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Locating line outages in a specific area of a power system with synchrophasors

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***Abstract*—We detect the location of line outages inside a specific area of the power system from synchrophasor measurements at the border of the area and inside the area. We process the area synchrophasor measurements using a DC load flow model of the area. The processed measurements do not respond to line trips or power redispatches outside the area. The method extends previous methods that locate line trips in an entire network so that they work in a particular area and also deals with cases of islanding. The method will be particularly useful when utilities or ISOs in large interconnections restrict their attention to network models and phasor measurements for only their own area.**

***Index Terms*—Phasor measurement, synchrophasor, event detection, power transmission, power system monitoring, smart grid**

I. Introduction

It is useful to detect and locate transmission line outages with synchrophasor measurements. The information can confirm topology changes available from the traditional, slower SCADA and state estimation methods and provide fast topology updates needed for other synchrophasor applications.

Tate and Overbye [1] give a method of detecting and locating line outages in an entire network interconnection by processing synchrophasor observations in an entire power grid. Edge detection methods are applied to detect changes in synchrophasor angles that exceed a threshold. The synchrophasor measurements are filtered to retain the changes in the steady state angle but suppress the settling transient that follows the line outage. Once a change in synchrophasor angles is detected and calculated, Tate and Overbye then give an algorithm to locate the line outage by comparing the observed change in angles to the changes in angle for all the possible line outages computed using a DC load flow model of the entire network.

For this paper, we assume that changes in synchrophasor angles are available using the detection and filtering methods in Tate and Overbye [1], and show how to adapt their line outage location method to an area within the interconnection with synchrophasor measurements inside the area and around the border of the area. The border measurements are used to decouple the area from the rest of the network so that the method detects whether the line outage occurs within the area and then locates a line outage within the area using a DC load flow model of the area. The motivation is that it is often convenient for utilities to maintain network models only for their own area, and line outage detection algorithms can work better if there are fewer candidate line outage to choose from. Our approach also confirms whether the line outage occurred inside the monitored area or not, giving a useful discrimination of the source of changes in the power system.

Line outage detection is more complicated in cases in which the line outage islands the system because the rebalancing of generation in the areas needs to be taken into account. We extend Tate and Overbye's method to accommodate islanding.

This paper begins with a review of the method established by Tate and Overbye, then discusses implementing the algorithm to a reduced area of the network for both non-islanding and islanding cases, and finishes with a simple example illustrating the concepts of the method. This paper rewrites material from the MS thesis [2].

II. Review of Tate and Overbye's method

We review Tate and Overbye's method [1] of using observed changes in synchrophasor measurements to estimate the most likely line outages. The method is based on the DC load flow model, and

$$B\Delta\theta = \Delta P, \tag{1}$$

where ΔP is the vector of the changes in real power injection at each bus after the line outage, B is the susceptance matrix, and $\Delta\theta$ is the vector of the changes in voltage phasor angle at each bus after the line outage. We note that in the DC load flow model, a line outage can be simulated as equal, but opposite power injections at the ends of the line so long as the system stays connected after the line outage [4]. (Modeling the line outage as power injections in a network with unchanged lines avoids the inconvenient recalculation of the susceptance matrix with the line removed.) Following a line outage, a change in the phasor angles is expected. A vector proportional to this change in angles is calculated for every possible line outage and compared to the observed change in angles. Since the synchrophasors are only at some of the buses, only some of the angle changes are known. The vector of the observed bus angles is θ_{obs} and the vector of calculated (predicted) angles is θ_{calc}. To relate the observed bus angles to all the buses, we introduce the matrix

$$K = [I_{\text{MxM}} \quad \mathbf{0}_{\text{Mx(N-M)}}] \tag{2}$$

where N is the number of buses in the system and M is the number of buses observed with synchrophasors. Then

$$\theta_{obs} = K\theta \tag{3}$$

Since a line outage can be modeled by equal and opposite power injections at the buses at each end of the line [1], the form of ΔP for the outage of a line l is[1]

$$\Delta P_{calc,l} = \begin{bmatrix} \mathbf{0} \\ \tilde{P}_l \\ \mathbf{0} \\ -\tilde{P}_l \\ \mathbf{0} \end{bmatrix} \begin{matrix} \\ \leftarrow \text{start bus} \\ \\ \leftarrow \text{end bus} \\ \\ \end{matrix} \tag{4}$$

