Transmission and Generation Expansion to Mitigate Seismic Risk

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Abstract—This paper develops a two-stage stochastic program and solution procedure to optimize the selection of capacity enhancement strategies to increase the resilience of electric power systems to earthquakes. The model explicitly considers the range of earthquake events that are possible and, for each, an approximation of the distribution of damage to be experienced. This is important because electric power systems are spatially distributed; hence their performance is driven by the distribution of damage to the components. We test this solution procedure against the nonlinear integer solver in LINGO 13 and apply the formulation and solution strategy to the Eastern Interconnect where the seismic hazard primarily stems from the New Madrid Seismic Zone. We show the feasibility of optimized capacity expansion to improve the resilience of large-scale power systems with respect to large earthquakes.

Index Terms—power transmission planning, power generation planning, strategic planning, earthquakes, optimization methods, systems engineering.

I. INTRODUCTION

Earthquakes pose a significant risk to electric power systems as illustrated by a range of recent events. For example, on January 17, 1994 the Northridge earthquake struck the city of Los Angeles and surrounding areas. Two and a half million customers lost power [1]. “Electricity was restored to virtually all customers within one week of the event” [2]. The Great Hanshin earthquake occurred a year later affecting Kobe, Japan. Twenty fossil-fire power generation units, six 275 kV substations, and two 154 kV substations were damaged. Approximately, 2.6 million customers were affected by outages [3]. Electricity was restored 7 days after the earthquake [4]. On May 18, 2008, the Wenchuan earthquake caused extensive damage to the local power transmission and distribution systems in Sichuan province, China. Approximately 900 substations and 270 transmission lines of the State Power Grid were damaged. It has been estimated that at least 90% of the damage could have been avoided by adopting new guidelines for seismic design [5]. One year after the earthquake electricity had been restored to virtually all permanent residences with only about 5% of individuals living in temporary houses and 2% of those living in tents with no access [6]. 90% of Chileans did not have electricity immediately following the February 27, 2010 8.8 Mw earthquake. The event caused the largest power transmission company in Chile to have direct losses of approximately US $ 6.5 billion [7]. 90% of the load was restored within the first week but it took several weeks to completely restore electricity supply [8]. 3,000 MW of generation capacity were unavailable right after the earthquake; it took about 30 days to restore another 2,257 MW of generation capacity and about 6 months for the other 693 MW to be restored. Restoring the remaining transmission capacity took several more months and only in the last stage of restoration, which involved restoring system reliability, the availability of spare parts, presented difficulties [8]. The devastating Tohoku Chiho – Taiheiyo-Oki earthquake on March 11, 2011 damaged 14 power plants, 70 transformers, and 42 transmission towers, among other failures. Outages stemming from the event affected 4.6 million residences and the April 7 aftershock affected an additional 4 million [9].

The objective of this paper is to develop a model to optimize the selection of mitigation strategies in the electric power system to control the consequences of earthquake events. These mitigation strategies include investments in additional transmission and generation margin to the system. It is important to realize that traditional research on mitigation against earthquake hazard only considers anchoring and reinforcement of electric power. Vanzi found optimized structural upgrading strategies for electric power networks using a new index to choose among critical nodes in the network [10]. The method was tested using a representation of the Sicily, Italy power network, which includes 181 nodes and 220 lines. Shumuta focused on upgrading substation equipment [11]. He evaluated the criticality of components with 4 indexes; two indexes represent earthquake resistance, the third index focuses on seismic performance, and the fourth index is cost-based. This method was tested on a hypothetical electric power system with 16 substations, located in the Nagoya region, Japan.

