

## Damping estimates of subsynchronous and power swing oscillations in power systems with thyristor switching devices

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**Abstract:** We extend the torque per unit velocity method for estimating the modal damping of subsynchronous oscillations to general power systems with thyristor switching devices. This allows damping of SSR by thyristor controlled series capacitors to be obtained from time domain simulations of only the electrical part of the system. Our method generalizes to the case of multiple torsional modes with the same natural frequency. A new method is used to estimate the damping and frequency of the swing mode. Torque per unit velocity methods are easier than exact eigenvalue analysis and testing on the IEEE first benchmark SSR model shows excellent agreement with exact eigenvalue analysis.

**Keywords:** subsynchronous resonance, SSR, perturbation methods, small signal stability, eigenvalue, thyristor controlled series capacitor, TCSC, damping.

### 1. Introduction

Subsynchronous resonance (SSR) is a phenomenon in which an electrical system compensated with series capacitors interacts with generator shafts to cause shaft fatigue or breakage. SSR in power systems without thyristor switching devices can be accurately analyzed by eigenvalue analysis (e.g. [1]), torque per unit velocity methods [2,3,4,5,6] and time domain simulation. Each of these approaches has strengths and weaknesses. Time domain simulation is accurate, applies to large, detailed system models and can be used to study large signal effects. However, estimating the damping of the various system modes can be difficult, especially when one of the modes is unstable. Complementary to time domain simulation and yielding different insights are the methods of eigenvalue analysis and torque per unit velocity. These methods are confined to small signal stability, but they both yield modal dampings. Eigenvalue analysis is exact but requires a linearized model of the entire electromechanical system to be developed. The torque per unit velocity method is a good approximation due to Bowler,

Hedin, Kilgore and others which only requires steady state simulation of the electrical part of the system.

A flexible AC transmission system such as the thyristor controlled series capacitor (TCSC) offers the possibility of power flow control together with suppression of SSR instabilities [7,8,9,10]. Thus it is necessary to analyze SSR in power systems with thyristor switching devices. The time variations and nonlinearities caused by the thyristor switchings can be overcome in time domain simulations such as EMTP, but they pose a challenge to the eigenvalue and torque per unit velocity methods. Eigenanalysis of the IEEE first benchmark model for SSR with a TCSC was achieved in [10,11] by computing the Jacobian of the Poincare map. Although this approach is general and exact, it would be arduous to form and solve the linearized system equations for large multimachine systems with controlled thyristor switching devices.

The objectives for this paper are threefold:

- (1) Justify and validate the extension of the torque per unit velocity method to general multimachine power systems with thyristor switching devices. This provides a practical alternative for estimating the damping of SSR modes.
- (2) Generalize the method to treat the case of multiple torsional modes having the same natural frequency in a multimachine system.
- (3) Present a new method for estimating the damping and frequency of the swing mode.

The paper focuses on presenting and illustrating torque per unit velocity methods achieving these objectives. Heuristic derivations are sketched in the Appendix and the methods are rigorously derived in [11,12] using an eigenvalue perturbation technique.

### 2. SSR analysis without switching devices

This section reviews the torque per unit velocity method for balanced power systems with no switching devices. When the power system operates under symmetrical and balanced three phase conditions, Park's transformation of the electrical system can be used to obtain system equations whose coefficients do not vary with time. Linearizing these equations yields a time invariant linear system.

The basic idea of the torque per unit velocity method is to trace the effect of a small sinusoidal mechanical disturbance of a generator rotor through the electrical network and determine the electromagnetic torque which

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acts either regeneratively (negative damping) or degeneratively (positive damping) on the initial displacement.

Suppose that the mode of interest has subsynchronous frequency  $\omega_i$  and the small disturbance of the rotor is represented by an input of  $e^{j\omega_i t}$  to the linearized electrical system. Then the electromagnetic torque response to this input has the form  $\Delta T = \Delta T(j\omega_i)e^{j\omega_i t}$ . The phasor  $\Delta T(j\omega_i)$  is a constant and the electromagnetic torque response  $\Delta T$  has the same frequency as the input because the linearized electrical system is time invariant. The damping of the torsional mode can then be estimated from  $\Delta T(j\omega_i)$ . The method works by cleverly exploiting the weak coupling from the electrical to the mechanical system.

