Hexagonal $\Sigma\Delta$ Modulators in Power Electronics

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***Abstract*—Design techniques for $\Sigma\Delta$ modulators from communications are applied and adapted to improve the spectral characteristics of high frequency power electronic applications. A high frequency power electronic circuit can be regarded as a quantizer in an interpolative $\Sigma\Delta$ modulator. We review one dimensional $\Sigma\Delta$ modulators and then generalize to the hexagonal sigma-delta modulators that are appropriate to three-phase converters. A range of interpolative modulator designs from communications can then be generalized and applied to power electronic circuits. White noise spectral analysis of sigma-delta modulators is generalized and applied to analyze the designs so that the noise can be shaped to design requirements. Simulation results for an inverter show significant improvements in spectral performance.**

***Index Terms*—Ergodic, power electronics, quantization, $\Sigma\Delta$ modulation, spectral analysis, voltage source inverter (VSI).**

I. INTRODUCTION

NUMEROUS advances in $\Sigma\Delta$ modulation technology have recently appeared in the communications literature. This paper generalizes and applies these improvements to the analysis and design of $\Sigma\Delta$ modulators for high frequency power electronic systems.

In communications, an analog signal is often converted (modulated) into a digital code using a quantizer, and then, after transmission, the digital signal is converted (demodulated) back to analog form. The analog-to-digital conversion can use a $\Sigma\Delta$ modulator [1], [2]. $\Sigma\Delta$ modulators or, more generally, oversampled analog-to-digital converters achieve the performance of high resolution quantizers by using low resolution quantizers in a feedback loop with linear filtering. These converters modulate an analog signal into a simple code, usually a single-bit, at a frequency much higher than the Nyquist rate. In this manner, the modulator can trade resolution in time for resolution in amplitude as well as employ simple and relatively high-tolerance analog components [2]–[4].

In power electronics, we view switching converters as analog-to-digital converters in which the analog input is the signal to be synthesized and the quantized digital output is the state of the circuit switches [5], [6]. For example, in a voltage source circuit, the switches impress discrete voltages on the load. Then the low-pass filtering action of typical loads removes the modulation frequencies and hence converts these discrete voltages back to analog form. In both communications theory and power electronics, the aim is to design the system so that the input signal is passed through the system with minimal distortion from noise. Moreover, switching converters typically switch at frequencies well in excess of the Nyquist rate of the reference signal. Therefore $\Sigma\Delta$ modulation techniques are pertinent.

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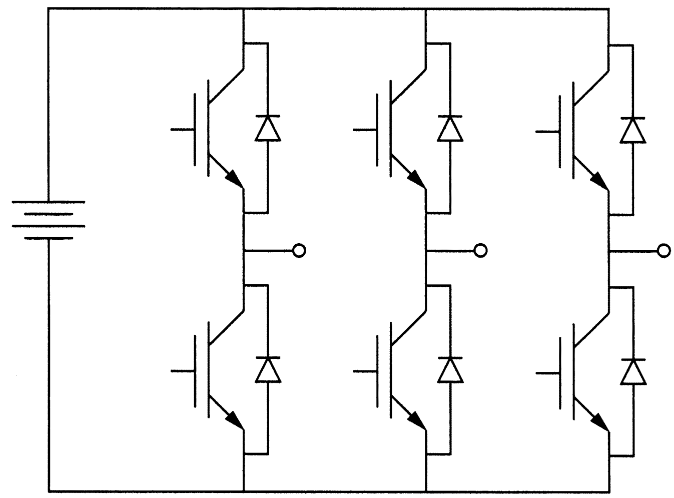

Fig. 1. Three phase voltage source inverter.

One consequence of this interpretation is that the power electronic switching states determine the possible "digitized states" or quantizer outputs. That is, the structure of the quantizer is determined by the power electronic circuit. For example, the three phase voltage source inverter (VSI) shown in Fig. 1 has seven switching states which correspond to the seven output vectors in Fig. 2. Similarly, other circuits such as the matrix converter, multilevel converters, and multiphase converters define particular quantizer outputs. Viewing high frequency power electronic circuits as contributing to quantization allows them to be regarded as part of the modulator topology (Fig. 5). It follows that noise-shaping methods of filter design may be applied to optimize the spectral characteristics. Hexagonal $\Sigma\Delta$ modulators have been designed using the methods of this paper and implemented in commercial power electronic products requiring high spectral performance.