The power injection $\tilde{P}_l$ is related to the pre-outage flow P_l on line l according to

$$\tilde{P}_l = \frac{-P_l}{1 + \mathrm{PTDF}_{l,l_{to}-l_{from}}}. \tag{5}$$

$\mathrm{PTDF}_{l,l_{to}-l_{from}}$ is the power transfer distribution factor giving the increment in real power on line l when 1 MW is injected at the "to" bus of line l and –1 MW is injected at the "from" bus of line l. It can be shown [4] that the injections $\tilde{P}_l$ make the powers entering the "to" bus from lines other than line l sum to zero and the powers entering the "from" bus from lines other than line l sum to zero, and hence have the same effect as removing line l. We will see that the following calculation does not depend on the value of $\tilde{P}_l$ or the value of P_l.

We now calculate based on (1) the change in the angle $\theta_{calc,l}$ for the outage of line l

$$\begin{aligned} \Delta\theta_{calc,l} &= KB^{-1}\Delta P_{calc,l} \\ &= \tilde{P}_l K B^{-1} \begin{bmatrix} \mathbf{0} \\ 1 \\ \mathbf{0} \\ -1 \\ \mathbf{0} \end{bmatrix} \\ &= \tilde{P}_l \widetilde{\Delta\theta}_{calc,l} \end{aligned} \tag{6}$$

Here it is convenient to write $B^{-1}\Delta P_{calc,l}$ for a solution $\Delta\theta$ to $\Delta P_{calc,l} = B\Delta\theta$. Recall that B is not invertible, and that one standard approach is to solve $\Delta P_{calc,l} = B\Delta\theta$ using the generalized inverse of matrix B [5]. We do not need to calculate the constant P_l as we can compare versions of $\Delta\theta_{obs}$ and $\widetilde{\Delta\theta}_{calc,l}$ that are normalized to unit length. That is, we compare $\widehat{\Delta\theta}_{obs}$ and $\widehat{\widetilde{\Delta\theta}}_{calc,l}$, where $\hat{x} = x/\|x\|$. This implies that we need to know neither the actual power flows on the line nor the magnitude of the power injections representing the line outage, because only the system's structure will determine the vector "shape," or direction in the vector space of angles. We seek to compute the magnitude of error between the shapes of the calculated and observed vectors. This corresponds to the least geometric distance in the vector space of angle changes called the Normalized Angle Distance (NAD). The NAD between vectors of angle changes a and b has the following definition[2]:

$$\mathrm{NAD} = \begin{cases} \|\hat{a} - \hat{b}\|, & a \cdot b \geq 0 \\ \|\hat{a} + \hat{b}\|, & a \cdot b < 0. \end{cases} \tag{7}$$

The NAD between the normalized computed line changes and the normalized observed angle changes is computed for every line in the system. The line outage with the lowest NAD is the one with the lowest error and hence the predicted line outage. We note that the calculation above assumes that the system stays connected. It is possible that a line outage could island the system. The most common case occurs when a generator supplies the rest of the grid through a single line. If this line outages, then the generator bus forms one island and the rest of the grid forms the other island. Different calculations are needed for islanding cases as discussed in Section III-B

III. Reduction to Specified Area

Tate and Overbye's method applies to the entire network. We now consider implementing the algorithm from the point of view of a specified area of the system, where only that part of the system is observed. This fits the perspective of an entity such as a utility or balancing authority, which has control over only part of a larger system. The observable area must take into account the effects from the other areas it connects to. Here we assume no knowledge of outside areas except measuring the power flows in the tie lines connecting the outside areas to the observed area. In order to illustrate the method, we make the following simplifying assumptions

- only synchrophasor data from the area is available
- there was only one line outage
- power flows of all tie lines to the area are observable
- all of the area is connected (i.e. not multiple islands) before the outage

With these assumptions, we aim to model the area as if it was isolated from the rest of the network.

