Applying synchrophasor computations to a specific area

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***Abstract*—Synchrophasor measurements at all the border buses of an area of a power system can be used to effectively decouple the area from the rest of the power system. Calculations combining models and synchrophasor measurements can then be applied using only models of the area and area measurements. We illustrate the decoupling for a line outage detection algorithm.**

***Index Terms*—Phasor measurement units, event detection, power transmission, smart grid.**

I. Area decoupling by superposition

There are good opportunities for calculations that combine synchrophasor measurements with system models. However, in large interconnections, it is often convenient for utilities or ISOs to maintain network models only for their own area. We suggest a way to do the calculations using synchrophasor measurements at the border of an area and superposition.

The area of interest is denoted by R. We first assume a DC load flow model of R and account for the effect of the rest of the network on R by a vector of power injections $P_r^{\rm into}$ into R at all the border buses of R that have tie lines connecting R to the rest of the network. For each border bus of R, the corresponding entry of the vector $P_r^{\rm into}$ is the sum of real power flows of the tie lines joined to that border bus. For each interior bus of R, the corresponding entry of the vector $P_r^{\rm into}$ is zero. We write $P_r^{\rm R}$ for the real power injections (generation or load) at buses of R. Then the total power injections at area buses are $P_r^{\rm R}+P_r^{\rm into}$. Write θ_r for the voltage phasor angles at buses of R, and $B_{rr}^{\rm R}$ for the susceptance matrix of area R considered as a stand-alone network. Then the DC load flow for area R is

$$B_{rr}^{\rm R}\theta_r = P_r^{\rm R}+P_r^{\rm into}. \tag{1}$$

Let $\theta_r^{\rm into}$ be the part of the angles corresponding to the tie line power flows so that

$$P_r^{\rm into} = B_{rr}^{\rm R}\theta_r^{\rm into} \tag{2}$$

Let $\theta_r^{\rm R}$ be the part of the angles corresponding to the power injections inside R so that

$$P_r^{\rm R} = B_{rr}^{\rm R}\theta^{\rm R}. \tag{3}$$

We call $\theta_r^{\rm R}$ the *internal voltage angles* for area R. We use a common angle reference for all angles. Then

$$\theta_r = \theta_r^{\rm into}+\theta_r^{\rm R}. \tag{4}$$

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To summarize, superposition enabled by the linearity of the DC load flow implies that area R angles θ_r are the sum of internal angles $\theta_r^{\rm R}$ due to the power injections inside R and angles $\theta_r^{\rm into}$ due to the power flows into R along the tie lines. Then (1) may be rewritten as

$$P_r^{\rm R} = B_{rr}^{\rm R}(\theta_r-\theta_r^{\rm into}) = B_{rr}^{\rm R}\theta_r^{\rm R}. \tag{5}$$

The internal angles $\theta_r^{\rm R}=\theta_r-\theta_r^{\rm into}$ are obtained by adjusting the area angles θ_r by subtracting $\theta_r^{\rm into}$. Equation (5) shows that the internal angles $\theta_r^{\rm R}$ satisfy the DC load flow equation of the area R as if area R were a stand-alone area decoupled from the rest of the network.

An alternative derivation of (5) starts from the DC load flow of the entire network. Write $\theta_{\overline{r}}$ and $P_{\overline{r}}$ respectively for the voltage angles and real power injections at buses outside area R. Order the buses so that the area R buses are first. Then the DC load flow $P=B\theta$ for the entire network is

$$\begin{pmatrix} P_r^{\rm R} \\ P_{\overline{r}} \end{pmatrix} = \begin{pmatrix} B_{rr} & B_{r\overline{r}} \\ B_{\overline{r}r} & B_{\overline{r}\overline{r}} \end{pmatrix} \begin{pmatrix} \theta_r \\ \theta_{\overline{r}} \end{pmatrix}. \tag{6}$$

The powers flowing into R along the tie lines are

$$P_r^{\rm into} = \sum_{j\in\overline{r}}(-B_{rj})(\theta_j-\theta_r) = -B_{r\overline{r}}\theta_{\overline{r}}+{\rm diag}\{B_{r\overline{r}}\mathbf{1}\}\theta_r.$$

