmic rays [16]. Our idealized probabilistic model of cascading failure [13] describes a general cascading process in which component failures weaken and further load the system so that subsequent failures are more likely. We have shown that this cascade model can be approximated by a Galton-Watson branching process [10], [8]. Moreover, some features of this cascade model are consistent with results from cascading failure simulations [4], [9], [19]. All of these models can show criticality and power law regions in the distribution of failure sizes or blackout sizes consistent with NERC data [6]. Initial work fitting branching process models to observed blackout data is in [11]. The first suggestion to apply branching processes to cascading blackouts appears to be in [10] and subsequent applications to blackouts appear in [9], [11], [12].

In the continuous state branching process the load shed is produced in stages. The propagation of the load shed is determined by the offspring distribution, which is the probability distribution of load shed that would occur if there was one unit of load shed in the previous stage. The mean of the offspring distribution is λ. It is also the case that if the amount of load shed in stage n is X_n, then the mean load shed in stage $n+1$

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is λX_n.

The eventual behavior of the branching process is governed by the parameter λ. In the subcritical case of $\lambda < 1$, the failures will die out and the mean load shed in each stage decreases exponentially. In the supercritical case of $\lambda > 1$, although it possible for the process to die out, often the load shed increases exponentially until the system size or saturation effects are encountered. The estimator for λ proposed in this paper is not restricted to subcritical λ, but the method for computing the probability distribution of blackout size is restricted to subcritical λ. We do not test the estimator for supercritical or saturating cases.

One direct way to estimate the probability distribution of load shed is simply to run the simulation or record real blackout data until sufficient data is accumulated to estimate the probability distribution of blackout sizes. This is straightforward but requires a large number of simulations or an impractically long observation time. If the distribution of line failures is near criticality and has a power law character, the probability distribution requires many observations to determine its form for the larger blackouts. For example, it can take of the order of 1000 to 10000 real or simulated blackouts to accurately estimate the probability distribution of blackout size. The near critical case is pertinent because there is some evidence and explanation indicating that the North American power transmission system is designed and operated near criticality [6], [5].

If one assumes the continuous state branching process model, one can compute the probability distribution of the load shed from the initial load shed distribution and the offspring distribution. In practice we assume a form for the initial and offspring distributions and estimate their parameters. This approach for estimating the distribution of load shed is much faster because the initial load shed distribution and the offspring distribution do not have heavy tails. That is, estimating the offspring and initial distributions and then computing the probability distribution of blackout size is much more efficient than directly estimating the probability distribution of blackout size.

II. Continuous State Branching Process

This section explains continuous state branching processes informally and states formulas for application in the following sections. See [17], [21] for a systematic account of continuous state branching processes.

The branching process starts with an initial amount of load shed X_0 in stage 0 and proceeds to generate a sequence of load shed amounts X_1, X_2, ... in stages 1, 2, ... respectively. The offspring distribution $H(x)$ is defined to be the probability density function (pdf) of load shed in any stage if the load shed in the preceding stage is 1. We write X for a random variable with pdf $H(x)$. The expected value of X is λ.

We will first assume that the initial load shed X_0 is a constant. The load X_1 shed in stage 1 is a random variable determined by the offspring distribution $H(x)$ in the following way: In the special case of $X_0 = 1$, X_1 has pdf $H(x)$. In general, X_1 has pdf $(H(x))^{*X_0}$ where $(H(x))^{*X_0}$ is the convolution of $H(x)$ with itself X_0 times and the pdf of the sum of X_0 independent copies of X. (The computation of $(H(x))^{*X_0}$ using Laplace transforms when X_0 is a noninteger positive real number is discussed below.) X_1 is realized by sampling from $(H(x))^{*X_0}$. Then the load X_2 shed in stage 2 has pdf $(H(x))^{*X_1}$ that is the pdf of the sum of X_1 independent copies of X. X_2 is realized by sampling from $(H(x))^{*X_1}$. Then the load X_3 shed in stage 3 has pdf $(H(x))^{*X_2}$, and so on.