The DC load flow (1) gives a linear relationship between changes of angles and changes of power injections. Hence we can relate the changes in phasor angles using a superposition of the relevant power injections. The area's observed change in phasor angles after a line outage, $\Delta\theta_{obs}$, can be broken into

[1] $\mathbf{0}$ is a column vector of all zeros with the number of components chosen to fit the context.

[2] This definition reformulates and simplifies the definition of [1] using the following result: For real vectors a and b, the sign of $a \cdot b$ determines the minimum of $\|a+b\|$ and $\|a-b\|$. Namely

$$\mathrm{Min}\,\{\|a+b\|, \|a-b\|\} = \begin{cases} \|a-b\|, & a \cdot b \geq 0 \\ \|a+b\|, & a \cdot b < 0 \end{cases}$$

The result can easily be proved by noting that

$$\|a+b\|^2 - \|a-b\|^2 = (a+b)\cdot(a+b) - (a-b)\cdot(a-b) = 4a \cdot b$$

two components:

$$\Delta\theta_{obs} = \Delta\theta_{area} + \Delta\theta_{into}. \tag{8}$$

Here $\Delta\theta_{area}$ is the change in phasor angles due to system changes within the specified area assuming the area was isolated, and $\Delta\theta_{into}$ is the change in phasor angles due to changes in power flow into the specified area through its tie lines. $\Delta\theta_{obs}$ is obtained from the synchrophasor measurements observed before and after the outage.

Since we observe all tie line flows, we also have the change in power flows into the area. We view these changes as power injections at the border buses. P_{into} is the vector of power injections representing the tie line flows into the area, which is only non-zero at border buses. Following the structure of (1), we have

$$\Delta\theta_{into} = B^{-1}\Delta P_{into}. \tag{9}$$

Note that even if there is no generation redispatch and the net power flow into the area remains the same, a line outage will generally change the power flow distribution in the tie lines. These changes in power flows into the area will affect voltage angles throughout the area. In order to recognize the angle changes only due to the area's line outage, $\Delta\theta_{into}$ is subtracted, leaving only the changes within the area

$$\Delta\theta_{area} = \Delta\theta_{obs} - \Delta\theta_{into}. \tag{10}$$

Removing the effects of the change in power flows into the area allows us to treat the observed area as an isolated area without need of information from inside the neighboring areas. This processing of the synchrophasor measurements so that they effectively decouple the area from the rest of the network is discussed in detail in [2, Chapter 2].

It is convenient to express all the angles relative to the same reference bus in the area, and the reference bus should be one of the area buses with phasor measurements. In reference-shifting, a constant value is added to all the angles so that the reference bus angle is zero. It follows that changes in angles at the reference bus are also zero, and that the corresponding component of all the vectors of angle changes in (10) will be zero.

The same basic steps are followed for all tested line outages, but there are differences in the calculations depending on whether the tested outage is non-islanding or islanding. Thus, we must first evaluate if a particular line outage would island the area. This is done by removing the line from the set of lines in the area and analyzing the buses that remain connected. Each set of connected buses is a separate island. If there is more than one such set of connected buses, that means the area has disconnected into separate islands after the line outage.

We mention that line outages are also detected in some state estimation methods by calculating the effect of each of all the single line outages, but different measures of fit are used; this approach derives from the classic paper [3].

A. Non-Islanding Outage

We begin with non-islanding outages. As the system stays connected, there is no generation redispatch. The outaged line's removal from the system is the only change within the area itself. The angles change only due to removing the line, and we obtain $\Delta\theta_{outage}$ as

$$\Delta\theta_{outage} = \Delta\theta_{area} \tag{11}$$

where we know $\Delta\theta_{area}$ from (10).