The purpose of this paper is to explain this view of switching converters as analog-to-digital converters and describe some inventions and design and analysis methods that follow from this view. In particular, we describe a vector $\Sigma\Delta$ modulator with a hexagonal quantizer (hexagonal $\Sigma\Delta$ modulator) that is the natural generalization of a conventional scalar $\Sigma\Delta$ modulator when the power electronics circuit is a VSI [7]. We extend the conventional white noise analysis of the scalar $\Sigma\Delta$ modulator to the hexagonal $\Sigma\Delta$ modulator and thereby derive useful design graphs and formulae. We use analysis and simulation to compare the single-loop and double-loop hexagonal $\Sigma\Delta$ modulators

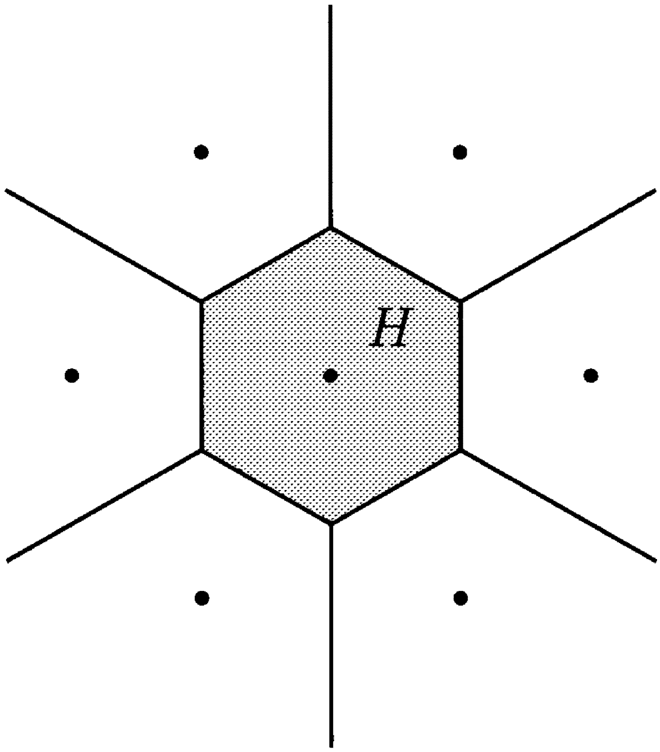

Fig. 2. Hexagonal vector quantizer, q.

in terms of spectra and switching rate and demonstrate the advantages of the double-loop hexagonal $\Sigma\Delta$ modulator.

Relative to our previous work in a conference paper [5] and a patent [8], this paper is a reworking of the explanation of our view of converters [5] and the invention of the hexagonal $\Sigma\Delta$ modulator [5], [8] and a significant extension of the design and analysis methods initially alluded to in [5]. Our previous paper [6] that derives the spectrum and switching rate of the hexagonal $\Sigma\Delta$ modulator uses entirely different nonlinear and exact analysis methods.

II. Literature Review

Oversampled analog-to-digital converters have been employed in power electronics for nearly two decades. However, attention to these converters has been sparse in comparison to the vast literature for pulse width modulators (PWM). The first reported power electronic application of an oversampled converter (delta modulator) was to a conventional three phase transistor inverter wherein the integration of the output voltage was calculated via the output inductors. The output current closed the feedback loop and thus could be controlled [9]. This so-called current controlled delta modulator exhibited a nonzero steady-state output current error which was improved by the addition of an integrator in the forward path [10].

