Criticality in a cascading failure blackout model

Dusko P. Nedic [a], Ian Dobson [b,*], Daniel S. Kirschen [a],
Benjamin A. Carreras [c], Vickie E. Lynch [c]

[a] *School of Electrical and Electronic Engineering, University of Manchester, Manchester M601QD, UK*
[b] *ECE Department, University of Wisconsin, 1415 Engineering Drive, Madison, WI 53706, USA*
[c] *Oak Ridge National Laboratory, Oak Ridge, TN 37831, USA*

Abstract

We verify and examine criticality in a 1000 bus network with an AC blackout model that represents many of the interactions that occur in cascading failure. At the critical loading there is a sharp rise in the mean blackout size and a power law probability distribution of blackout size that indicates a significant risk of large blackouts.
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1. Introduction

Cascading failure is the main mechanism of large blackouts. Failures successively weaken the system and make further failures more likely so that a blackout can propagate to disable large portions of the electric power transmission infrastructure upon which modern society depends. Here the term ''failure'' refers to power system components being removed from service by a variety of means, including action or malfunction of the protection system, automatic or manual controls, and physical breakdown.

There are many different types of interactions by which failures can propagate during the course of a blackout. For example, a transmission line tripping can cause a transient, the overloading of other lines, the operation or misoperation of relays, reactive power problems, or can contribute to system instabilities or operator stress. However, for all these types of interaction, the risk of cascading failure

generally becomes more severe as overall system loading increases. But exactly how does cascading failure become more likely as loading increases? Previous work suggests that a cascading failure does not gradually and uniformly become more likely; instead there is a transition point at which its likelihood sharply increases. This transition point has some of the properties of a critical transition or a phase transition in statistical physics. That is, there is a critical loading at which the probability of large blackouts and the mean blackout size start to rise quickly and at which the probability distribution of blackout sizes has a power law dependence [1–4]. The significance of the power law in the probability distribution of blackout size is that it implies a substantial risk of large blackouts near the critical loading [5]. (For example, a power law dependence with exponent −1 implies that blackout probability is inversely proportional to blackout size. Then, if blackout cost is proportional to blackout size, the blackout risk, which is the product of blackout size and blackout probability, remains constant so that the risk of large blackouts is comparable to that of small blackouts.) The power law is also consistent with the observed frequency of large blackouts in North America [6]. It is important to verify and compare criticality in a variety of power system blackout models

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* Corresponding author. Tel.: +1 608 262 2661; fax: +1 608 262 1267.
 E-mail addresses: dusko.nedic@tnei.co.uk (D.P. Nedic), dobson@
engr.wisc.edu (I. Dobson), D.Kirschen@manchester.ac.uk (D.S.
Kirschen), carrerasba@ornl.gov (B.A. Carreras).

to find out the extent to which it is a phenomenon universally associated with cascading failure of power systems. Models of cascading failure in power systems are described in [7,8,1,9,3,4,10]. The conference version of this paper is [11].

We investigate criticality in a sizable network with a fairly detailed blackout model and measure blackout size by energy unserved. At a given loading level, initial outages are chosen randomly and the consequences of each initial outage are simulated to estimate the distribution of blackout size. Gradually increasing the loading yields estimates of the probability distribution of blackout size at each loading. The mean blackout size is also computed at each loading.

Blackouts are traditionally analyzed after the blackout by a detailed investigation of the particular sequence of failures. This is good engineering practice and very useful for finding and correcting weaknesses in the power system. We take a different and complementary approach and seek to analyze the overall probability and risk of blackouts from a global perspective. An analogy can highlight this bulk systems approach and the role of phase transitions: If we seek to find out how close a beaker of water is to boiling, we do not compute in detail the vast number of individual molecule movements and collisions according to Newton's deterministic laws. A bulk statistical approach using a thermometer works well and we know that the phase transition of boiling water occurs at 100 °C. The analogy suggests that to manage the risk of blackouts, we first need to confirm, understand and detect the criticality phase transition in blackout risk that seems to occur as power systems become more stressed.

2. Power system blackout model

The AC power blackout model developed at the University of Manchester represents a range of interactions, including cascade and sympathetic tripping of transmission lines, heuristic representation of generator instability, under-frequency load shedding, post-contingency redispatch of active and reactive resources, and emergency load shedding to prevent a complete system blackout caused by a voltage collapse. The amount of time required for restoration is assumed to depend on the amount of load to be reconnected.