Here $\mathbf{1}$ is a column vector of all ones. The first block row of (6) may be rewritten as

$$P_r^{\rm R} = B_{rr}^{\rm R}\theta_r - P_r^{\rm into}, \tag{7}$$

where $B_{rr}^{\rm R}=B_{rr}+{\rm diag}\{B_{r\overline{r}}\mathbf{1}\}$ is the susceptance matrix for the area R considered as a stand-alone network. ($B_{rr}^{\rm R}$ is different than the submatrix B_{rr} of the susceptance matrix for the entire network, because the diagonal entries of $B_{rr}^{\rm R}$ do not include the tie line susceptances.) Then (2) can be used to rewrite (7) as (5).

II. Measurements to get internal angles $\theta_s^{\rm R}$

We suppose that the following are available:

- Synchrophasor measurements θ_s of voltage angles at some buses inside area R (θ_s is a vector that is some of the components of θ_r).
- Synchrophasor measurements of the entering currents and voltages at all the tie lines joined to area R border buses. This yields the powers $P_r^{\rm into}$ entering the border buses of R along all the tie lines.
- Area susceptance matrix $B_{rr}^{\rm R}$. This can be calculated from the current area topology [1].

Then the internal voltage phasor angles $\theta_s^{\rm R}$ at the buses measured with synchrophasors can be computed as follows:

1) Obtain $\theta_r^{\rm into}$ from (2). $\theta_s^{\rm into}$ is some components of $\theta_r^{\rm into}$.
2) Obtain internal angles $\theta_s^{\rm R}$ using $\theta_s^{\rm R}=\theta_s-\theta_s^{\rm into}$, which corresponds to some of the components of (4).

III. Tate's method of line outage detection

It is useful to detect and locate transmission line outages with synchrophasor measurements. The information can confirm topology changes available from traditional, slower SCADA and state estimation methods and provide the fast topology updates needed for other synchrophasor applications. Tate and Overbye [1] describe a method to detect and locate line outages in an entire network interconnection using synchrophasor measurements and a DC load flow model of the entire network. They model the effect of a line outage by a change ΔP in power injections that consists of equal and opposite power injections at the ends of the line. The change in bus voltage angles $\Delta\theta$ corresponding to the line outage is computed from the DC load flow

$$\Delta P = B\Delta\theta. \tag{8}$$

The effect of each line outage on $\Delta\theta$ is computed at the synchrophasor buses, and compared to the observed changes in synchrophasor voltage angle measurements. The line that outaged is identified as the line with the computed effect closest to that observed. Applying Tate's method to a large interconnection requires maintaining a large DC load flow model and discriminating among many possible line outages, so here we show how to adapt the method to a specific area within the interconnection.

IV. Example: applying Tate's method to an area

Fig. 1 shows an example of an area R in the 39 bus New England test system (network data are given in [2]). Bus 32 is the reference bus. There are synchrophasor voltage measurements at area border buses 4, 8, 14 and at buses 12 and 32 inside the area. There are synchrophasor measurements of the currents in the area tie lines 8–9, 4–3, 14–15. Assume that a change in voltage angles is detected, and that differences in the measurements before and after the change are obtained [1]. In particular, the change in the measured angles $\Delta\theta_s$ and the change in the tie line powers $\Delta P_r^{\rm into}$ are obtained. $\Delta\theta_r^{\rm into}$ corresponding to $\Delta P_r^{\rm into}$ is calculated using $\Delta P_r^{\rm into} = B_{rr}^{\rm R}\Delta\theta_r^{\rm into}$. Then the measurement-based changes in the internal voltage angles at the measured buses are $\Delta\theta_s^{\rm R} = \Delta\theta_s - \Delta\theta_s^{\rm into}$. Then $\Delta\theta_s^{\rm R}$ is compared with the computed internal angle changes for each line obtained using $\Delta P_r^{\rm R} = B_{rr}^{\rm R}\Delta\theta_r^{\rm R}$, the incremental version of (5).