### 3. SSR analysis with switching devices

This section explains how thyristor switching devices cause the system linearization to be time varying and the response of the electrical system to subsynchronous rotor oscillations to be complicated. Whenever some thyristors are conducting and other thyristors are off, the 3 phase network is instantaneously unbalanced and this causes the system to be periodically time varying even when Park's transformation is used. Unbalanced electrical networks without switching devices have been proposed for SSR mitigation [13] and these also lead to a periodically time varying system. When these systems are linearized about their periodic steady state a linear periodically time varying system results. That is, the coefficients of the linear system vary periodically with time at some frequency  $\omega_0$ . The effect of the time varying linear system is that the steady state torque response to small rotor input  $e^{j\omega t}$  has the general form (see [14]):

$$\Delta T_i = \Delta T_i(j\omega_i)e^{j\omega_i t} + g_{\text{other}}(t) \quad (1)$$

where the term  $\Delta T_i(j\omega_i)e^{j\omega_i t}$  is the response at frequency  $\omega$  and  $g_{\text{other}}(t)$  contains the remaining frequency components of the response. While  $\Delta T_i(j\omega_i)$  can be thought of as the fundamental frequency part of the response, the term  $g_{\text{other}}(t)$  is quite complicated; for example, if  $\omega_0$  and  $\omega$  are not in an integer ratio,  $g_{\text{other}}(t)$  is not even periodic in  $t$ !

Frequency domain analysis of SSR in systems containing thyristors has been attempted for the Kayenta system in [9] and for the Slatt substation in [7,8]. In the Kayenta system, the frequency response of a TCSC was measured using time domain simulation. For the Slatt system, the effect of the TCSC was evaluated by representing the TCSC by a linear transfer function obtained from simulations. A frequency domain analysis of the TCSC was also carried out in [15]. These studies measured the response of the TCSC only at the frequency of the input and other frequency components involved in the response were not discussed.

### 4. Description of system

This section describes the assumptions about the electrical system and the equations of the generator shafts.

We consider a general multimachine power system which contains thyristor switching devices and their controls. The system is assumed to be operating at a periodic steady state (typically with a 60 Hz frequency) and to be linearizable about the periodic steady state.

The electrical parts of the system and its controls are modeled in a standard way [1]. Full account must be taken of the thyristor switchings and their controls. For example, the thyristors which are off are effectively removed from the circuit and the thyristors switch off when their current becomes zero. (See [10,16] for modeling the electrical system with detailed representation of thyristor switchings and controls.)

We assume that a simulation of the electrical system and controls is available. In particular, this simulation must be able to determine the steady state torque response of any generator to a sinusoidal displacement of any generator rotor about the steady state operating condition. Then the steady state torque response to the signal  $e^{j\omega_i t}$  can be evaluated as (response to  $\cos \omega_i t$ ) +  $j$ (response to  $\sin \omega_i t$ ).

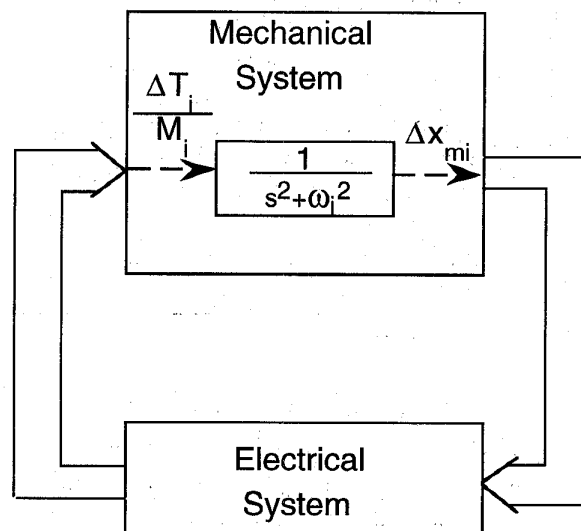


Figure 1. Electromechanical System Interaction.