The Manchester model used in this paper was designed to provide a realistic representation of the behavior of a power system when subjected to disturbances [7,8]. For example, the Manchester model represents various adjustments that are made by automatic control systems and operators when trying to return the system to a stable operating condition. It also incorporates mechanisms that can amplify a relatively benign initial disturbance and cause a cascading outage.

Because it is impossible to predict the nature or the location of the initiating fault or failure, the model relies on a state sampling Monte Carlo simulation of the behavior of the system. Simulations start from a normal system state

that can be varied to study the resilience of the system under various conditions. As Fig. 1 illustrates, the system is returned to this normal state at the beginning of each trial. For each trial, an initial disturbance is created by simulating random outages of system components. This is achieved by generating a random number between 0 and 1 for each component in the system. If the random number is below the probability of outage over the period considered for the component under consideration, this component is taken out of service. Otherwise, it is assumed to remain in service. This approach generates a new system state with either no outage, a single outage or, occasionally, multiple independent outages. Some of the faults that these simulated outages represent can cause generators to go unstable. A simple procedure has been implemented to identify circumstances under which one or more generators might be disconnected because of transient instability caused by the initiating failures [7]. If this procedure determines that a generator would go unstable following the initiating fault, it is disconnected from the system.
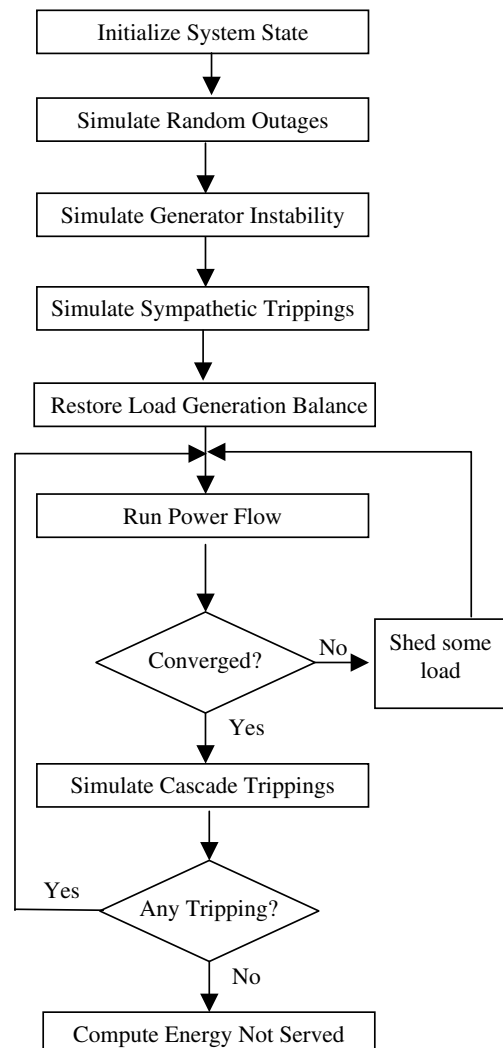

Fig. 1. Cascading failure power system model.

It has been recognized that malfunctions in the protection system are an important contributing factor in major blackouts [12,13]. Hidden failures in the protection system cause intact equipment to be unnecessarily disconnected following a fault on a neighboring component. These sympathetic failures have the potential to transform a routine outage into a major incident. Because it is impossible to know which protection device might malfunction, a probabilistic approach has been adopted. The component affected by the initiating failure is assumed to be at the centre of a "vulnerability region" that contains all the components that might be tripped by a malfunction of the protection system in response to the initiating failure. "Sympathetic trippings" are then simulated when a random number generated for each component in the vulnerability region turns out to be smaller than the assumed probability of protection malfunction.

The initiating failure and the possible subsequent generator instabilities and sympathetic trippings may cause an imbalance between load and generation. Such an imbalance would cause a change in frequency that triggers under-frequency load shedding or generation outages due to overspeed. Once the load/generation balance has been restored, a power flow is run taking into consideration all the changes to the base case triggered by the initiating failure. If this power flow does not converge, this is taken as an indication that the system would suffer from a voltage collapse if the operator did not take action. The model assumes that the operator has enough time and understanding of the situation to implement load shedding in an attempt to arrest this voltage collapse. Rather than use an optimal power flow to determine the minimum amount of load to be shed, the model sheds load in blocks of 5% in the area with the worst mismatches until convergence is achieved. If shedding load in one area is not sufficient, further disconnections are made in neighboring areas. If convergence is not achieved after a certain number of load shedding steps, it is assumed that a complete system blackout has occurred. Once the power flow has converged, the power flow in each branch is checked against its emergency rating. If this rating is exceeded, the branch is assumed to have tripped. The power flow is recalculated and further load shedding implemented if needed to achieve convergence.