Table I shows how the base case angles θ_s at synchrophasor buses split into internal angles $\theta_s^{\rm R}$ and angles $\theta_s^{\rm into}$ due to the tie line flows. A line outage inside the area such as outaging line 10–11 causes changes in internal angles $\Delta\theta_s^{\rm R}$ that can be used in Tate's method. Table I also shows that a line outage outside area R such as outaging line 2–3 leads to changes in area angles θ_s, but the internal angles $\theta_s^{\rm R}$ do not change. In general, line outages or power redispatches outside the area do not affect the area power injections P_r and therefore, according to (5), the internal angles $\theta_s^{\rm R}$ do not change. Thus changes or not in internal angles can be used to detect whether the line outage occurs inside the area or not.

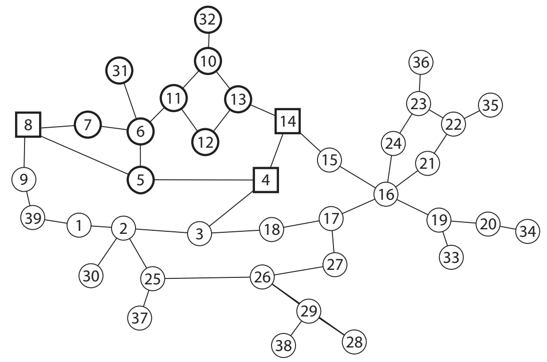

Fig. 1. Area R consisting of buses $4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 31, 32$ shown with thicker circles or squares in the 39 bus New England test system. Border buses $4, 8, 14$ are shown with squares.

TABLE I
AREA ANGLES

	base case			line 10–11 out		line 2–3 out	
bus	θ_s	$\theta_s^{\rm into}$	$\theta_s^{\rm R}$	$\Delta\theta_s$	$\Delta\theta_s^{\rm R}$	$\Delta\theta_s$	$\Delta\theta_s^{\rm R}$
4	-11.68	0.37	-12.23	-3.62	-3.68	-0.59	0
8	-12.66	0.21	-12.87	-5.21	-5.40	0.63	0
12	-8.35	0.01	-8.36	-3.70	-3.70	0	0
14	-9.91	0.06	-9.98	-2.20	-2.07	-0.23	0
32	0	0	0	0	0	0	0

all angles in degrees

V. Conclusion

We compute internal voltage angles from synchrophasor measurements that only respond to changes inside the area and correspond to models of the area that are effectively decoupled from the rest of the network. We illustrate the use of internal voltage angles by applying Tate's line outage algorithm to an area. The method will be particularly useful when utilities or ISOs in large interconnections restrict their attention to network models and synchrophasor measurements for only their own area. The results show the value of synchrophasor measurements at all the tie lines of an an area. Moreover, according to [2], synchrophasor measurements at all the tie lines also enable area stress angles to be calculated.

Our example of the decoupling uses voltage angles and a DC power flow. However, since the decoupling depends only on linear circuit laws, we emphasize that exactly similar results apply to complex voltages in an AC load flow [2]. That is, we define internal complex voltages $V_r^{\rm R}$ that only respond to changes inside the area and correspond to a decoupled area model. The internal complex voltages are computed using $V_r^{\rm R} = V_r - V_r^{\rm into}$, where V_r are synchrophasor measurements of complex voltages inside the area and $V_r^{\rm into}$ is obtained from complex current injections $I_r^{\rm into}$ measured along the area tie lines using $I_r^{\rm into} = Y_{rr}^{\rm R}V_r^{\rm into}$, where $Y_{rr}^{\rm R}$ is the complex admittance matrix of the area.

References

[1] J.E. Tate, T.J. Overbye, Line outage detection using phasor angle measurements, IEEE Trans. Power Syst., vol 23, no 4, Nov 2008, pp 1644-1652.
[2] I. Dobson, Voltages across an area of a network, IEEE Trans. Power Systems, vol 27, no 2, May 2012, pp 993-1002.