Transmission expansion planning for power networks includes a variety of approaches with exact or heuristic solution procedures, and for static or dynamic planning horizons. An extensive literature review can be found in Latorre et al. [12]. Work in this area does not specifically consider the benefits under seismic risk and normally applies to small scale networks. Samarakoon et al. used a mixed
integer linear programming model to solve the transmission and generation expansion problem [13]. Prior to the model application, they identified all possible single circuit outages associated with all possible expansion investments. The critical contingencies (possible single circuit outages) are included in the model as constraints. The method is tested on the Sri Lankan power network, a 38 node model, with 8 new nodes and 19 new branches considered among the expansion options. Alguacil et al. used a revised mixed integer linear formulation of the static transmission expansion problem that is computationally efficient using conventional solvers [14]. Both Samarakoon et al. [13] and Alguacil et al. [14] use a “disjunctive parameter to allow enough degrees of freedom for the voltage angle difference between every disconnected node” [14]. They [14] tested their model in the One Area IEEE reliability test system [15] which has 24 nodes and 38 links. With the same problem formulation, Carrión et al. presented a mixed-integer linear programming solution to expand the transmission network in order to reduce its vulnerability to intentional attacks [16]. This formulation includes not only the enhancement of current lines but the development of new lines, which is achieved by including prospective transmission lines with an initial capacity of zero. Carrión et al. [16] tested their methodology on the Two Area IEEE reliability test system [15]; the test system has 48 nodes and 79 lines, and the model considered 20 prospective new lines; this solution procedure would not be applicable to large scale problems because the resultant mixed-integer linear problem is currently too computationally intensive if approached directly. Georgilakis presented an improved differential evolution solution to the transmission expansion problem [17]. The methodology uses a reference network subproblem which is topologically identical to the expanded network, and with generation and load unchanged. The reference network subproblem is used to find the optimal capacities of transmission lines; this subproblem includes a set of contingency scenarios as constraints of the individual load flow problem. The optimum network problem is the same as the subproblem with the additional difficulty of having to choose the lines to add to the network. Georgilakis [17] tested the methodology in a 30-bus IEEE reliability test system with the addition of 9 prospective transmission lines. Aguado et al. recently presented a transmission expansion model that explicitly considers a multi-year planning horizon [18]. The model is formulated as a mixed-integer linear problem and the method is tested on a 6 bus system and applied to the transmission system of mainland Spain, which includes 86 buses and 168 circuits.

This paper focuses on transmission and generation expansion as a way to mitigate seismic risk, which can enhance traditional seismic mitigation strategies, such as structural reinforcement.

The contribution of this paper is two fold. First, this is the first model to optimize capacity enhancement opportunities for seismic mitigation. Second, this is the first solution procedure for the optimization of transmission system capacity expansion that can be applied to very large problem instances. We focus on the Eastern Interconnect Power Grid (EI) in its entirety; hence the problem instances have about 15,000 nodes and 23,000 arcs. This is about two orders of magnitude larger than the biggest network found in the literature.

Our model and solution procedure is illustrated through its application to the EI to investigate opportunities to mitigate the risks generated by the New Madrid Seismic Zone (NMSZ). The power network used is a 1998 representation of the EI. We model the NMSZ risk using HAZUS’ regional loss estimation method vulnerability and repair data for generic components [19]. The performance of the power grid is evaluated using a dc load flow economic dispatch model, which estimates the load shed at each node on the grid.

We implement a knapsack-based heuristic to solve the nonlinear integer programming problem (NLIP) to optimize the selection of mitigation strategies for electric power system components. To model the seismic risk, we use a suite of earthquake scenarios that nearly replicates the exceedance curves for peak ground acceleration (PGA) as measured at 81 control locations across the NMSZ. Since the electric power system is a spatially distributed system, we create a suite of consequence scenarios for each earthquake scenario where each consequence scenario identifies the resulting damage state of each component. Once the damage state of a component is known, the expected time required for the component to be operational again used in the model is as given in HAZUS. The construction of these consequence scenarios provides an implicit representation of the distribution of damage for each earthquake scenario. The damage to the power grid considered is limited to transmission lines and substations. The input data to the model does not capture additional damage from local site conditions, or specific structural vulnerability of each component. An important future contribution to the quality of the recommendations given by a tool like this is the inclusion of detailed soil information for study area and structural design of the components located in critical areas.

As mentioned previously, the operation of the power grid is modeled using an economic dispatch model and it is assumed that the operator of the network has a limited budget to invest in mitigating the risk. The same formulation and solution approach is applied to a significantly smaller problem instance so that the solution approach can be tested against the commercial solver Lingo 13. The results using Lingo and our proposed heuristic are compared for different mitigation budgets to gain a sense of the performance of the heuristic. This is done because the EI problem is too large for a commercial solver.

This paper is divided into four additional sections: formulation, solution procedure, case study, and conclusions. The solution procedure section includes a comparison of the performance of the solution procedure developed to LINGO 13, a commercial solver, for a significantly simplified problem instance. The case study section presents the outcome of the application of the tools developed in the formulation and solution procedure to the EI. The conclusions summarize the key elements of the paper and next steps for future research.
II. FORMULATION

The key question addressed by the formulation is how to optimally invest in capacity enhancement to mitigate seismic risk on power networks. We measure the performance of the power network as the sum of the power generating costs and load shed costs under a set of consequence scenarios, that model the seismic hazard in the region and vulnerability of the network to that hazard.