$\Sigma\Delta$ modulators have been applied successfully to systems such as resonant dc link converters (RDCL) where the discrete timing of the circuit switching precludes the use of conventional modulation techniques such as pulse-width modulation (PWM) [5], [11]. The RDCL uses zero voltage switching to limit switching losses and attain relatively high switching frequencies [12]. Interest in applying $\Sigma\Delta$ modulators to the RDCL was fostered since both require constrained discrete switching instants. Studies of $\Sigma\Delta$ modulators applied to the RDCL that considered their spectral performance and harmonic distortion using simulation, experiment, and basic analysis were reported in [13]. A three-fold $\Sigma\Delta$ modulator was used which employed three identical independent scalar modulators to control an inverter leg voltage rather than the output voltage vector. This modulator has reduced dynamic range and nonadjacent state switching as compared to the hexagonal $\Sigma\Delta$ modulator of [5]. The use of zero output voltage state (i.e., all switches high/low) was introduced in [11] to obtain adjacent state switching. This so-called modified $\Sigma\Delta$ modulator is a three-fold $\Sigma\Delta$ modulator with the provision that nonadjacent states are overridden by a zero state. This work differs from the hexagonal $\Sigma\Delta$ modulator of [5] in that the zero vector is not chosen unless a nonadjacent state is selected. Seidl [14] derived the hexagonal quantizer based on its one-step ahead optimality properties (minimum squared error) and developed a neural network delta modulator employing the hexagonal quantizer. An alternative to current controlled delta modulators using a one-step ahead minimization of the infinity norm of the current error was proposed in [15]. Another possibility based on sliding modes is presented in [16]. Attempts to combine $\Sigma\Delta$ modulation with space vector modulation [7] are developed in [17] and [18]. Summaries of the application of current controlled delta modulators and (to a lesser extent) $\Sigma\Delta$ modulators to the RDCL prior to 1994 are found in [14], [17]. Applications of $\Sigma\Delta$ modulation to the mitigation of EMI and harmonic spikes in switched-mode power supplies can be found in [19] and [20].

A coherent analysis of $\Sigma\Delta$ modulators applied to the RDCL was reported by Mertens in [21]. This work drew from the basic reference in communications for the behavior of quantization noise with dc input of Candy and Benjamin [3]. They extended methods of Iwersen [22] from Δ modulation. Their approach was based on an approximate continuous time model for a $\Sigma\Delta$ modulator and their results well matched experimental results (for a continuous time system). They applied their approximations to evaluate the mean squared error when an ideal low-pass filter is used as a decoder. Iwersen applied Fourier series expansion of the quantizer error function to a specific input to obtain a Fourier series for the error sequence from which the spectrum was deduced [22].

Our conference paper [5] and patent [8] introduced the $\Sigma\Delta$ modulator with hexagonal quantization as well as the extension to double-loop and interpolative $\Sigma\Delta$ modulators. This work represents a significant improvement in spectral performance over prior work. A novel insight put forth in [5] is that a power electronic circuit may be thought of as an A/D converter in which the analog input is the signal to be synthesized and the quantized digital output is the state of the circuit switches. One consequence of this interpretation is that the power electronic switching states determine the possible “digitized states” or quantizer outputs. Similarly, other circuits such as the matrix converter, multilevel converters, and multiphase converters define particular quantizer outputs. We have also analytically derived the exact output spectrum (no white noise approximation) of the hexagonal $\Sigma\Delta$ modulator with a constant input using ergodic theory and hexagonal Fourier series [6], [23]. The switching rate of the modulator is important for power electronic design and formulas for the average switching rate are derived for constant and slowly varying sinusoidal inputs [6].

Recently, Nieznański [24]–[26] compared the hexagonal $\Sigma\Delta$ modulator to the modified $\Sigma\Delta$ modulator of [11] and the threefold-scalar $\Sigma\Delta$ modulator [13]. This work builds on the comparisons made in [17]. Via simulation, this work showed the hexagonal $\Sigma\Delta$ to have lower distortion power and device switching rate. Additionally, this study compared the hexagonal $\Sigma\Delta$ to space vector PWM [7]. For inputs less than 90%, the space vector PWM was shown to outperform the hexagonal $\Sigma\Delta$ in the sense that it requires lower device switching frequency to obtain a given spectral performance. However, for inputs exceeding 90% the situation is reversed.

There has been extensive design and analysis of scalar $\Sigma\Delta$ modulators for applications in communications and signal processing [1]. Also vector quantization is applied (but not to $\Sigma\Delta$ modulation) in a number of applications in signal processing [27]. The power electronic application combines specific vector quantizers with $\Sigma\Delta$ modulation and requires a significant generalization of the scalar work. Of course, vector quantization can be used in $\Sigma\Delta$ modulation simply by applying scalar $\Sigma\Delta$ modulation to each component; but the analysis of these systems is straightforward. In contrast, the hexagonal quantizer cannot be reduced to two independent scalar quantizers, so that the required generalization is not trivial. The vector generalization motivated by the power electronic application is natural enough in communications and signal processing since, for example, the nearest neighbor quantizer is one of the simplest vector quantizers. However, it appears that the use of hexagonal quantizers in $\Sigma\Delta$ modulators has not been studied previously and that this type of generalization has not been explored.