If this trial resulted in the outage of one or more components and if these outages caused the disconnection of some load, the energy not served is then computed and recorded. Going from the load disconnected to the energy not served requires a model of the restoration process. The model that was adopted [14] assumes a fixed time during which the operators take stock of the situation, get organized and then reconnect the load at a constant rate. With this model all complete system blackouts result in the same amount of energy not served.

The Manchester model has been exhaustively tested on a 53-bus system [7] and on the large power system used in this paper [8].

3. Results

The test case represents a large European power system and has 1000 busses, 1800 transmission lines and transformers, and 150 generating units. The base case for the studies is derived from a snapshot of the output of the state estimator running at the control center that manages this system. The base case load has an active power demand of 33 GW and a reactive power demand of 2.5 GVAr.

The Manchester model was run with randomized initial outages at a series of load levels starting from 1.20 times base case load and ending at 2.00 times base case load.

The sharp rise in expected energy unserved in Fig. 2 suggests a critical loading at 1.94 times the base case loading. The corresponding probability distribution (probability density function) obtained by binning the data for loadings at the critical loading is shown in Fig. 3. (The bins are chosen to require a minimum number of points per bin. The
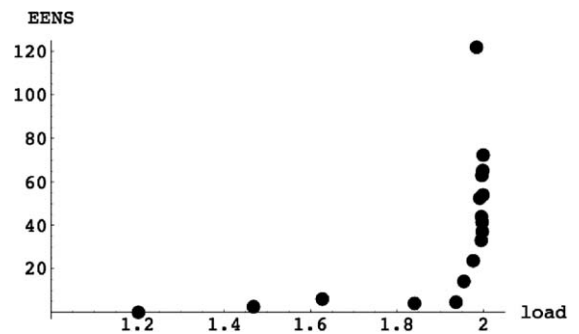
Fig. 2. Expected energy not served as a function of the loading factor with respect to the base case. There is a sharp increase in blackout size at the critical loading of 1.94 times the base case loading.
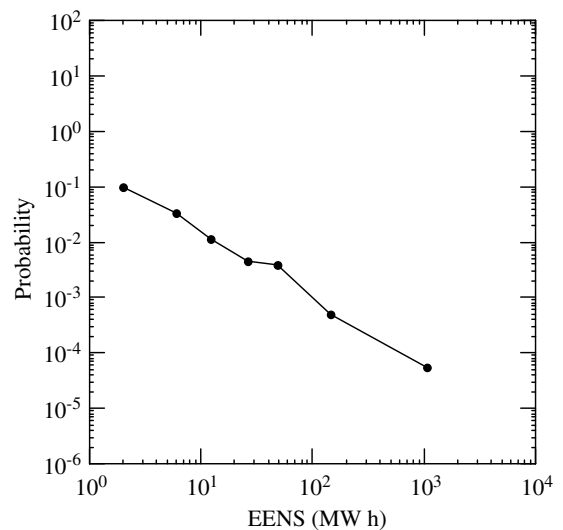
Fig. 3. Probability distribution of expected energy not served at the critical loading of 1.94 times the base case loading. Note the log–log scale; a straight line on this plot indicates a power law relation. The straight line slope of approximately -1.2 indicates that the blackout probability is proportional to $(\text{EENS})^{-1.2}$.
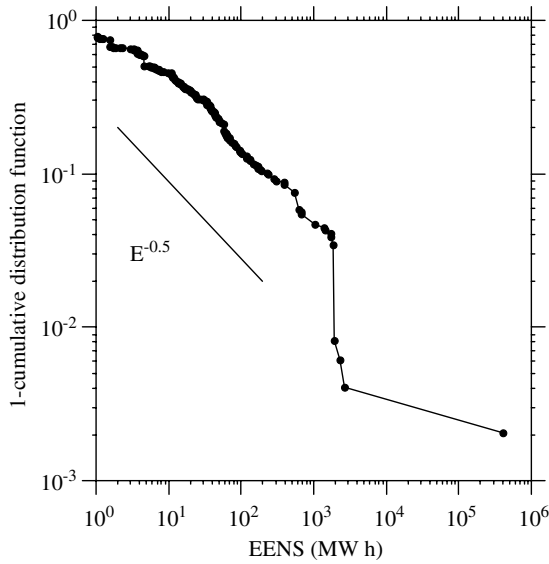
Fig. 4. Probability that blackout exceeds a given expected energy not served (complementary cumulative probability distribution) at the critical loading of 1.94 times the base case loading. The straight line slope of approximately −0.5 indicates that the blackout probability is proportional to $(EENS)^{-1.5}$.
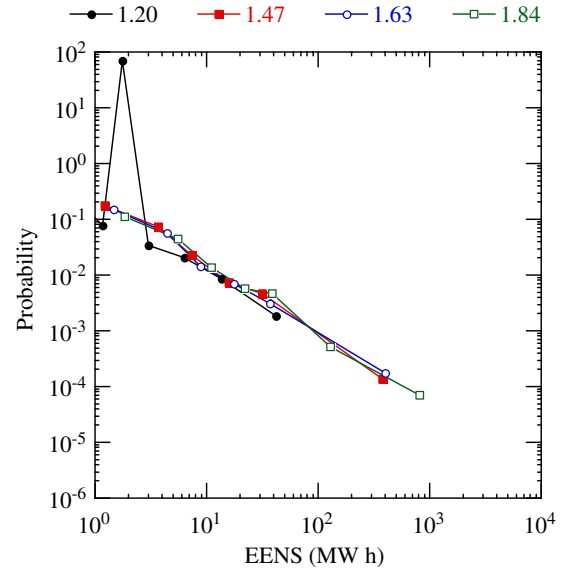