As discussed in Section II, the outages are modeled using equal and opposite power injections. Following (6), $\widetilde{\Delta\theta}_{calc,l}$ is

$$\widetilde{\Delta\theta}_{calc,l} \quad = \quad KB^{-1}\begin{bmatrix} \mathbf{0} \\ 1 \\ \mathbf{0} \\ -1 \\ \mathbf{0} \end{bmatrix} \tag{12}$$

We define reference-shifting as

$$\bar{\theta} = \theta - \theta^{\text{ref}} \tag{13}$$

where θ^{ref} is the angle at the reference bus, which is chosen to be one of the buses in the area with phasor measurements. We express all angles with respect to the same reference bus θ^{ref}.

After reference-shifting, the normalized vectors are used to compare the shape of the calculated angle change vector to that of the observed angle change vector. That is, we compare the reference-shifted, normalized angles to get the NAD for each line l in the area using

$$\text{NAD}_l = \begin{cases} \|\widehat{\overline{\Delta\theta}}_{outage} - \widehat{\overline{\widetilde{\Delta\theta}}}_{calc,l}\|, & \overline{\Delta\theta}_{outage} \cdot \overline{\widetilde{\Delta\theta}}_{calc,l} \geq 0 \\ \|\widehat{\overline{\Delta\theta}}_{outage} + \widehat{\overline{\widetilde{\Delta\theta}}}_{calc,l}\|, & \overline{\Delta\theta}_{outage} \cdot \overline{\widetilde{\Delta\theta}}_{calc,l} < 0 \end{cases} \tag{14}$$

B. Islanding Outage

Since the area is assumed to be initially connected, a single line outage islanding the area must produce exactly two islands, I_1 and I_2. The islanding line must be the only line which connects I_1 and I_2.

The method is more complicated when a line outage islands the area, and a different approach to modeling the line outage is needed. To accurately estimate the change in the bus phase angles in such a case, two factors must be taken into account

Redispatch: The islanding line outage generally results in an imbalance in generation and load in each island. If there is no load shed, the steady-state reached after the outage must have resolved these imbalances by generation redispatch.

Island Referencing: Due to the islanding, buses on the two islands can vary independently, but the bus phase angles are still related to each other within each island and a reference bus for each island is needed.

To estimate the correct angle relations for an islanding outage we determine the redispatch effects and then shift the angles according to each island's reference.

We note that the islanding considered here is islanding of the area considered separately from the entire network. When the area is islanded, it is possible that the entire network is islanded or that the entire network remains connected. If the entire network remains connected, the generation is not redispatched because the generation imbalance in the islands is supplied from the rest of the network through the area tie lines.

1) Line Outage Modeling: The islanding case cannot model the line outage using the method of the connected case [6], [7]. (Since the islanding line l must be the only line which connects the two areas, its power transfer distribution factor for the equal and opposite power injections at the ends of the line is $PTDF_{l,l_{to}-l_{from}} = -1$ and the required power injection $\tilde{P}_l$ in (5) becomes infinite.)

When the line outages, there are three components to the changes in the overall power balance in each island. First, there is the change in tie line flows, ΔP_{into}, as previously discussed. Secondly, the outage of the line changes the overall island power balance by the amount of power flow on the line P_l, requiring a redispatch of that magnitude in each island. We refer to this portion of the redispatch as ΔP_{outage}. Lastly, the change in the net power entering each island along the tie lines creates an additional imbalance requiring another component of generation redispatch, ΔP_{in}. These three elements make up the power injections to model an islanding line outage in the subnetwork:

$$\Delta P = \Delta P_{into} + \Delta P_{outage} + \Delta P_{in}. \tag{15}$$

The rest of the subsection specifies ΔP_{outage} and ΔP_{in} in more detail.

We write Δp_1 for the total change in the power entering island 1 and Δp_2 for the total change in the power entering island 2; these are calculated by summing the power flows entering each island along tie lines:

$$\Delta p_I = \sum_{\substack{j \text{ in border} \\ \text{of island } I}} \Delta P_{into,j}, \quad I = 1, 2. \tag{16}$$