The electric power transmission system investment planning model is formulated as a two-stage stochastic program. A two-stage stochastic program is an optimization model formulation that incorporates uncertainty in the parameters of the model. The two-stage structure assumes that all decisions are made at one time instant prior to the resolution of all uncertainty. In this case, the uncertainty revolves around what damage will occur to each component in the electric power system. This uncertainty is expressed through the use of a number of consequence scenarios, where each consequence scenario gives the damage to each component. The decisions are made in what is termed the “first-stage” of the model. In our formulation, the first-stage is the identification of what components in the electric power system should be enhanced. The consequences of those decisions, under each consequence scenario, occur in the “second-stage” of the model. In this problem formulation, the second-stage is the power flow across each component including what demands for power are not satisfied under each consequence scenario.

We first introduce the topology of the power network. Let $\Pi$ be the set of transmission lines. Let $S$ be the set of substations. Let $G$ be the set of generators. Let $B$ be the set of buses. Let $I(i)\ be the set of generators connected to bus $i$. We define the first-stage binary decision variables as follows. Let $z_g$ take integer values 0, 1 or 2 representing the number of capacity increments to add to the capacity of power generator $g$. The cost to add a discrete increment $\mu$ to the existing capacity of generator $g$ is $c_{\mu}$. Let $w_{ij}$ take integer values from 0 to 4 representing the number of increments to add to the transmission capacity for line $(i,j)$. The cost to add a discrete capacity increment $\rho$ to the existing capacity of transmission line $(i,j)$ is $h_{ij}$. We assume that the total available budget for transmission capacity and power generation enhancement is $M^C$. Since the total investments to add capacity to the components cannot exceed the available budget, then equation (1) must hold.

$$\sum_{(i,j) \in \Pi} h_{ij} w_{ij} + \sum_{g \in G} c_{\mu} z_{g} \leq M^C$$

In practice, transmission line and generation enhancement increment units and maximums are specific to each component. When available, the proposed model can be easily adjusted to include the specific incremental units for each component. In the absence of the exact information we propose to use the total component's capacity as reference to the incremental unit. The percentage of the capacity and maximum capacity to add for transmission lines and generators where selected to reflect the sense that there are more variations in the transmission line upgrade. Transmission line enhancement is modeled as discrete increments of a quarter of total original capacity of the line. Generation enhancement is modeled as discrete increments of a fifth of initial capacity. The cost of the enhancement is modeled as a percentage of the total cost of the line or generator. Capital costs and operational unit cost of generation unit were obtained from [20]. Line capacity enhancement costs were extracted from [21]. All the costs were converted to 2002 U.S. dollars and adjusted to make them consistent among sources. Requiring the enhancements to be discrete is more widely representative of practically feasible upgrades such as reconductoring, changes in operational limits with improved coordination or controls, or adding another generating unit.

Based on the HAZUS seismic risk assessment methodology [19], five damage states are defined for electric power components: none, minor, moderate, extensive and complete. Of those five, we disregard minor because the damage is not significant in this context. Moderate damage generates a repair cost of 40% of substation cost and does not affect any of the transformers in the substation. Extensive damage is assumed to imply damages costing 70% of the value of the substation including impacting 50% of the transformers in the substation. Complete damage causes the complete loss of the substation including all the transformers. HAZUS presents restoration curves for substations, lines and generators for different damage stages [19]. The mean time for repair is 3 days for moderate damage and a week for extensive damage. For substation restoration under complete damage, the mean value is 30 days; however, repairs can vary depending on the difficulty with which some components such as transformers can be replaced. We assume that the average lead-time for medium and high voltage transformers is 6 months. For low voltage transformers, we assume that the operator would have access to spares within a month. Therefore, all the components in substations under complete damage are back to normal within a month with the exception of medium and high voltage transformers which is 6 months. For transmission lines we only model two levels of damage: extensive and complete. Extensive damage for a transmission line corresponds to a damage ratio of 50% of the total cost of the line and complete damage results in costs totaling the full cost of the line. Transmission lines under extensive damage can be repaired within 3 days and under complete damage within a week. This implies that by the end of 6 months, in the worst case, the system is back to normal. The analysis focuses on damage to transformers not damage to other substation components because transformers typically drive the restoration process due to their long repair lead-times. The analysis does not include damage to generators because they generally perform better than the rest of components in power networks.

From a modeling perspective, this implies that the repair process is composed of 4 time periods. The first period extends from the event to the end of the third day. By then transmission lines that have experienced extensive damage have been restored. Also, substations under moderate damage have been repaired. The second time period extends from the beginning of day four to the end of the first week. By then
transmission lines that have experienced complete damage have been repaired as well as substations under extensive damage. The third time period extends from the end of the first week to the end of the first month. By the end of this time period, low voltage transformers will have been replaced. The final time period extends from one month to six months. Six months after the event, medium and large voltage transformers will have been replaced. The following notation encapsulates these time period definitions. Let $t_0 = 0$, $t_1 = 3$ days, $t_2 = 1$ week, $t_3 = 1$ month, and $t_4 = 6$ months, then $t_k - t_{k-1}$ is the time length in days of period $k$ for $k = 1, 2, 3, 4$.