III. $\Sigma\Delta$ Modulators

This section explains a conventional scalar $\Sigma\Delta$ modulator [1], and how a half-bridge converter may be embedded in the modulator structure. These ideas are then generalized to the vector modulator of central interest in this paper.

A. Scalar $\Sigma\Delta$ Modulator

The simplest form of a $\Sigma\Delta$ modulator is shown in Fig. 3. x is the input signal, u is the integrator state, and y is the latch output. The comparator is thought of as a quantizer whose output $q(u)$ is ± 1 according to the the sign of the integrator state u. The latch samples the comparator or quantizer output $q(u)$ at the sampling frequency f_s and holds that value until the next sampling instant.

Intuitively, the $\Sigma\Delta$ modulator uses feedback to lock onto a band-limited input signal $x(t)$. As explained in [28], "Unless the input signal $x(t)$ exactly equals one of the discrete quantizer output levels, a tracking error results. The integrator accumulates the tracking error over time and the quantizer and latch feed back a value that will minimize the accumulated tracking error. Thus, the quantizer output $y(t)$ toggles about the input signal $x(t)$ so that the average quantizer output is approximately equal to the average of the input."

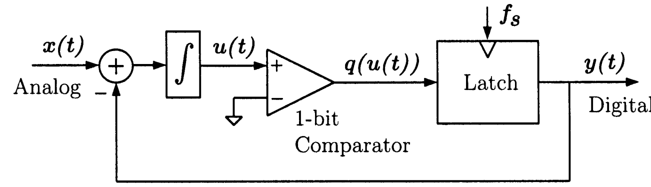

Fig. 3. Conventional sigma-delta modulator.

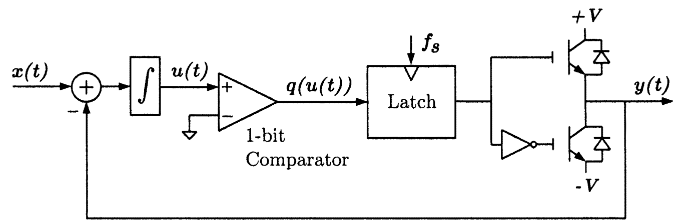

Fig. 4. Half-bridge embedded in $\Sigma\Delta$ modulator loop.

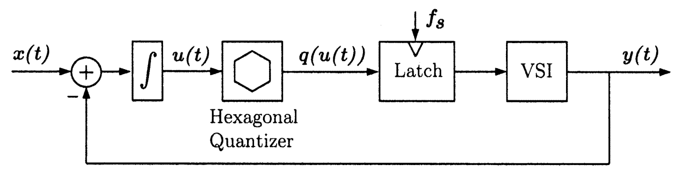

Fig. 5. VSI embedded in hexagonal $\Sigma\Delta$ modulator loop.

To illustrate how a power electronic circuit can be embedded in a $\Sigma\Delta$ modulator, consider the modulator for the half-bridge converter shown in Fig. 4. In this arrangement the gating circuitry and half-bridge are embedded into the loop following the latch in Fig. 3. The comparator and latch set the switch state for each sampling period according to the sign of the comparator input u at the sampling instant. The switch state impresses the voltage $\pm V$ on the output, $y(t)$. Since Figs. 3 and 4 are different implementations of the same overall quantizing and latch functions, the corresponding modulators have identical behavior. Thus, by taking the input signal $x(t)$ to be the desired output voltage, the actual output voltage $y(t)$ will approximate the desired output voltage. As will be seen, this approximation can be improved by generalizing the integrator in Fig. 3 to a linear filter or by increasing the sampling rate f_s.