The mechanical turbine-generator rotors of all the generators are modeled as lumped masses connected by lossless linear torsional springs. The natural mechanical dampings of the shafts are small and are assumed to be zero (if the natural dampings are available, we add them to the dampings due to the electromechanical interaction to get total damping). It is convenient to write the mechanical equations in modal form. The standard modal transformation [1,3] is achieved by linearly transforming rotor angles and velocities using a matrix whose columns represent the mode shapes of all mechanical modes of vibration. In modal coordinates, the mechanical equations become a set of second order oscillators (cf. Fig. 1) and

the  $i$ th modal equation is

$$\Delta \ddot{x}_{mi} = -(K_i/M_i)\Delta x_{mi} - \Delta T_i/M_i \quad (2)$$

$\Delta x_{mi}$  is the angular position of mode  $i$  and  $M_i$  and  $K_i$  are the modal inertia and spring constant of mode  $i$ . The modal frequency (or oscillator natural frequency)  $\omega_i = \sqrt{K_i/M_i}$  is in the subsynchronous range. A SSR torsional mode has  $\omega_i \neq 0$  and a power swing mode has  $\omega_i = 0$ .

$\Delta T_i$  is the modal torque; it is the torque fed back to the  $i$ th mode. The modal inertias  $M_i$  are large so that the electromagnetic acceleration  $\Delta T_i/M_i$  is small (typically two orders of magnitude smaller than other terms in (2)). The smallness of  $\Delta T_i/M_i$  also causes the SSR modal frequency to remain very close to the rotor natural frequency.

### 5. Modal damping formulas for SSR modes

This section presents and explains formulas for estimating the damping of SSR modes in general multi-machine power systems with thyristor switching devices. The natural mechanical dampings of the machines are assumed to be zero throughout. We first compute the damping of the torsional SSR mode  $i$  whose subsynchronous frequency  $\omega_i$  is distinct (well separated) from other modal frequencies. In the open loop, i.e., without the feedback term  $\Delta T_i/M_i$ , the steady state response of the oscillator described by equation (2) is  $\Delta x_{m1} = e^{j\omega_i t}$ . It follows that the open loop modal eigenvalue is  $j\omega_i$ , the open loop modal damping is zero and modal frequency is  $\omega_i$ . If the term  $\Delta T_i/M_i$  is now included, it will act to either stabilize or destabilize this response. Recall from equation (1) of Section 3 that the modal torque has the form

$$\Delta T_i = \Delta \mathcal{T}_i(j\omega_i)e^{j\omega_i t} + g_{\text{other}}(t)$$

where  $\Delta \mathcal{T}_i(j\omega_i)$  is the response of  $\Delta T_i$  at frequency  $\omega_i$  and  $g_{\text{other}}(t)$  contains the other frequency components. According to the Appendix and [11,12],  $\Delta \mathcal{T}_i(j\omega_i)$  can be used to estimate the closed loop modal damping  $\gamma_i$  with the formula

$$\gamma_i = \text{Real} \left\{ \frac{\Delta \mathcal{T}_i(j\omega_i)}{2j\omega_i M_i} \right\} \quad (3)$$

Formula (3) is accurate to first order in the small quantity  $\Delta T_i/M_i$  [11,12]. To this accuracy, the other frequency components  $g_{\text{other}}(t)$  do not appear in formula (3) and have no effect on the damping estimate. The term  $g_{\text{other}}(t)$  also has no effect on the damping estimates of other torsional modes [11,12].

For time invariant systems,  $\Delta \mathcal{T}_i(j\omega_i)$  is the phasor of the torque response in modal coordinates and (2) agrees with the result presented in [3] for a single machine power system without switching devices (in the notation of [3],  $\Delta \mathcal{T}_i(j\omega_i) = Q_{ii}^2 \Delta T_e(j\omega_i)$ ).