It is important to observe that the uncertainty in the repair times could have been integrated into the scenario definitions. This would likely lead to more than four time periods. For simplicity this was not done. However, the inclusion of this uncertainty is likely to prove useful and is therefore a worthy subject for future study.

We assume that there are $N$ earthquake consequence scenarios, i.e., $n = 1, \ldots, N$. The associated annual probability of scenario $n$ is $Pr(n)$. Let $c^l$ be the per unit load shed cost. Let $c^G$ be the per unit power generation cost of generator $g$. Note that the first-stage reinforcement decisions and the earthquake scenario determine the level of damage in the component; hence, the length of time from the earthquake that the component is unavailable is known. Let $\Delta^k_s = 1$ if substation $s$ is not functional in period $k$ under earthquake scenario $n$ and $\Delta^k_s = 0$ otherwise. Similarly, let $\Lambda^k_{ij} = 1$ if transmission line $(i,j)$ is not functional in period $k$ under scenario $n$ and $\Lambda^k_{ij} = 0$ otherwise.

Now we define the second-stage decision variables. Let $\theta^k_i$ be the voltage phase angle in bus $i$ and time period $k$ under scenario $n$. Let $P^k_{ij}$ be the real power flow in transmission line $(i,j)$ in period $k$ under scenario $n$. Since the electric flows can go in both directions, $P^k_{ij}$ can be positive or negative. Let $G^k_{g}$ be the nonnegative generation output from generator $g$ in period $k$ under scenario $n$. Let $U^k_{ij}$ be the nonnegative load shed in bus $i$ in period $k$ under scenario $n$.

Let $m_{ij}$ be the reactance of transmission line $(i,j)$. Let $T_{ij}$ be an indicator parameter with value 1 when a spare transformer for transmission line $(i,j)$ can be obtained “quickly”, which is defined as on the order of a month or less, and 0 otherwise.

$$
(\theta^1_i - \theta^1_j) \left(1 - \Lambda^1_{ij}\right) \left(1 - \Delta^1_{ij}\right) \times \\
\left(1 + \mu_{w_{ij}}\right) = m_{ij} P^{a1}_{ij}, \forall (i,j), n
$$

(2)

$$
(\theta^2_i - \theta^2_j) \left(1 - \Lambda^2_{ij}\right) \left(1 - \Delta^2_{ij}\right) \times \\
\left(1 + \mu_{w_{ij}}\right) = m_{ij} P^{a2}_{ij}, \forall (i,j), n
$$

(3)

$$
(\theta^3_i - \theta^3_j) \left(1 - \Lambda^3_{ij}\right) \left(1 - \Delta^3_{ij}\right) \times \\
\left(1 + \mu_{w_{ij}}\right) = m_{ij} P^{a3}_{ij}, \forall (i,j), n
$$

(4)

Constraints (2), (3), (4), and (5) approximate the active power flows on the transmission lines in the four periods of the repair process.

$$
\sum_{g \in ^G_i(i)} G^k_{g} - \sum_{(i,j) \in ^D(i)} P^k_{ij} + \sum_{(i,j) \in ^C(i)} P^k_{ij} = D_i - U^k_{ij}, \forall i,k,n
$$

(6)

If the per day demand at bus $i$ is $D_i$, and $\delta^i$ is the set of the transmission lines such that $(i,j) \in \delta^i$ is the set of transmission lines such that $(j,i) \in \Pi$, then (6) state flow conservation at each bus under each earthquake scenario.

The load shed at a bus cannot exceed the demand at the bus.

$$
0 \leq U^k_{ij} \leq D_i, \forall i,k,n
$$

(7)

$$
0 \leq G^k_{g} \leq G^\mu \left(1 + \mu z_{g}\right), \forall g,k,n
$$

(8)

$$
\left| P^{a1}_{ij} \right| \leq P^m_{ij} \times \\
\left(1 + \mu_{w_{ij}}\right) \left(1 - \Lambda^1_{ij}\right) \left(1 - \Delta^1_{ij}\right), \forall (i,j), n
$$

(9)

$$
\left| P^{a2}_{ij} \right| \leq P^m_{ij} \times \\
\left(1 + \mu_{w_{ij}}\right) \left(1 - \Lambda^2_{ij}\right) \left(1 - \Delta^2_{ij}\right), \forall (i,j), n
$$

(10)

$$
\left| P^{a3}_{ij} \right| \leq P^m_{ij} \left(1 + \mu_{w_{ij}}\right) \left(1 - \Lambda^3_{ij}\right) \left(1 - \Delta^3_{ij}\right), \forall (i,j), n
$$

(11)

$$
\left| P^{a4}_{ij} \right| \leq P^m_{ij} \left(1 + \mu_{w_{ij}}\right) \left(1 - \Lambda^4_{ij}\right) \left(1 - \Delta^4_{ij}\right) \left(1 - T_{ij}\right), \forall (i,j), n
$$

(12)

where $s_i$ is the substation to which bus $i$ belongs.