B. Hexagonal $\Sigma\Delta$ Modulator

We now consider a vector $\Sigma\Delta$ modulator (Fig. 5) which may be applied to a VSI [7]. We assume balanced three-phase signals represented by vectors with three coordinates which sum to zero. The outputs of the VSI are the line-to-neutral voltages which may equal one of seven possible values according to the switch state. These seven space vectors are shown as dots in Fig. 2 and can be thought of as the possible output vectors of a quantizer. Here we choose the quantizer so that a quantizer input vector u maps to the dot nearest to u. The broken lines in Fig. 2 delimit the regions which map to each dot. This "hexagonal" vector quantizer is a nearest neighbor quantizer and is well known in communications [29], [30].

To apply the conventional $\Sigma\Delta$ modulator with binary quantization to the VSI requires some generalization. First, the output

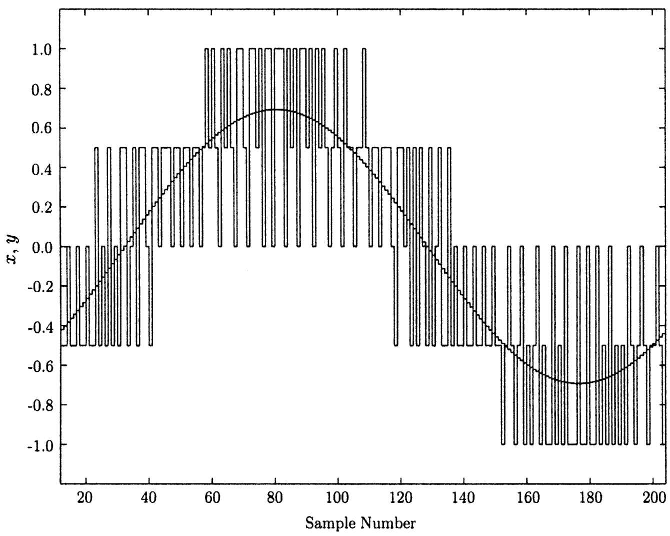

Fig. 6. Waveform of hexagonal $\Sigma\Delta$ modulator.

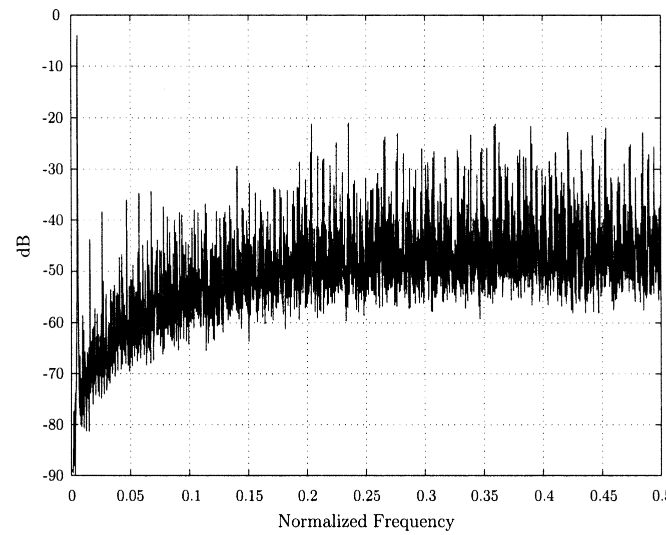

Fig. 7. Spectrum of hexagonal $\Sigma\Delta$ modulator.

voltages of the VSI are limited to a set of seven output vectors which form the truncated hexagonal lattice. If we assume a nearest neighbor partition as in the binary case, the appropriate generalization is the truncated hexagonal vector quantizer of Fig. 2. Second, all modulator signals are augmented from scalar quantities to vectors and a vector integrator replaces the scalar integrator. In an analogous manner to the half-bridge circuit of Section III-A, the VSI may be embedded in the vector $\Sigma\Delta$ loop with hexagonal quantization (Fig. 5). We call the system of Fig. 5 with the VSI omitted a hexagonal $\Sigma\Delta$ modulator.

A typical output line-neutral waveform (reference superimposed) and spectrum for the hexagonal $\Sigma\Delta$ modulator are shown in Figs. 6 and 7, respectively. The oversampling ratio is 64 and the input amplitude is 80% of full-scale linear range. Oversampling ratio (OSR) is defined as ratio $f_s/(2f_o)$ of the sampling frequency, f_s to the reference Nyquist frequency, $2f_o$. For practical converters, the OSR typically ranges from 16 to 256. However, higher OSR entails higher switching frequencies, but it should be noted that average switching frequency is less than the sampling frequency and may be calculated using the formulas of [6].