The computation of these pdfs is simplified by working in terms of their cumulant generating functions (cgf's). The cgf $h(s)$ of the offspring distribution is the negative logarithm of the Laplace transform of $H(x)$:

$$h(s) = -\ln \int_0^\infty e^{-sx} H(x)dx = -\ln Ee^{-sX}$$

The Laplace transform of $(H(x))^{*X_0}$ is the Laplace transform of $H(x)$ to the power X_0:

$$Ee^{-sX_1} = (Ee^{-sX})^{X_0}$$

Hence the cgf of the load X_1 shed in stage 1 is

$$h_1(s) = -\ln Ee^{-sX_1} = -\ln\left((Ee^{-sX})^{X_0}\right) = X_0 h(s)$$

The cgf of the load X_2 shed in stage 2 is

$$\begin{aligned} h_2(s) &= -\ln Ee^{-sX_2} \\ &= -\ln E[E[(e^{-sX})^{X_1}|X_1]] \\ &= -\ln E[e^{-h(s)X_1}] \\ &= h_1(h(s)) = X_0 h(h(s)) \end{aligned} \tag{1}$$

Similar reasoning shows that the cgf of X_3 is $X_0 h(h(h(s)))$ and that the cgf of X_n is

$$h_n(s) = -\ln Ee^{-sX_n} = X_0 h^{(n)}(s) \tag{2}$$

where $h^{(n)}$ is the n-fold functional composition of h.

As the cascade proceeds, the load shed accumulates and the running total of the shed at stage n is given by

$$Y_n = X_0 + X_1 + \ldots + X_n.$$

If $\lambda < 1$, the cascade will die out and Y_n converges to the total load shed or blackout size

$$Y = \lim_{n\to\infty} Y_n.$$

Assuming the subcritical case $\lambda < 1$, the distribution of Y can be computed from the offspring distribution. First consider the case of $X_0 = 1$ and let $k_\bullet(s)$ be the cgf of Y when $X_0 = 1$. Then $k_\bullet(s)$ satisfies the implicit equation

$$k_\bullet(s) = s + h(k_\bullet(s)). \tag{3}$$

If we assume the subcritical case of $\lambda < 1$, (3) can be solved by the Lagrange inversion method [23]:

$$k_\bullet(s) = s + \sum_{a=1}^{\infty} \frac{1}{a!} \frac{d^{a-1}}{ds^{a-1}} (h(s))^a \tag{4}$$

In practice we use 15 terms of the infinite sum in (4) to obtain a good approximation for $k_\bullet(s)$.

Equation (3) can be understood as follows. Consider $Y-1 = Y - X_0 = X_1 + X_2 + X_3 + ...$ If $X_1 = 1$, then the cgf of $Y - 1$ is $k_\bullet(s)$. If X_1 were a constant, then the cgf of $Y - 1$ would be $X_1 k_\bullet(s)$. This follows from the independence of the branching process generated by different portions of X_1. For example, if $X_1 = 2$, $Y - 1$ can be regarded as being produced by the sum of two independent branching processes with $X_1 = 1$ so that the cgf of $Y - 1$ is $2k_\bullet(s)$. If X_1 has cgf $h(s)$, as it does when $X_0 = 1$, then the cgf of $Y - 1$ is

$$\begin{aligned} -\ln Ee^{-s(Y-1)} &= -\ln E[E[(e^{-s(Y-1)}|X_1]] \\ &= -\ln E[e^{X_1 k_\bullet(s)}] \\ &= h(k_\bullet(s)) \end{aligned} \tag{5}$$