We assume that generator $g$ has capacity $G^m_{g}$ and transmission line $(i,j)$ has capacity $P^m_{ij}$. Equations (8)-(12) reflect the capacity constraints in each generator and each transmission line in each time period under each earthquake scenario. Notice that as stated before, in (9)-(12) the flow in a line goes to zero when it is connected to a non-operational component for a given scenario and time period.

Constraints (9), (10), (11), and (12) operate in conjunction with (2), (3), (4), and (5). Constraint (9), for example, focuses on the first time period after the event and says that a line is unavailable and hence its associated power flow is zero if the line itself is unavailable and/or the substations at either end of the line are unavailable. If none of these conditions are true, that is, the line itself is available as well as the substations at both ends, then the power flow can be nonzero. Similarly, in the fourth time period, as represented by (12), only high voltage and customized transformers are assumed not to have been repaired and hence only those components, if they were damaged in the event, have power flows that are constrained to be zero. When the power flow is constrained be zero in (9)
through (12), it is important that the phase angle constraints given in (2) through (5) are removed from the optimization. Therefore, the terms that indicate which components are no longer available that impact each line are included in the left-hand sides of (2) through (5). It is important to also notice that the left-hand side of (2) through (5) includes the term \((1 + \rho W_{ij})\). This is done to rescale the reactance as capacity is added to lines.

The objective function of the two-stage stochastic program is to minimize the expected generation, load shed and repair costs in the four recovery periods as given in (13).

\[
\sum_{n=1}^{N} \Pr(n) \sum_{k=1,2,3,4} (t_k - t_{k-1}) \left( \sum_{i \in B} c_i^B U_{i nk} + \sum_{g \in G} c_g^G G_{g nk} \right)
\]  

(13)

Note that the two-stage stochastic program (1) – (13) is a nonlinear mixed integer stochastic program.

### III. SOLUTION PROCEDURE

This is a two-stage mixed integer nonlinear stochastic program for which realistic instances will be very large (on the order of many hundreds to thousands of integer variables, each with small values) therefore; we develop a heuristic solution procedure. The key idea that underlies the heuristic is to construct a knapsack problem with a linear objective function so that the solution of the knapsack problem is also a good solution for capacity expansion.

The third step is to identify a subset of enhancements that maximizes the consistency of the recommendations from the previous step, with the budget constraint given in equation (1).

To reduce computational time and memory requirements, we split the problem by scenario \(n\) and time period \(k\). Let \(W_{nk}^k\) be a continuous variable representing the capacity enhancement of transmission line \((i,j)\) under scenario \(n\) and time period \(k\). In the same way, let \(z_{g nk}\) be a continuous variable representing the capacity enhancement of generator \(g\) under scenario \(n\) and time period \(k\). Notice there is no requirement that these variables be the same across scenarios and time periods. Of course these recommendations cannot be implemented directly. The heuristic will integrate these decisions together and draw conclusions that are implementable.

This is an iterative procedure, therefore we define \(M_{nk}^{*}\) as the budget that has been allocated in previous iterations. At the beginning of the procedure, this value is zero. Also, let \(l\) be the iteration number.

**Step i.** Initialize parameters. Let \(l = 0\), \(W_{nk}^l = 0\) for all transmission lines \((i,j)\), and \(z_{g nk}^l = 0\) for all generators \(g\). Also \(M_{nk}^{*} = 0\). Select the maximum budget, \(\overline{C}\), to be allocated in enhancement units per iteration. Also select the \(\overline{W}\) and \(\overline{Z}\), the upper bounds for \(w\) and \(z\). Notice that these constants place an upper limit of the investment decisions that can be made in each iteration.