We call the frequency band $0 \leq f < f_0$ the baseband; it includes the frequency of the input signal and the band over which we wish to reduce noise in the output. The baseband is chosen according to the application specifications. The input x_n is sampled by the modulator at a frequency much higher than the baseband f_0.

IV. Linear Analysis of Scalar $\Sigma\Delta$ Modulators

This section reviews the linear analysis of scalar $\Sigma\Delta$ modulators so that the generalization to vector $\Sigma\Delta$ modulators can be made in Section V.

A. White Noise Approximation

Rigorous analysis of $\Sigma\Delta$ modulators is challenging because the quantizer nonlinearity occurs inside a feedback loop. However an approximate approach based on the results of Bennett [31] is very useful for many practical design purposes [4], [32], [28]. In this approach, the quantization noise is approximated by an input-independent additive noise source having a similar long-term sample distribution and power spectrum. The simplest noise model is white noise with a uniform distribution. Under such an approximation, the nonlinear $\Sigma\Delta$ modulator becomes a linear system with a stochastic input, and the performance can easily be derived.

For a single-loop scalar $\Sigma\Delta$ modulator without overload, the white noise approximation gives good estimates of the mean-square quantization error [33], [34] and SNR. On the other hand, the white noise approximation fails to predict idle tones in the output spectrum [35] or modulator instability [36]. However, the white-noise approximation is exact for higher order scalar $\Sigma\Delta$ architectures provided the quantizer does not overload [35].

Analysis of the output spectrum of scalar modulators without making the white noise approximation has been accomplished by Gray, Delchamps, and He [35], [37]–[40]. This exact analysis is generalized to the vector modulator with hexagonal quantization in [6], [23].

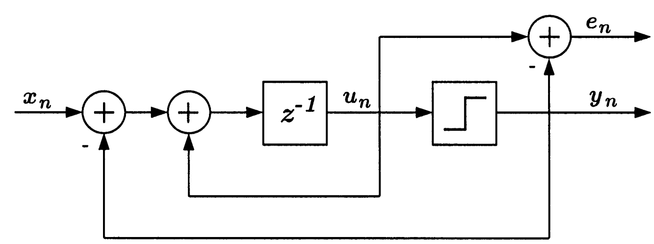

Fig. 8. Single-bit, single-loop $\Sigma\Delta$ modulator.

B. Single-Loop $\Sigma\Delta$ Modulator

The linear discrete-time model of the scalar $\Sigma\Delta$ modulator (Fig. 3) is shown in Fig. 8. The integrator is modeled with its discrete-time equivalent and the quantization process is modeled as an additive noise source $e_n = u_n - y_n$. As discussed above, e_n is assumed to be a white, uniform noise source that is statistically uncorrelated with the input.

From Fig. 8, one can write the following difference equations that describe the $\Sigma\Delta$ modulator

$$y_n = z^{-1}x_n + (1 - z^{-1})e_n. \quad (1)$$

Intuitively, the quantizer output y is the sum of the input signal x (delayed), plus a difference (or discrete time derivative) of the quantization noise e. The principle is that this difference will be a high-frequency term that can be removed by low-pass filtering to obtain the original signal.

If the input signal x is a tone in the baseband, the modulator's output spectrum is similar to that shown in Fig. 7. We see that the low-frequency quantization noise power in the baseband is attenuated relative to its total power. That is, the quantization noise will be pushed or shaped to higher frequencies. This shaping of the noise is advantageous since the high frequency noise can be removed by a low-pass filter.

C. Double-Loop $\Sigma\Delta$ Modulator

Another popular $\Sigma\Delta$ system is the double-loop $\Sigma\Delta$ introduced by Candy [4] and first rigorously analyzed by He [40]. Here, a single-loop $\Sigma\Delta$ is embedded in a second loop with an integrator in the feedforward path. It can be interpreted as a single-loop with the original input replaced by the integrated error between the input and the quantized output. Observe that output of the double-loop $\Sigma\Delta$ modulator is

$$y_n = z^{-1}x_n + (1 - 2z^{-1} + z^{-2})e_n. \quad (2)$$

In contrast to (1) this has the interpretation of being the original signal plus a second-order difference instead of a first-order difference of the single-loop system. A well-known difficulty with the double-loop $\Sigma\Delta$ modulator is the potential for quantizer overload [4].