Now (3) follows since the cgf of $Y - 1$ is also

$$-\ln Ee^{-s(Y-1)} = -\ln Ee^{-sY} - s = k_\bullet(s) - s. \tag{6}$$

This section has so far assumed that the initial load shed X_0 is a constant whereas the branching process model of this paper assumes that X_0 is a random variable with cgf $m(s)$. That is, stage 0 of the branching process is generated using $m(s)$ and all subsequent stages are generated using $h(s)$. When X_0 has cgf $m(s)$, the cgf of X_n in (2) becomes

$$h_n(s) = m(h^{(n)}(s)) \tag{7}$$

Let $k(s)$ be the cgf of Y when X_0 has cgf $m(s)$. Then

$$k(s) = m(k_\bullet(s)) \tag{8}$$

The expected value of X_0 is θ and the expected value of the offspring distribution X is λ. The expected value of load shed in stage n can be evaluated by differentiating (7) and setting $s = 1$ to obtain

$$EX_n = \theta\lambda^n \tag{9}$$

Once $k(s)$ has been obtained as an explicit function of s using (8), the pdf $K(x)$ of the total load shed Y is obtained as the inverse Laplace transform of $e^{-k(s)}$ using the Post-Widder method [24]:

$$K(x) = \lim_{a\to\infty} \frac{(-1)^a}{a!}\left(\frac{a}{x}\right)^{a+1}\left(\frac{d^a}{ds^a}e^{-k(s)}\bigg|_{s=a/x}\right) \tag{10}$$

In practice we use $a = 15$ in (10) to obtain a good approximation for $K(x)$. The cumulative distribution function of Y is similarly obtained as the inverse Laplace transform of $e^{-k(s)}/s$.

In applying the branching process to a real power system, the total load shed Y is of course limited by the total power system load. Moreover, it is also conceivable that there could be effects that may tend to inhibit the spread of the blackout when the blackout reaches a certain size. We refer to both these limitations as “saturation”. Saturation effects are considered in applying a Galton-Watson branching process to transmission line outages in [12], but we do not consider these effects in this paper. Saturation is less likely in the subcritical case $\lambda < 1$.

III. Estimating λ and the blackout size pdf

This section details how the propagation λ and the pdf of blackout size can be estimated from cascading blackout data.

A. Cascading Blackout Data

A cascading failure simulation is assumed to produce samples of cascading blackouts and for each blackout the size of the blackout as well as the size at each intermediate stage of the blackout is recorded. Specifically there are J separate cascades, and X_n^i denotes the load shed at stage n of cascade i. The accumulated blackout data then looks like this:

$$\begin{array}{lcccc} & \text{stage } 0 & \text{stage } 1 & \text{stage } 2 & \cdots \\ \text{cascade } 1 & X_0^{(1)} & X_1^{(1)} & X_2^{(1)} & \cdots \\ \text{cascade } 2 & X_0^{(2)} & X_1^{(2)} & X_2^{(2)} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \text{cascade } J & X_0^{(J)} & X_1^{(J)} & X_2^{(J)} & \cdots \end{array}.$$

Likewise, $Y_n^{(i)}$ refers to the cumulative load shed

$$Y_n^{(i)} = X_0^{(i)} + X_1^{(i)} + \ldots + X_n^{(i)}$$

at stage n of cascade i.

The data must be handled in such a way that each cascade starts with a nonzero amount of shed. For example, cascades with no load shed are discarded. One effect of this is that the computed pdf of load shed is conditioned on the cascade starting. Moreover, if the simulation produces some cascades with no load shed in initial stages and load shed in subsequent stages, then we choose to discard the initial stages with no load shed so that stage 0 starts with a positive amount of load shed. Then cascade i has $N(i)$ stages. $N(i)$ is determined either by the maximum number of simulated stages being reached or the amount of load shed in a stage being zero or negligible.

B. Estimating λ and θ

The estimator for the propagation λ is

$$\begin{aligned} \hat\lambda &= \frac{\displaystyle\sum_{i=1}^{J}\left(X_1^{(i)} + X_2^{(i)} + \ldots + X_{N(i)}^{(i)}\right)}{\displaystyle\sum_{i=1}^{J}\left(X_0^{(i)} + X_1^{(i)} + \ldots + X_{N(i)-1}^{(i)}\right)} \\ &= \frac{\displaystyle\sum_{i=1}^{J} Y_{N(i)}^{i} - X_0^{(i)}}{\displaystyle\sum_{i=1}^{J} Y_{N(i)-1}^{(i)}}. \end{aligned}$$

This is a variant of the maximum likelihood estimator

$$\frac{\sum_{i=1}^{J} Y_N^i - X_0^{(i)}}{\sum_{i=1}^{J} Y_{N-1}^{(i)}}$$

when each cascade has the same number of stages N. This maximum likelihood estimator is consistent and asymptotically unbiased as $J \to \infty$ [7], [14].