It is convenient in the explanation that follows to assume that the orientation of line l is chosen so that the "from" bus of line l is in area 2. Before the islanding outage, the islanding line has power flow P_l that is transferring power from area 2 to area 1. After the line outage, there is a deficit of power $P_l - \Delta p_1$ in area 1 and a surplus of power $P_l + \Delta p_2$ in area 2. To reach steady state, there must be power redispatch in each area to restore balance. Let g_1 be the total generation in island 1 and g_2 the total generation in island 2. Then the required changes in total generation in each island are

$$\Delta g_1 = P_l - \Delta p_1 \tag{17}$$

$$\Delta g_2 = -P_l - \Delta p_2, \tag{18}$$

Implementing the generation redispatch will balance the power flow in each island. Moreover, in the network model of the area that includes the outaged line, the generator redispatch causes the line flow to be zero. And the powers entering the "to" bus of line l from the lines of island 1 sum to zero and the powers entering the "from" bus of line l from the lines of island 2 sum to zero. Thus the redispatch correctly models the effect of the islanding line outage *inside* each island. However, the islands are no longer synchronized, and differences between an angle in one island and an angle in the other island are not well defined. This problem is addressed below by choosing angle references in each island.

To implement the redispatch in each island, it is both necessary and part of the modeling to define the participation of the generators in each island. Any specific participation can be assumed, but here for definiteness we assume that the generators in each island participate in the redispatch in proportion to their generation. Then, recalling that the total generation in island I is g_I, the participation of each area bus in the redispatch is given in the vector γ, where

$$(\gamma)_i = \begin{cases} (\text{generation at bus } i)/g_1, & \text{bus } i \text{ a generator in island 1} \\ -(\text{generation at bus } i)/g_2, & \text{bus } i \text{ a generator in island 2} \\ 0 & \text{otherwise} \end{cases} \tag{19}$$

Let $\Delta\vec{g}$ be the vector with entries Δg_1 corresponding to the buses of island 1 and $-\Delta g_2$ corresponding to the buses of island 2.

$$(\Delta\vec{g})_i = \begin{cases} \Delta g_1, & \text{bus } i \text{ in island 1} \\ -\Delta g_2, & \text{bus } i \text{ in island 2} \end{cases} \tag{20}$$

It is now notationally convenient to reorder the buses so that the buses in island 1 come first. Then[3]

$$\Delta\vec{g} = \begin{pmatrix} \Delta g_1 \mathbf{1} \\ -\Delta g_2 \mathbf{1} \end{pmatrix} \tag{21}$$

and

$$\gamma = \begin{pmatrix} \gamma_1 \\ -\gamma_2 \end{pmatrix}, \tag{22}$$

Then the power injections that specify the redispatch in the area buses are ΔP_{area} where

$$\Delta P_{area} = \begin{pmatrix} \Delta g_1 \gamma_1 \\ -\Delta g_2 \gamma_2 \end{pmatrix} = \Delta P_{in} + \Delta P_{outage} \tag{23}$$

where

$$\Delta P_{outage} = P_l \gamma \tag{24}$$

and

$$\Delta P_{in} = -\begin{pmatrix} \Delta p_1 \gamma_1 \\ \Delta p_2 \gamma_2 \end{pmatrix}. \tag{25}$$

[3] $\mathbf{1}$ is a column vector of all ones with the number of components chosen to fit the context.

2) Redispatch: Now we turn to the actual algorithm to detect the line outage in this case, first addressing the redispatch aspect of the outage. We turn again to (8), reformulating it similarly to the division in (23)

$$\begin{aligned} \Delta\theta_{obs} &= \Delta\theta_{area} + \Delta\theta_{into} \\ &= \Delta\theta_{in} + \Delta\theta_{outage} + \Delta\theta_{into}. \end{aligned} \tag{26}$$

$\Delta\theta_{outage}$ is the change in area angles due to power imbalance directly caused by the line outage. $\Delta\theta_{into}$ is known from the tie line power changes using (9). ΔP_{in} is known by summing the tie line power changes to obtain Δp, and then $\Delta\theta_{in} = KB^{-1}\Delta P_{in}$. We rearrange (26) to express the desired angle change component in terms of the changes in angles that can be obtained from the measurements.

$$\begin{aligned} \Delta\theta_{outage} &= \Delta\theta_{obs} - \Delta\theta_{into} - \Delta\theta_{in} \\ &= \Delta\theta_{obs} - KB^{-1}(\Delta P_{into} + \Delta P_{in}) \end{aligned} \tag{27}$$

$\Delta\theta_{outage}$ is to be compared to the calculated results from the line topology.