D. Interpolative Modulators

The $\Sigma\Delta$ modulator is one of a variety of specific devices for achieving oversampled analog-to-digital conversion. $\Sigma\Delta$ modulators [41], [42] employ ideal integrators as linear filters. These analog-to-digital converters can be embedded within additional feedback loops to form high-order or multiloop $\Sigma\Delta$ modulators [4] or cascaded to form multistage $\Sigma\Delta$ modulators [43]. In the single-loop feedback case, a general system can have linear filters in both the feedforward and feedback paths. When the filtering is entirely in the feedback path, the system is called a *predictive coder* [44] of which the delta modulator [45] is a special case. When the filtering is entirely in the feedforward path, the system is called a *noise shaping coder* [44] or *interpolative coder* [28]. The $\Sigma\Delta$ modulator is an interpolative coder with the linear filter specialized to an ideal integrator.

Greater suppression of the quantization noise can be achieved by replacing the integrator with more complex higher-order filters, but the stability of the resulting system must be carefully considered. The resulting architecture in known as an interpolative modulator and is shown in Fig. 9. The input feeds a loop filter $G(z)$ that is followed by a binary quantizer. The quantized output y is fed back and subtracted from the input. This forces the average value of the quantized output to follow the average value of the input. Equation (1) generalizes to

$$Y(z) = \frac{G(z)}{1+G(z)}X(z) + \frac{1}{1+G(z)}E(z) \quad (3)$$

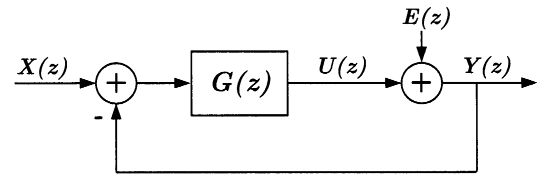

Fig. 9. Interpolative $\Sigma\Delta$ modulator.

where X, U, E, Y are the z-transforms of x, u, e, y, respectively. For example, the choice $G(z) = 1/(z-1)$ yields the modulator of Fig. 8. Equation (3) is a fundamental relation describing interpolative modulators. It states that the output of the modulator consists of the sum of two terms: the input signal x filtered by the signal transfer function (STF), $G(z)/(1+G(z))$ and the quantizer noise e filtered by the noise transfer function (NTF), $1/(1+G(z))$. If G is designed properly, the NTF will have a high-pass response, and the STF will be approximately unity in the baseband. In this manner the output SNR in the baseband can be made large [32]. An important consideration in the design of interpolative modulators, especially those of higher order (> 2), is their stability, for large-amplitude low-frequency oscillations can appear [36]. These oscillations can drive the modulator into sustained modes of integrator saturation. Another potential drawback is that higher order filters may require increased hardware complexity [46].

The noise-shaping filter G is usually designed in discrete-time. However, analog modulators have been implemented with op-amps, comparators, and latches. A discrete-time modulator can be converted to an equivalent continuous-time modulator using the impulse-invariant transformation [2], [47], [48].

V. Linear Analysis of Vector $\Sigma\Delta$ Modulators With Hexagonal Quantization

A popular measure of modulator performance is the output SNR over the baseband. SNR is mainly governed by the order of the loop filter G and the OSR. The advantage of choosing a higher order filter is an improved SNR for a given OSR. The white noise model can be used to evaluate the spectral density of the modulation noise and the total modulation noise power in the baseband for the single-loop and double-loop $\Sigma\Delta$ modulators. This calculation is done in [1] for single-bit $\Sigma\Delta$ modulators and is presented in this section for vector $\Sigma\Delta$ modulators.

A. Linear Model

We describe the three dimensional coordinates used for subsequent calculations. Let x, y, z be the three line-neutral voltages assuming a dc bus voltage of 1 V. Since these voltages are balanced they sum to zero; and in geometric terms the three dimensional vector (x, y, z) lies on the plane through the origin that is shown in Fig. 2. The inverter output line-neutral voltages, $\{(0,0,0), \pm(2,-1,-1)/3, \pm(-1,2,-1)/3, \pm(-1,-1,2)/3\}$, are the vertices of the hexagon H. The seven inverter line-line output voltages are the large dots in Fig. 2. The area of H is $|H| = \sqrt{3}$.