Fig. 5. Probability distributions of expected energy not served for four loadings below the critical loading, namely 1.20, 1.47, 1.63 and 1.84 times the base case loading.
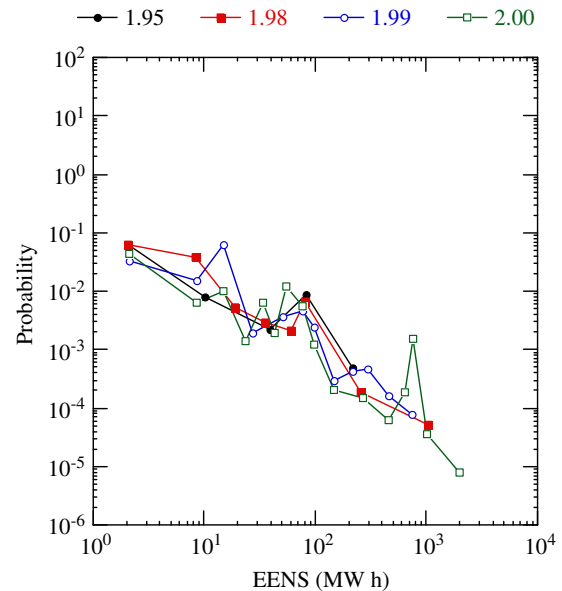

Fig. 6. Probability distributions of expected energy not served for four loadings above the critical loading, namely 1.95, 1.98, 1.99 and 2.00 times the base case loading.

probability density estimated in each bin is then the fraction of points in the bin divided by the bin width.) The probability distribution at critical loading in Fig. 3 shows evidence of a power law region over almost three decades of expected energy not served.

The exponent of the power law relationship is approximately −1.2. We also process the data at the critical loading in a different way to get another estimate of the power law exponent. The relative frequency of blackouts exceeding a given expected energy not served gives an estimate of the complementary cumulative distribution function as shown in Fig. 4. This complementary cumulative distribution function shows a power law region of slope approximately −0.5, and, since a cumulative distribution function is the integral of a probability distribution function, this corresponds to a power law in the (non-cumulative) probability distribution function of exponent −1.5. Both methods give a power law exponent for the probability distribution function between −1 and −2. Both the sharp rise in average blackout size and the power law in the probability distribution of expected energy unserved near loading 1.94 are evidence of criticality.

Loadings below the critical loading show a similar power law behavior in Fig. 5 except that there seems to be a tendency for the power law to extend to lower maximum values of blackout size. A sample of loadings above the critical loading yield the probability distributions of Fig. 6. Fig. 6 shows an overall approximate power law behavior together with peaks at which blackouts are concentrated at particular sizes. These results have some uncertainty due to noise in the data and the peaks show some sensitivity to the precise loading level.