**Step ii.** Run a linearized version of the dc load flow economic dispatch defined by (1) – (12) and objective function defined in (13) but decomposed by scenario and time period. We solve \(NK\) linear problems choosing variables \((P_{g nk}, G_{g nk}, U_{i nk}, W_{nk}^k, z_{g nk})\) that minimize

\[
\sum_{i \in B} c_i^B U_{i nk} + \sum_{g \in G} c_g^G G_{g nk}
\]  

subject to

\[
(\theta_i^{n1} - \theta_j^{n1}) \left(1 - \Lambda_{ij}^{n1}\right) \left(1 - \Delta_{ij}^{n1}\right) x_{ij} + (1 + \rho W_{ij}) = m_{ij} P_{ij}^{n1}, \quad \forall (i,j) \]  

(15)

\[
(\theta_i^{n2} - \theta_j^{n2}) \left(1 - \Lambda_{ij}^{n2}\right) \left(1 - \Delta_{ij}^{n2}\right) x_{ij} + (1 + \rho W_{ij}) = m_{ij} P_{ij}^{n2}, \quad \forall (i,j) \]  

(16)

\[
(\theta_i^{n3} - \theta_j^{n3}) \left(1 - \Lambda_{ij}^{n3}\right) \left(1 - \Delta_{ij}^{n3}\right) x_{ij} + (1 + \rho W_{ij}) = m_{ij} P_{ij}^{n3}, \quad \forall (i,j) \]  

(17)

\[
(\theta_i^{n4} - \theta_j^{n4}) \left(1 - \Lambda_{ij}^{n4}\right) \left(1 - T_{ij}\right) x_{ij} + (1 + \rho W_{ij}) = m_{ij} P_{ij}^{n4}, \quad \forall (i,j) \]  

(18)

\[
\sum_{g \in E(i)} G_{g nk} - \sum_{(i,j) \in B'(i)} P_{ij}^{nk} + \sum_{(i,j) \in B(i)} P_{ij}^{nk} = D_i - U_i^{nk},
\]  

\[
\sum_{g \in E(i)} G_{g nk} - \sum_{(i,j) \in B'(i)} P_{ij}^{nk} + \sum_{(i,j) \in B(i)} P_{ij}^{nk} = D_i - U_i^{nk},
\]

![Fig. 1. Iterative heuristic to select enhancement strategies](image-url)
maximum enhancement units for generators, $\bar{Z}$. This ratio scales the coefficients so there is balance between generator and transmission enhancements encouraging coordination. The enhancement coefficient in transmission line $(i,j)$ is defined as follows:

$$\beta_{ij}^C = \sum_{k=1,2,3,4} \Pr(n) \sum_{k=1,2,3,4} \left( \left( t_k - t_{k-1} \right) \left( w_{ij}^{nk} - w_{ij}^l \right) \right)$$

The enhancement coefficient in generator, $g$ is defined as follows:

$$\beta_{g}^C = \left( \frac{W}{Z} \right) \sum_{n=1}^N \Pr(n) \sum_{k=1,2,3,4} \left( \left( t_k - t_{k-1} \right) \left( z_{g}^{nk} - z_{g}^l \right) \right)$$

**Step iv.** Select a subset of the capacity enhancement strategy of the network. Run the integer program to determine binary variables $(\bar{w}, \bar{z})$ that maximize

$$\sum_{(i,j)\in\Pi} \beta_{ij}^C \bar{w}_{ij} + \sum_{g\in G} \beta_{g}^C \bar{z}_{g}$$

subject to

$$\sum_{(i,j)\in\Pi} h_{ij}^C \bar{w}_{ij} + \sum_{g\in G} o_{g}^C \bar{z}_{g} \leq M - M^*$$

$$\sum_{(i,j)\in\Pi} h_{ij}^C \bar{w}_{ij} + \sum_{g\in G} o_{g}^C \bar{z}_{g} \leq C$$

Let $(\bar{w}, \bar{z})$ be the solution.

This problem is formulated as a simple knapsack problem using $\beta_{ij}^C$ and $\beta_{g}^C$ as coefficients in the objective function for each $w_{ij}$ and $z_{g}$ respectively. There are two constraints to the problem, (32) represents the budget constraint for the iteration, and (33) the maximum number of selected enhancements per iteration. Notice that we only add one discrete unit of capacity per component per iteration; therefore, it is important that the number $C$, which limits budget allocated per iteration, is small enough in relation to the number of enhancements we can choose from the total possible for the available budget. Further, it is important to realize that $C$ must be larger than the cost of any feasible individual enhancement. For the EI case study, $C$ ranged from 10 to 50 million, depending on the total available budget.

**Step v.** Update the solutions and the budget as follows:

$$w_{ij}^{l+1} = w_{ij}^l + \bar{w}_{ij}$$

$$z_{g}^{l+1} = z_{g}^l + \bar{z}_{g}$$

$$M^* = \sum_{(i,j)\in\Pi} h_{ij}^C w_{ij}^{l+1} + \sum_{g\in G} o_{g}^C z_{g}^{l+1}$$

**Step vi.** Check stopping conditions: If $M^* - M^* > \varepsilon$, let $l = l + 1$ and go back to step ii. Otherwise, report the enhancement
solutions \(w, z = (w', z')\). In the presented case study, \(l\) ranged from 10 to 200.