The mean initial load shed θ is estimated by the sample mean

$$\hat\theta = \frac{1}{J}\sum_{i=1}^{J} X_0^i.$$

C. Estimating blackout size pdf

The general procedure for estimating the blackout size pdf $K(x)$ is

1) Assume a parameterized form for the initial load shed cgf $m(s)$ and offspring cgf $h(s)$.
2) Estimate the parameters of $m(s)$ and $h(s)$ from the data.
3) Compute the blackout size cgf $k(s)$ from $m(s)$ and $h(s)$ using (3) and (8)
4) Compute the inverse Laplace transform of $e^{-k(s)}$ to obtain the blackout size pdf $K(x)$ using (10).

The procedure estimates parameters of an explicit form of $m(s)$ and $h(s)$ so that the computation of $k(s)$ and the Laplace inversion can be done using computer algebra.

We choose to assume gamma distributions for the initial load shed and offspring distributions. Then the corresponding cgf's are

$$m(s) = \frac{\theta^2}{\sigma_{\text{init}}^2} \ln\left(1 + s\frac{\sigma_{\text{init}}^2}{\theta}\right) \tag{11}$$

and

$$h(s) = \frac{\lambda^2}{\sigma_{\text{off}}^2} \ln\left(1 + s\frac{\sigma_{\text{off}}^2}{\lambda}\right). \tag{12}$$

The parameters of the initial load shed cgf are the mean θ and the variance σ_{init}^2. The parameters of the offspring cgf are the mean λ and the variance σ_{off}^2.

The means λ and θ are estimated from the data as described in the previous subsection. The variance of the initial load shed σ_{init}^2 is estimated using

$$\hat{\sigma}_{\text{init}}^2 = \frac{1}{J}\sum_{i=1}^{J}(X_0^{(i)})^2 - \hat{\theta}^2.$$

The variance of the offspring distribution σ_{off}^2 is estimated by applying the method of moments to X_1. The second moment of X_1 is

$$EX_1^2 = \frac{d^2}{ds^2}\, e^{-m(h(s))}\big|_{s=0} = \lambda^2\left(\theta^2 + \sigma_{\text{init}}^2\right) + \theta\sigma_{\text{off}}^2 \tag{13}$$

Then the estimator $\hat{\sigma}_{\text{off}}^2$ may be found by solving:

$$\frac{1}{J}\sum_{i=1}^{J}(X_1^{(i)})^2 = \hat{\lambda}_{\text{init}}^2\left(\hat{\theta}^2 + \hat{\sigma}_{\text{init}}^2\right) + \hat{\theta}\hat{\sigma}_{\text{off}}^2$$

where

$$\hat{\lambda}_{\text{init}} = \frac{1}{\hat{\theta}}\left(\frac{1}{J}\sum_{i=1}^{J}X_1^{(i)}\right).$$

IV. Results

The OPA model produces cascading transmission line outages and load shed in stages resulting from a random initial set of line failures. The power transmission system is modeled using DC load flow and LP generator dispatch and cascading line overloads and failures are represented. The power system is assumed to be fixed with no transmission line upgrade process. For details of the OPA model see [3].

The method is tested on the IEEE 118 bus system. $J = 5000$ cascades were simulated for each of the four loading levels $L = 0.85$, $L = 0.90$, $L = 0.95$, and $L = 1.0$, where L is the fraction of the base case loading. These loading levels are chosen so as to avoid significant saturation effects.

Ten stages of each cascade are simulated. The number of stages is reduced in some cases: For example, initial stages with zero or negligible load shed are discarded (the threshold for negligible load is 10^{-15}). The load shed is measured as a fraction of the total load so that the maximum load shed possible is 1, or total blackout.

The estimated propagation $\hat{\lambda}$ and blackout size pdf are estimated for the OPA data according to the methods described above.