We now compute the dampings of two or more torsional modes having the same natural frequency. This

case can occur when a natural frequency of one generator turbine system coincides with a natural frequency of another generator turbine system. We assume that the first two torsional modes have the same nonzero natural frequency so that  $\omega_1 = \omega_2$ ; the analysis for more than two such modes is similar. Now there are two independent modes having natural frequency  $\omega_1$ ; in general, the first two oscillators will participate in both modes. This implies that the output of both oscillators must be taken into account in determining the damping of these SSR modes. Let  $\Delta \mathcal{T}_{ik}(j\omega_k)$  be the response of  $\Delta T_i$  at frequency  $\omega_i$  due to the  $k^{\text{th}}$  oscillator input  $x_{mk} = e^{j\omega_k t}$ . (As before, the frequency components not at frequency  $\omega_k$  can be neglected.) Define the matrix

$$B = \frac{1}{2j} \begin{pmatrix} \Delta \mathcal{T}_{11}(j\omega_1)/(M_1\omega_1) & \Delta \mathcal{T}_{12}(j\omega_2)/(M_1\omega_1) \\ \Delta \mathcal{T}_{21}(j\omega_1)/(M_2\omega_2) & \Delta \mathcal{T}_{22}(j\omega_2)/(M_2\omega_2) \end{pmatrix} \quad (4)$$

Then [11,12] prove that the damping estimates of the first two torsional modes are the real parts of the eigenvalues of  $B$ . (If  $\omega_1 \neq \omega_2$  and the frequency separation  $\omega_1 - \omega_2$  is more than the bandwidth of each oscillator's response, then the off diagonal terms of  $B$  are zero and the multiple mode case reduces to two instances of formula (3). However, if  $\omega_1 \neq \omega_2$ , but  $\omega_1$  is close to  $\omega_2$ , then the practical computation of  $B$  can yield nonzero off diagonal terms and the multiple mode method applies.) The computation for  $k$  torsional modes with the same frequency is similar except that  $B$  becomes a  $k \times k$  matrix.

### 6. Damping formulas for the swing mode

This section presents special formulas and iterative methods to estimate the damping of a swing mode. To avoid the complications of multiple swing modes, only the single generator case is treated. As is apparent from the derivations in the Appendix and [12], formula (3) for SSR modes does not apply to the swing mode. Instead the swing modal eigenvalue  $\gamma$  is estimated by

$$\gamma = \sqrt{-\Delta \mathcal{T}_i(j0)/M_i} \quad (5)$$

where  $\Delta \mathcal{T}_i(j0)$  is the zero frequency or constant part of the modal torque response to an input  $\Delta x_{mi} = e^{j0t} = 1$ .  $\Delta \mathcal{T}_i(j0)/M_i$  is a real number. Moreover, for typical power systems,  $\Delta \mathcal{T}_i(j0)$  is positive and  $\gamma = \sqrt{-\Delta \mathcal{T}_i(j0)/M_i}$  is purely imaginary. Thus we estimate the closed loop frequency of swing mode as  $\beta = \sqrt{\Delta \mathcal{T}_i(j0)/M_i}$  and the closed loop damping of the swing mode as zero.  $\beta$  is typically in the range 1–2 Hz.

To better estimate both the damping and frequency of the swing mode, we use an iterative technique [12]. Let  $\Delta \mathcal{T}_i = \Delta \mathcal{T}_i(j\beta)e^{j\beta t} + g_{\text{other}}(t)$  be the response to the input  $\Delta x_{mi} = e^{j\beta t}$ . Let the estimate of the closed loop modal eigenvalue after  $k$  iterations be  $\gamma^{[k]} = \alpha^{[k]} + j\beta^{[k]}$ . Start with the open loop eigenvalue estimate  $\gamma^{[0]} = 0$  and compute the  $k^{\text{th}}$  closed loop eigenvalue estimate as:

$$\gamma^{[k+1]} = \sqrt{-\Delta \mathcal{T}_i(j\beta^{[k]})/M_i} \quad (6)$$

$(\mathcal{T}_i(j\beta^{[k]}))$  is used in the right hand side of (6) because for practical power systems  $\alpha$  is much smaller than  $\beta$  so that  $\Delta\mathcal{T}_i(j\gamma^{[k]})$  is well approximated by  $\Delta\mathcal{T}_i(j\beta^{[k]})$ .) We have found the iteration to converge to a good estimate of the swing mode damping in about three iterations.