We used the one area IEEE Reliability Test System (RTS) – 1996 [15] to test the solution procedure. We formulated the problem for a hypothetical case in which no seismic risk was modeled but the system was stressed by doubling the demands. The nonlinear integer programming problem (NLIP) was evaluated for 4 different budgets: US$50 million, US$100 million, US$500 million, and US$1 billion using two methods: the proposed heuristic coded in C++ with IBM ILOG Optimization Studio CPLEX 12.2 serving as an LP solver, and the full NLIP in Lingo 13 (using the global solver) in a Dell Precision T5500, Intel® Xeon® X5650 with 2 processors of 2.66 GHz., and 6.00 GB of total RAM memory. The four solutions are within a 0.1% error from the optimal solution found using Lingo. Lingo requires between 20 seconds to 3 minutes depending on the investment budget, and the heuristic, less than 20 seconds for all experiments.

IV. CASE STUDY

To demonstrate the formulation and illustrate insights from its application, we focus on questions of seismic mitigation of the EI under limited budget using illustrative data. The representation of the EI is a 2003 Summer Peak ECAR case as of 1998 with demands reflective of a prediction of the summer of 2003. This case includes direct representation of every region in the EI, which extends approximately from the Rocky Mountains to the East Coast excluding Texas. Detailed representation is for voltages greater than 100 kV. It includes information for 23,416 transmission lines and 14,957 buses. These buses are grouped in 2,765 substations with two or more buses and 6,448 single buses. Load shed, generation output, repair, and mitigation costs were estimated in 2002 U.S. dollars.

We only consider the seismic risk from the NMSZ. The hazard is modeled by a set of earthquake scenarios selected using the mathematical optimization method developed by Vaziri et al. [22]. The method assigns a probability to each scenario minimizing the discrepancy with seismic behavior as represented in the exceedance curves for PGA at discrete locations (control points) throughout the NMSZ. To do the selection of events from the candidate set of events as well as the probability identification for those events, we located 81 control points in the NMSZ area and obtained the PGA exceedance curves for each point from the U.S. Geological Survey (USGS) Seismic Hazard Maps [23]. Eight earthquake scenarios were found to form a reasonable approximation to the seismic risk.

Fig. 2 and Fig. 3 compare the exceedance curves for PGA from USGS and the estimated exceedance probability after selecting eight of scenarios for two different control points (from the set of 81). The difference between the two curves is the error. For PGA values larger than 0.10g, we compute the difference between the values provided by USGS and those that result from the scenarios selected. Fig. 4 shows a histogram of those values across all control points.

A key input to that optimization is the identification of the candidate set of earthquake events. We used two sources to create the candidate set: the earthquake catalog from the USGS website [24], and 20 synthetic scenarios created using code provided by USGS [25]. The earthquake catalog includes 433 earthquakes that occurred within the NMSZ. The magnitudes were converted from \(m_{blg}\) to \(M_w\) as described in Petersen et al. [26]. In addition to the 433 earthquakes identified in the Central-East Unites States earthquake scenarios catalog, we use 20 synthetic events on 5 synthetic faults created by USGS to represent the hazard in New Madrid. The 20 scenarios correspond to each of 4 possible magnitudes (7.3, 7.5, 7.7 and 8) for ruptures in the 5 different branches described in Petersen et al. [26]. USGS provides computer code that can be compiled and run to generate each of these deterministic scenarios [25].
Using the fragility curves in [19], we estimate the probability that each component sustains specific levels of damage under each scenario. The probability that each component sustains the different levels of damages is modeled as a set of consequence scenarios, each one with an adjusted occurrence probability. We use the optimization method introduced by Brown et al. [27] to develop the consequence scenarios and their hazard-consistent probability of occurrence. The objective of the optimization is to create consequence scenarios and their hazard-consistent probability of occurrence so the implied vulnerabilities of each component match the “true” (input) vulnerability (as given by the marginal distribution of damage) as closely as possible.

Of the eight earthquake scenarios that were found to approximate the seismic risk, only two scenarios resulted in considerable physical damage to the electric grid. For each of these events, we generate 6 consequence scenarios using the marginal distribution of damage for each component based on [19].