4. Comparison with other models

4.1. Abstract cascading failure model

The power system results can be compared to the corresponding results from the idealized CASCADE model of cascading failure [2]. Here CASCADE has 1000 identical components randomly loaded. An initial disturbance adds load d to each component and causes some components to

fail by exceeding their loading limit. Failure of a component causes a load increase p for all the other components. As components fail, the system becomes more loaded and cascading failure of further components becomes likely. The model is simple enough that there is an analytic formula for the probability distribution of the number of failed components. CASCADE is an abstract model with a minimum representation of cascading failure and it has no power systems modeling. However, it can be used to help understand cascading failure in more complicated models.

We increase the average initial loading L to obtain the results shown in Figs. 7 and 8 (parameters are $d = p = 0.0004/(2 - 2L)$ and more modeling details are given in [2]). The critical loading occurs near $L = 0.8$ as is seen by the change in gradient in both the mean number of components failed in Fig. 7 and the probability of more than 500 components failing in Fig. 8.

The probability distribution of the number of failed components at criticality is shown in Fig. 9 and the exponent of the power law region is approximately -1.4. Branching process models that approximate CASCADE show similar results [15]. For single-type branching
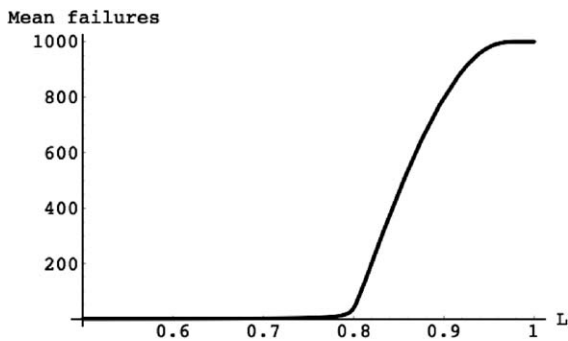

Fig. 7. Mean number of failed components as a function of average initial loading L in CASCADE model. There is a sharp increase at the critical loading $L = 0.8$. There is saturation with all 1000 components failed at loading $L = 0.96$.
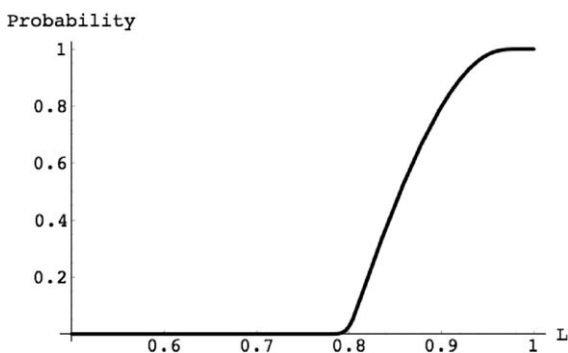

Fig. 8. Probability of more than 500 components failed as a function of loading L in CASCADE model. There is a sharp increase at the critical loading $L = 0.8$.
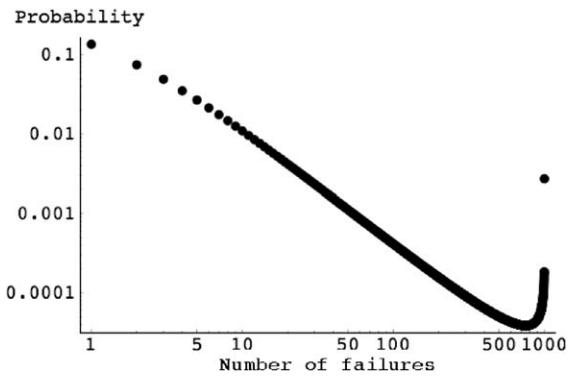

Fig. 9. Probability distribution of number of failed components near criticality in CASCADE model.

processes it is known analytically that the exponent of the power law region is -1.5.

4.2. Power system blackout models

The OPA power system blackout model [1] represents transmission lines, loads and generators with the usual DC load flow assumptions. Starting from a solved base case, blackouts are initiated by random line outages. Whenever a line is outaged, the generation and load is redispatched using standard linear programming methods. The cost function is weighted to ensure that load shedding (blackout) is avoided where possible. If any lines were overloaded during the optimization, then these lines are outaged with a fixed probability. The process of redispatch and testing for outages is iterated until there are no more outages. (This describes OPA with a fixed network; OPA can also represent the complex dynamics of load increase and network upgrade [16].)