A. Estimated propagation $\hat{\lambda}$

The estimated propagation at each load level computed from the load shed data is shown in the second column of Table I. As expected, $\hat{\lambda}$ increases with loading. Table I also compares, using the same OPA cascades, $\hat{\lambda}$ computed from the load shed with $\hat{\lambda}$ computed from the transmission lines outaged using the methods of [12]. The $\hat{\lambda}$ for load shed matches quite well with the $\hat{\lambda}$ for line outages. This match tends to support the assertion that $\hat{\lambda}$ for load shed quantifies the cascading process in OPA.

TABLE I

ESTIMATED PROPAGATION $\hat{\lambda}$ FROM LOAD SHED AND LINE OUTAGE DATA

loading factor	load shed $\hat{\lambda}$	line outages $\hat{\lambda}$
0.85	0.128	0.115
0.9	0.159	0.188
0.95	0.264	0.288
1.0	0.429	0.430

B. Blackout size pdf

Table II shows the parameters of the initial load shed and offspring distributions estimated from the load shed data. All cases considered are subcritical ($\lambda < 1$) as required for the Lagrange inversion (4).

Figure 1 compares the empirical and estimated pdfs for loading level $L = 0.85$, and Figure 3 compares the empirical and estimated pdfs for loading level $L = 1.0$. The blackout size is plotted on a log scale over two decades, from a small blackout $Y = .01$ (shedding of 1% of total load) to $Y = 1$ (shedding of 100% of total load and total blackout). The corresponding cgfs are also plotted in Figure 2 and Figure 4 to give another view of how well the empirical and estimated distributions match.

TABLE II

ESTIMATED INITIAL LOAD SHED AND OFFSPRING DISTRIBUTION PARAMETERS

loading factor	$\hat{\lambda}$	$\hat{\theta}$	$\hat{\sigma}_{\text{init}}^2$	$\hat{\sigma}_{\text{off}}^2$
0.85	0.128	0.0520	0.00198	0.00431
0.9	0.159	0.0482	0.00195	0.00568
0.95	0.264	0.0445	0.00182	0.00995
1.0	0.429	0.0383	0.00160	0.01230

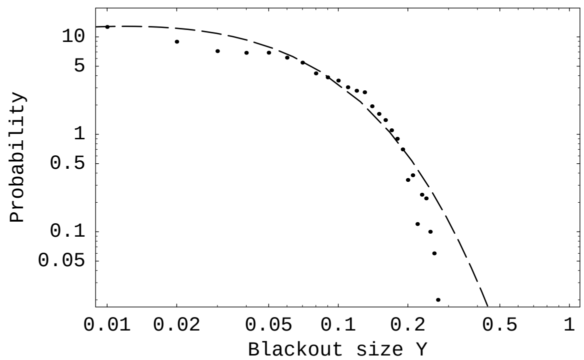

Fig. 1. Probability density function of blackout size Y on log-log plot. Empirical pdf shown as dots, estimated pdf shown as dashed line. IEEE 118 bus system with loading $L = 0.85$.

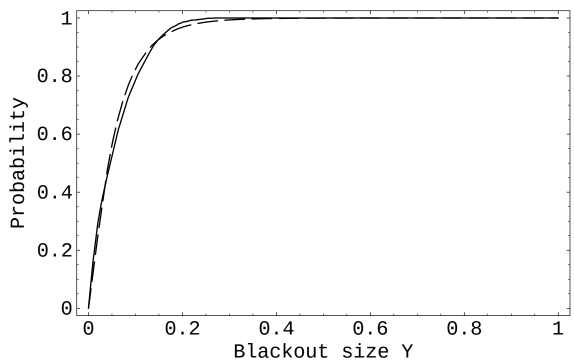

Fig. 2. Cumulative distribution function of blackout size Y. Empirical cdf shown as solid line, estimated cdf shown as dashed line. IEEE 118 bus system with loading $L = 0.85$.