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Damage level probability (“True”/Estimated)</th>
<th>Consequence Scenario ID</th>
<th>Scenario probability</th>
<th>Damage level</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.08/0.92/0.14</td>
<td>1</td>
<td>0.16</td>
<td>Com.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>0.20</td>
<td>Com.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>0.08</td>
<td>Ext.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>0.18</td>
<td>Com.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5</td>
<td>0.24</td>
<td>Com.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6</td>
<td>0.14</td>
<td>Com.</td>
</tr>
<tr>
<td>2</td>
<td>0.02/0.98/1.00</td>
<td>7</td>
<td>0.07</td>
<td>Com.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8</td>
<td>0.22</td>
<td>Com.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9</td>
<td>0.16</td>
<td>Com.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10</td>
<td>0.19</td>
<td>Com.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11</td>
<td>0.24</td>
<td>Com.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12</td>
<td>0.12</td>
<td>Com.</td>
</tr>
</tbody>
</table>

Table I shows the probability that a substation near New Madrid, Missouri has moderate, extensive, or complete damage for the two earthquake scenarios that resulted in considerable damage to the grid. It also gives the probability of each consequence scenario (given the earthquake scenario occurs), the damage level assigned to the substation for each consequence scenario, in addition to the true and estimated damage level probabilities. For example, for the second earthquake scenario the 6 consequence scenarios (IDs 7-12) imply a 100% probability of complete damage whereas the “true” probability is 98%. Table II presents the 12 consequence scenarios, the adjusted occurrence probability for each, and the number of transmission lines and substations that fall into each of the possible damage states. The occurrence probability for each consequence scenario combines the scenario probability from TABLE I, and the probability assigned to each of the two earthquake scenarios after selecting the earthquake scenarios to approximate the seismic risk in the NMSZ.

We used the heuristic to find optimum enhancement strategies to mitigate the consequences on the EI of a seismic event in the NMSZ. The problem included the 12 consequence scenarios over 30,000 components yielding over 110,000 decision variables. We found solutions for three enhancement budgets, US$ 100 million, US$ 1 billion, and US$ 10 billion. Computational time varies significantly depending on the relation between the total budget and parameter $\bar{C}$, with the smallest computation times on the order 2 hours and the longest on the order of 12 hours. These problem instances cannot be solved using Lingo. The evolution of the objective function with respect to the number of iterations, $l$, for the three defense budgets are presented in Fig. 5.
100 million), 226 lines and 1 generator (US$ 1 billion), and 448 lines and 52 generators (US$ 10 billion), respectively. An investment of US$ 100 million represents a 25% savings in load-shed over the 6 months repair period. For a budget of US$ 1 billion and US$ 10 billion, the load shed savings are 37% and 48% respectively.

This formulation uses the stochastic information summarized in earthquake scenarios and consequence scenarios directly. Alternatively, this information could be summarized by using the probabilistically weighted average of the 12 consequence scenarios. Once this single deterministic scenario is identified, the same heuristic procedure can be invoked. Fig. 7 presents the reduction in load shed costs when the mitigation is found based on the proposed stochastic model, versus when the mitigation is found based on this single consequence scenario. In other words Fig. 7 gives the value of the stochastic programming solution in contrast to deterministic solution [28]. Fig. 8 shows the load shed costs after implementing the mitigation strategies based on the 12 scenarios stochastic model and the average scenario for a mitigation budget of 1 billion. As an illustration of the value of the stochastic solution, the probability that the loss exceeds $5 billion is reduced by about 42% when the stochastic solution is used (in contrast to the solution created using the average scenario).

V. CONCLUSIONS

This paper develops a computationally effective procedure, using stochastic programming, to optimize seismic mitigation (capacity expansion for transmission and generation) in electric power systems for large-scale application. The solution procedure was tested using smaller problem instances. It was then applied to perform illustrative seismic mitigation planning for EI, which has almost 15,000 nodes and 23,000 links. As mentioned previously, this is the first paper to model the opportunity to add operating margin to address seismic hazards. Further this is the first attempt to address capacity planning for very large transmission systems.

Future work is valuable in at least three related areas. First and foremost, it is useful to refine the data to do these types of analyses. For instance, ideally there would be more detailed information on each component, especially structural vulnerability, local site conditions, and restoration times. Additionally, the model can be improved by including a representation of the distribution system so that the interdependencies between the generation, transmission and distribution systems can be considered. Further, we assume that the restoration process is deterministic as a simplification. In practice there is considerable uncertainty in these times. Hence, it would be very useful to augment our scenario definitions with uncertainty in the repair process as well. Second, when considering capacity expansions, it is important to consider all the motivations for expansion so that appropriate decisions can be made. This includes growth in demand, resilience during periods of stress, etc. Hence expanding this modeling to include a range of hazards is useful. Finally, it is desirable to extend the formulation to be dynamic so as to improve the representation of the performance of the system over time.

REFERENCES


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