### 7. Illustration of using the damping formulas

The electrical torque response  $\Delta\mathcal{T}(j\omega_i)$  used in the formulas of Sections 5 and 6 can be computed by using a simulator. (System simulation is usually required for other purposes anyway such as checking large signal performance.) The electrical part of the system used to compute the steady state electrical torque response is typically stable with well damped modes. Therefore it is straightforward to compute the steady state torque response from simulation. This advantage applies even when the entire electromechanical system is unstable.

We now give an example of estimating damping of torsional modes and the power swing mode for the IEEE SSR first benchmark model [17] with a TCSC as the thyristor conduction angle  $\sigma$  varies. The electrical part of the system is shown in Fig. 2. There are five torsional modes TM1 through TM5 for the mechanical system with respective frequencies 16, 20, 25, 32 and 47 Hz and a rigid body mode TM0 corresponding to power swings of the system. The TM4 mode is highly unstable when the series capacitor compensation level is 29% of the combined transmission and transformer impedances. Each phase of the TCSC consists of a fixed capacitor with a parallel connected thyristor controlled reactor as shown in Fig. 2.

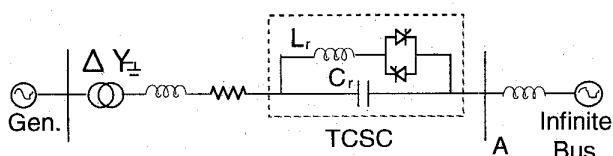


Figure 2. IEEE First Benchmark Model with TCSC.

The thyristor switch on times are determined by the equal distance firing synchronization method, though our methods in this paper are valid for general thyristor firing control schemes. The case  $\sigma = 0^\circ$  corresponds to blocking the thyristors. Detailed system description and parameter values are provided in [17,10].

Figure 3 shows the net damping of the modes TM0 through TM4. (The mode TM5 damping is zero throughout and obscured by the horizontal axis of Fig 3.) Negative modal damping implies modal instability. The circles are the estimates of this paper and the solid lines are obtained by exact eigenanalysis using the methods of [10]. (The net modal damping was obtained by adding the natural modal damping to the modal damping due to SSR.) We refer to [10] for discussion of the results; the purpose of this section is to demonstrate the close reproduction of the results of exact eigenanalysis with torque

per unit velocity methods. These results are also validated by time domain simulations in EMTP [10]. The larger errors in the SSR and swing modes occur far from zero damping and these modes are estimated accurately when their damping is small. This is to be expected because the method effectively perturbs about the small natural mechanical damping.

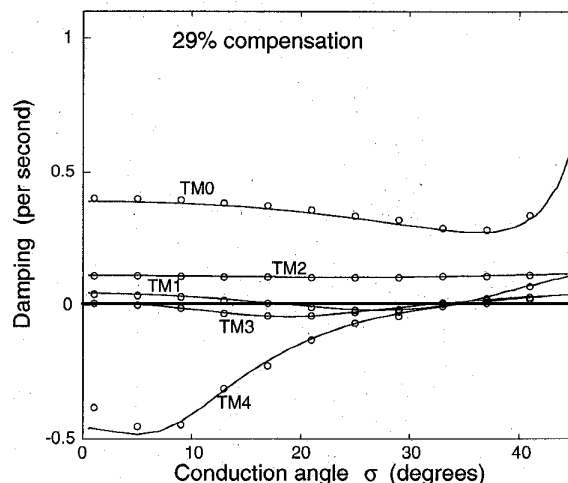


Figure 3. Modal Dampings.

These results were obtained by time domain simulation using the software [18]. Formula (3) was used for the damping estimation of the torsional modes and three iterations of (6) were used for the power swing mode. For each value of  $\sigma$ , the time for computing modal dampings was roughly ten minutes on an HPRISC machine.