We write $\Delta P_{calc,l}$ for the calculated change in power injections directly caused by the outage of line l so that $\Delta P_{calc,l} = \Delta P_{outage}$ as given in (24). We use (24) to reformulate (6) for the islanding case

$$\Delta\theta_{calc,l} = KB^{-1}\Delta P_{calc,l} = P_l KB^{-1}\gamma_{calc,l} \tag{28}$$

which gives us

$$\widetilde{\Delta\theta}_{calc,l} = KB^{-1}\gamma_{calc,l}. \tag{29}$$

3) Island Referencing: $\Delta\theta_{outage}$ and $\widetilde{\Delta\theta}_{calc,l}$ must now be appropriately reference-shifted to compare their shapes. $\widetilde{\Delta\theta}_{calc,l}$ cannot be used exactly the same same way as the connected network case because the islands are decoupled. The phases of one island may shift with respect to the phases of the other island while maintaining their proper relation between phases within each island. Such shifts affect the normalization of overall system. To avoid any such shift between the observed angles and the calculated angles, we assign a reference bus for each island and express each angle in the area with respect to the reference bus angle for its island. We indicate the shifting of the angles with respect to the appropriate reference by an overbar:

$$(\bar{\theta})_i = \begin{cases} \theta_i - \theta_1^{\text{ref}}, & \text{bus } i \text{ in island 1} \\ \theta_i - \theta_2^{\text{ref}}, & \text{bus } i \text{ in island 2} \end{cases} \tag{30}$$

where θ_I^{ref} is the reference bus angle of island I. The shifting of (30) is applied to $\Delta\theta_{calc,l}$ and $\Delta\theta_{outage}$. Then the NAD of the outage of islanding line l can be calculated as

$$\text{NAD}_l = \begin{cases} \|\widehat{\overline{\Delta\theta}}_{outage} - \widehat{\overline{\widetilde{\Delta\theta}}}_{calc,l}\|, & \overline{\Delta\theta}_{outage} \cdot \overline{\widetilde{\Delta\theta}}_{calc,l} \geq 0 \\ \|\widehat{\overline{\Delta\theta}}_{outage} + \widehat{\overline{\widetilde{\Delta\theta}}}_{calc,l}\|, & \overline{\Delta\theta}_{outage} \cdot \overline{\widetilde{\Delta\theta}}_{calc,l} < 0 \end{cases} \tag{31}$$

C. Example

We test the implementation of the algorithm with computer calculations using Mathematica. A reduced New England system DC load flow model with 39 buses is used as illustrated in Fig. 1. The bolded area in the figure is selected for study due to the different categories of line outages available within the area. We illustrate the dependency of the algorithm on the synchrophasor bus selection by fixing the number of synchrophasors in the area to 5 and comparing the results of two selections of sets of 5 synchrophasors.

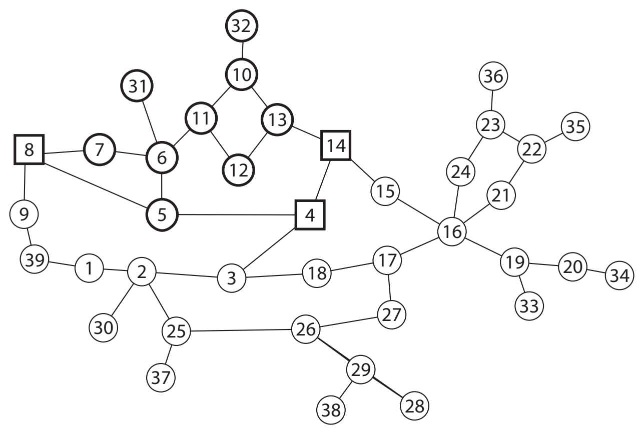

Fig. 1. 39 bus New England system. Area buses are shown with bold circles and squares.