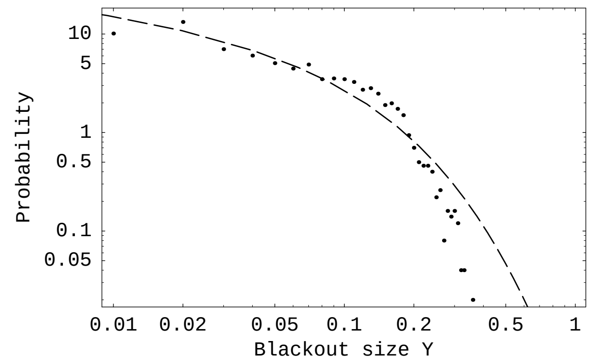

Fig. 3. Probability density function of blackout size Y on log-log plot. Empirical pdf shown as dots, estimated pdf shown as dashed line. IEEE 118 bus system with loading $L = 1.0$.

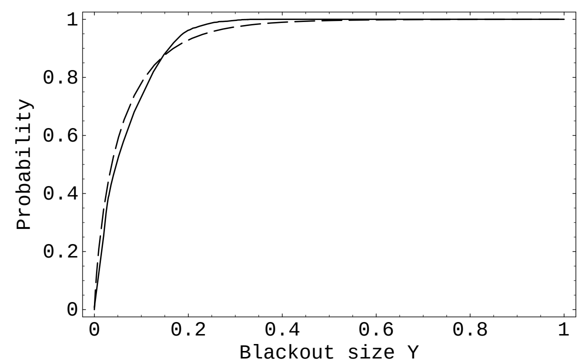

Fig. 4. Cumulative distribution function of blackout size Y. Empirical cdf shown as solid line, estimated cdf shown as dashed line. IEEE 118 bus system with loading $L = 1.0$.

C. Initial load shed and offspring distributions

We discuss the choices of the forms of initial load shed and offspring distributions that are assumed in the computations.

The initial load shed gamma distribution parameters $\hat{\theta}$ and $\hat{\sigma}^2_{init}$ shown in Table 2 are relatively insensitive to loading changes. For all these cases $\hat{\sigma}^2_{init} \approx \hat{\theta}^2$ and hence the initial load shed is approximately exponentially distributed. Figure 5 shows estimated and empirical initial failure distributions for loading $L = 1.0$.

Figure 6 shows the estimated offspring distribution pdf for loading $L = 1.0$. This is a gamma distribution with mean 0.0383 and variance 0.00160 that is approximately a normal distribution. However, the offspring pdf becomes more asymmetrical when the loading L is decreased. Any parameterized nonnegative distribution that is infinitely divisible is a candidate to describe the offspring distribution and we have not found general arguments supporting our specific choice of the gamma distribution.

V. Conclusions

In this paper we show how load shed data from a subcritical cascading failure blackout simulation can be processed to estimate the extent λ to which load shed propagates and the probability distribution of the total load shed. Total load shed is a useful measure of blackout size. The simulation is assumed to produce amounts of load shed in stages for each of a series of cascades. A continuous state branching process model described by an initial load shed distribution and an offspring distribution is used as a high level probabilistic model of the cascading. The method is illustrated using the OPA model of cascading line outages on some subcritical cases on the IEEE 118 bus system.

The results for load shed propagation λ match well the corresponding results for transmission line outage propagation computed using the discrete state branching process method of [12]. This suggests that the cascading process in the OPA model can be thought of a single cascading process that can be monitored by either line outages or load shed. If this conclusion holds more generally, then it could make possible the estimation of propagation of load shed by monitoring line outages. Note that line outages are an "internal" measure of blackout size of interest to the power industry and that load shed is an "external" measure of blackout size of interest to our entire society. Line outages may be easier to monitor and may occur in precursor events in which no load is shed.