### 8. Conclusions

This paper derives simple formulas for estimating the damping of SSR and power swing modes when the power system includes thyristor switching devices. In the electrical part of these systems, a small sinusoidal perturbation of a machine rotor at a modal frequency leads to a complicated electrical torque response with many frequency components. However, we show that the damping may be estimated from the component of the electrical torque response at the modal frequency. The formulas extend to the case of multiple torsional modes having the same natural frequency. The formulas are heuristically derived in the Appendix and rigorously derived in [11,12]. The derivation of the formulas justifies the extension of the torque per unit velocity method to systems with thyristor switchings.

We also propose a new method to estimate the damping and frequency of the power swing mode and to improve the estimates by a simple iteration.

The formulas require the steady state electrical torque response of the electrical system to sinusoidal machine rotor perturbations to be computed and this can be done by time domain simulation. The method is simpler than exact eigenanalysis of the entire electromechanical sys-

tem and testing on the IEEE SSR first benchmark model with a TCSC shows excellent agreement with exact eigenanalysis.

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### Appendix

The damping estimates (3) and (5) are informally derived. See [11,12] for rigorous proofs. The oscillator equation (2) of the  $i^{\text{th}}$  mode is

$$\Delta \ddot{x}_{mi} + \omega_i^2 \Delta x_{mi} = -\Delta T_i / M_i \quad (\text{A.1})$$

The solution to  $\Delta \ddot{x}_{mi} + \omega_i^2 \Delta x_{mi} = 0$  is  $\Delta x_{mi} = e^{j\omega_i t}$ . Since (A.1) has the periodically time varying term  $\Delta T_i / M_i$ , its solution has the form

$$\Delta x_{mi} = e^{(j\omega_i + \gamma)t} (1 + g(t)) \quad (\text{A.2})$$

where  $g(t)$  is periodic with period  $T_0 = 2\pi/\omega_0$  [14]. Since the acceleration term  $\Delta T_i / M_i$  is small, the solution (A.2) is a small perturbation of  $e^{j\omega_i t}$  and  $\gamma$  and  $g(t)$  are small. It can be shown [12] that  $\Delta x_{mi} = e^{j\omega_i t}$  causes a torque  $\Delta T_i = e^{j\omega_i t} \Delta T_i(j\omega_i) + g_{\text{other}}(t)$ . This torque is used to approximate the right hand side of (A.1) so that

$$\Delta \ddot{x}_{mi} + \omega_i^2 \Delta x_{mi} \approx -(e^{j\omega_i t} \Delta T_i(j\omega_i) + g_{\text{other}}(t)) / M_i \quad (\text{A.3})$$

Substitution of the assumed solution (A.2) in (A.3) and dividing by  $e^{(j\omega_i + \gamma)t}$  gives

$$\begin{aligned} (j\omega_i + \gamma)^2 (1 + g(t)) + 2(j\omega_i + \gamma)\dot{g}(t) + \ddot{g}(t) + \omega_i^2 (1 + g(t)) \\ \approx -(e^{-\gamma t} \Delta T_i(j\omega_i) + e^{-(j\omega_i + \gamma)t} g_{\text{other}}(t)) / M_i \\ \approx -(\Delta T_i(j\omega_i) + e^{-j\omega_i t} g_{\text{other}}(t)) / M_i \end{aligned} \quad (\text{A.4})$$

where the last step follows since  $\gamma$  is small. Now  $\dot{g}(t)$ ,  $\ddot{g}(t)$  and  $e^{-j\omega_i t} g_{\text{other}}(t)$  are periodic functions with average value zero and  $g(t)$  is small [12]. Taking the average over the period  $T_0$  of both sides of (A.4) and discarding the products of small terms yields

$$2j\omega_i \gamma + \gamma^2 \approx -\Delta T_i / M_i$$

In the case of an SSR mode with  $\omega_i \neq 0$ , the smaller term  $\gamma^2$  is neglected and the modal eigenvalue is  $j\omega_i + \gamma \approx j\omega_i - \Delta T_i / (2j\omega_i M_i)$  and taking the real part yields formula (3). In the case of a swing mode with  $\omega_i = 0$ , we obtain formula (5) and hence (6).

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