The results of the algorithm in simulations of all 14 possible line outages within the area are listed in Table I. For synchrophasor Set 1, we have synchrophasors at buses 4, 8, 14, 31, and 32. With this synchrophasor set, the algorithm specified the exact line outage for every line except lines 6-7, 7-8, 12-11, and 12-13. With synchrophasor Set 2 that has synchrophasors at 4, 8, 14, 12, and 32, we have the same results as the synchrophasor Set 1 except that the ambiguities for the outage of lines 12-11 and 12-13 disappear, while gaining an ambiguity for the outage of 31-6.

To explain the ambiguities, we begin by focusing on the difference between the results of the synchrophasor sets. By simply moving a synchrophasor from bus 31 to bus 12, we remove ambiguity from two line outage cases and added an ambiguity to one line outage case. For the observed area, there are only two generator buses, 31 and 32, and they are both at the end of radial lines. The outage of one of the radial lines will island generation, necessitating the redispatch of the other. From the point of view of the rest of the system, the loss of one of these radial lines will be a power loss where it connected to the rest of the system, but there will also be a power injection at the connection of the other radial line due to the redispatch of the other generator to make up for the power loss.

The outage of a radial line will cause the bus at the end of it to become a disconnected island with one isolated bus. Only

TABLE I
PREDICTED OUTAGES

Actual Outage	Synchrophasor Bus Set 1: 4, 8, 14, 31, 32	Synchrophasor Bus Set 2: 4, 8, 14, 12, 32
4-5	4-5	4-5
4-14	4-14	4-14
5-6	5-6	5-6
5-8	5-8	5-8
6-7	6-7, 7-8	6-7, 7-8
6-11	6-11	6-11
7-8	6-7, 7-8	6-7, 7-8
10-11	10-11	10-11
10-13	10-13	10-13
13-14	13-14	13-14
31-6	31-6	31-6, 10-32
10-32	10-32	10-32
12-11	12-11, 12-13	12-11
12-13	12-11, 12-13	12-13

the differences in phasor angles between buses in an island have value, hence a lone phasor angle gives no information about that island and we can detect no error from it. However, if we have a synchrophasor at the other radial line's bus, we can detect the error from the redispatch as it will generally not fit the redispatch if it were disconnected from the system. This is why synchrophasor Set 2, which has no synchrophasor at bus 31, cannot detect the error from the outage of line 31-6.

The ambiguity between lines 12-11 and 12-13 for synchrophasor Set 1 is due to the lines being in series. Viewing a line outage as a pair of equal and opposite power injections at the ends of the line, we see that a line outage of either 12-11 or 12-13 will be a pair of such power injections between the series pair of lines between buses 11 and 13. From the point of view of the system outside the two lines, these power injections will be equivalent to power injections at the ends of the two lines at buses 11 and 13. This is because the pair of lines has a set load which is balanced before the outage with a certain power flow from outside the pair of lines. After the pair of injections are added, the system outside the pair of lines is still balanced with the same flows as before, but is now superimposed on the flows from the added pair of injections from the two lines in series. The magnitude of the power injections may differ depending on the line, but because they are equal and opposite, they have proportional effects from the point of view of the system outside the pair of lines. Hence, synchrophasor Set 1 cannot distinguish between the outage of line 12-11 and the outage of line 12-13. On the other hand, synchrophasor Set 2 has a synchrophasor at bus 12 within the series of lines, which allows the algorithm to distinguish the change in angles within the pair of lines. Note that the ambiguity of lines 6-7 and 7-8 is of the same type.

IV. Acknowledgements

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V. Conclusions

It is often convenient and pragmatic to work with models and measurements within a specific power system area inside a larger grid interconnection. We have shown how synchrophasor measurements of voltage and current around the border of an area can be used together with synchrophasor voltage measurements inside the area and a DC load flow model of the area to detect single line outages in the area and discriminate which line outaged. The method extends Tate and Overbye's method of line outage identification [1] to apply to a specific area.

We also give a way to model the effect of line outages that island the area, since the usual modeling of line outages by power injections at the ends of the line does not work for the islanding case. Hence we are able to also discriminate these islanding line outages with synchrophasor measurements.

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