The hidden failure power system blackout model [3] has a similar level of modeling detail as OPA for cascading line outages but also models in detail hidden failures of relays by increasing the probability of failure of loaded lines "exposed" by adjacent line trips. Probabilities of blackout are determined using importance sampling.

Both OPA [1] and the hidden failure model [3] show a change in gradient as overall system load is increased, although the effect is less clear-cut in the case of the hidden failure model. Some approximate power law exponents obtained at criticality for all the models discussed in this paper are shown in Table 1.

Table 1
Approximate power law exponents at criticality for several cascading failure models

Model	Exponent	Test case	Reference
Manchester	$-1.2, -1.5$	1000 bus	This paper
OPA	$-1.2, -1.6$	"tree" 382	[1,16]
Hidden failure	-1.6	WSCC179	[3]
CASCADE	-1.4	–	[2]
Branching	-1.5	–	[15]

4.3. Further discussion of criticality

In complex systems and statistical physics, a critical point for a type-2 phase transition is characterized by a discontinuity of the gradient in some measured quantity. (For example, for the Manchester model results the measured quantity is mean blackout size (expected energy not served), and for the CASCADE model the measured quantity is the mean number of failed components or the probability of the number of failures exceeding half the components.) (There are other types of transition possible: type-1 phase transitions have a discontinuity in the measured quantity at the phase transition.) At the critical point, fluctuations of the measured quantity can be of any size and their correlation length becomes of the order of the system size. In terms of blackouts, this means that a range of blackout sizes up to the system size are possible and that an initial failure can possibly propagate to black out most of the power system. A consequence of criticality is that the probability distribution of the fluctuations or the blackout size has a power tail.

CASCADE and especially OPA show complicated critical transition behavior [1,2]. The parameter values for the CASCADE results above correspond to a small initial disturbance; this gives qualitatively similar results to power system cascading failures with a few initial line outages. In CASCADE, the mean number of failures can fail to detect the criticality at other parameter values. The loading at which an initial failure can with positive probability propagate to failure of the entire system is called the percolation threshold. A measure for criticality that corresponds more exactly to a percolation threshold is the probability of a large fraction of components failing and this measure was illustrated for the CASCADE model. The CASCADE results show that for the parameter values chosen, the mean number of failures detects criticality very similarly to the probability of a large fraction of components failing.

5. Discussion and conclusions

The main conclusion is that the Manchester model of cascading failure on a 1000 bus power system test case shows evidence of a critical loading at which there is a sharp rise in mean blackout size and a power law in the probability distribution of blackout size. In addition to being another and needed verification of the criticality phenomenon for cascading blackout risk, this is an advance upon previous results because of the variety of cascading failure interactions represented in the Manchester model and because of the test case size and realism.

The details of the observed criticality are broadly consistent with previous results. For example, the Manchester model on the 1000 bus test case gives an approximate power law exponent of -1.2 or -1.5 at criticality and observed NERC data and power system and abstract models of cascading failure also give power law exponents between -1 and -2 [6,1,16,5,3,17,2]. There is some uncertainty in obtaining precise values of the power law exponent for any of the real or simulated power system data and considerable idealization in applying the more abstract models of cascading failure, so a more accurate comparison of the exponents is not feasible at present. More quantitative statistical estimation would be desirable.

The importance of the critical loading is that it defines a reference point for increasing risk of cascading failure. Monitoring the proximity to the critical loading will allow the risk of large, cascading failure blackouts to be assessed. In terms of the temperature analogy discussed in the introduction, determining the proximity to the critical loading is akin to measuring temperature to determine the proximity to boiling. The emerging methods to make these bulk statistical measurements include the correlated sampling Monte Carlo methods in [8] and the measurement of average propagation of failures in a branching process approximation suggested in [18].

Although there is a substantial risk of large blackouts near criticality, it cannot be assumed based on the present knowledge about blackout risk that power system operation near criticality is necessarily undesirable; there are also substantial economic benefits in maximizing the use of the power transmission system that will also have to be considered in a comprehensive risk analysis. A key part of such a risk analysis is obtaining the probability distribution of blackout size from simulations or measurements and criticality plays an important role in the form of the probability distribution. Such a comprehensive risk analysis of blackouts of all sizes is an overall research goal and the work in this paper confirming criticality with a fairly detailed model in a 1000 bus test case contributes to this goal.

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