The estimated total load shed pdf is compared to the empirical total load shed pdf obtained by exhaustively running the OPA simulation and the results show some qualitative agreement. This estimation of the blackout size pdf requires the assumption of analytic forms for the initial and offspring distributions and the estimation of the mean and variance of these distributions. Although not addressed in this paper, we

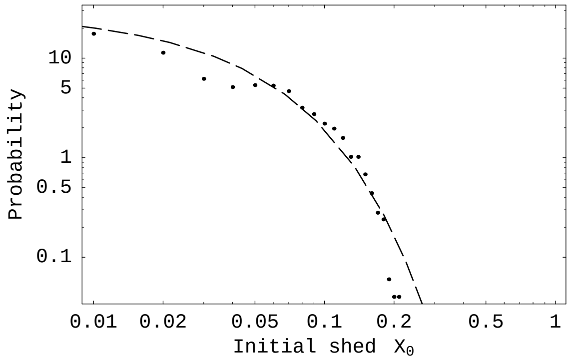

Fig. 5. Probability density function of initial load shed X_1 on log-log plot. Empirical pdf shown as dots, estimated pdf shown as dashed line. IEEE 118 bus system with loading $L = 1.0$.

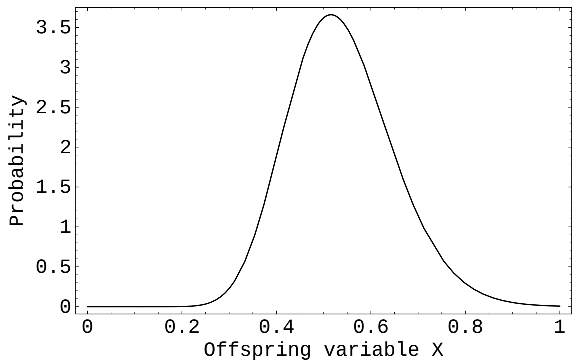

Fig. 6. Probability density function $H(x)$ of offspring distribution that is a gamma distribution with mean $\lambda = .429$ and variance $\sigma^2_{\rm off} = .0123$. Parameters computed from data on IEEE 118 bus system with loading $L = 1.0$.

expect similarly to [12] that this estimation via the initial and offspring distributions is substantially more efficient in that estimates may be obtained with far fewer simulated cascades.

The paper is initial work showing the possibility of the computations of λ and the blackout size pdf. To fully establish these methods, there is more work needed such as testing on wider range of cases, testing on more detailed cascading failure simulations (e.g. [18], [15]), optimizing the method, and determining its accuracy. Supercritical cases and the issue of blackout saturation are not yet addressed. While branching process models do seem to be the most promising high-level probabilistic models of cascading failure, their descriptive power for cascading failure blackouts remains under scrutiny. To extend the methods from simulated blackout data to real power system data it is necessary either to group the real blackout data in stages (for a first attempt see [11]) or to apply branching process models that incorporate time.

Nevertheless, the possibility of estimating cascade propagation λ and blackout size pdfs is significant in that it opens up opportunities in quantifying overall blackout risk. Quantifying blackout propagation with λ gives a measure of system stress with respect to cascading. In particular, $\lambda = 1$ indicates criticality and power laws in the pdf of blackout size. Quantifying the pdf of total load shed gives the pdf of one measure of blackout size. If the total load shed can be mapped to energy unserved, or if the blackout cost can be expressed as a function of total load shed, then the pdf of blackout risk can be estimated from the pdf of total load shed. The ability to efficiently quantify the blackout risk pdf from a modest number of simulated or observed cascading blackouts could have several useful applications. The changes in blackout risk due to transmission system upgrades and particularly the change in the risk of larger blackouts could be quickly estimated from cascading failure simulations. The overall blackout risk could be estimated and monitored by recording events on the real power system.

This paper develops methods to quantify cascading failure blackouts of the electric power transmission system. However, the electric power infrastructure interacts with other infrastructures and failures in one infrastructure can propagate to other infrastructures [20]. The cascading can greatly magnify the extent of damage to society and both accidental cascading failures and those caused by sabotage or terrorism are of concern. While progress has been made in describing and simulating these interactions, it would also be valuable to better quantify the extent to which failures propagate by cascading both within and between infrastructures [25]. If the size of the different sorts of failures can be measured in some common way, such as cost, then it might be feasible to extend the methods initiated in this paper to quantify these failure propagations and the risks of large cascading failures.

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