Let $e_n \in H$ be a sequence of uncorrelated, zero mean random vectors (i.e., white noise process) that is uniform over

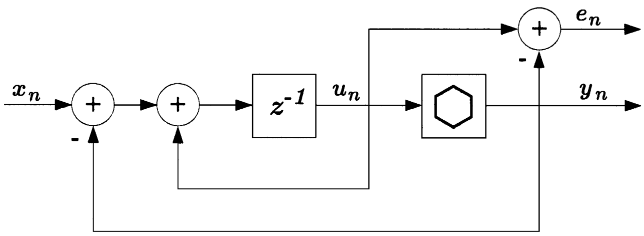

Fig. 10. Discrete time model of the single-loop hexagonal $\Sigma\Delta$ modulator.

the hexagon H and statistically uncorrelated with the input. The power spectral density matrix of e is

$$S_e = \frac{2}{f_s}\frac{1}{|H|}\int_H ee^* de = \frac{2}{f_s}\frac{5}{72}P \tag{4}$$

where

$$P = \frac{1}{3}\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} \tag{5}$$

f_s is the sampling rate, and $*$ denotes complex conjugate transpose.

This modulator's input/output relation with input vector x_n and output vector y_n is given by

$$y_n = z^{-1}x_n + (1 - z^{-1})e_n. \tag{6}$$

The output sequence y_n is the sum of the input (delayed) plus a high-frequency term.

As in the single-bit case, we can achieve greater noise suppression by replacing the integrator in Fig. 10 with an appropriately designed linear filter as in Fig. 9. The operation of this interpolative $\Sigma\Delta$ modulator can be analyzed quantitatively by modeling the quantization process by an additive white noise vector $E(z)$. Then the modulator of Fig. 9 is described by the linearized equation

$$Y(z) = (I + G(z))^{-1}G(z)X(z) + (I + G(z))^{-1}E(z) \tag{7}$$

where the inverse is a matrix inverse. Similar to the case of a single-bit $\Sigma\Delta$ modulator, (7) states that the output vector Y consists of two terms: The input signal X filtered by the signal transfer function (STF) matrix $(I+G(z))^{-1}G(z)$ and the quantizer noise E filtered by the noise transfer function (NTF) matrix $(I+G(z))^{-1}$. G can be designed so that the NTF is small in the baseband and the STF is approximately unity in the baseband. In this manner the SNR in the baseband can be made large [32]. Two possible choices of the filter G are

$$G_1(z) = \frac{z^{-1}}{1 - z^{-1}}P \tag{8}$$

$$G_2(z) = \frac{z^{-1}(2 - z^{-1})}{1 - 2z^{-1} + z^{-2}}P. \tag{9}$$

These are vector generalizations of the conventional single-loop modulator, and (up to a prefilter on the input) the conventional double-loop $\Sigma\Delta$ modulator.

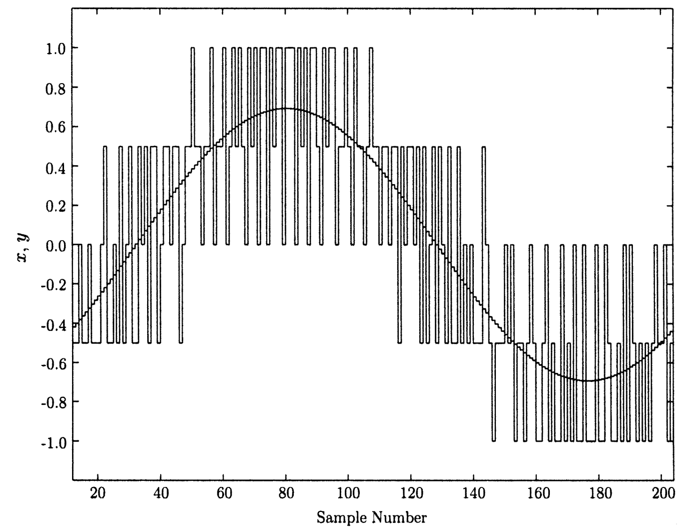

Fig. 11. Waveform of double-loop hexagonal $\Sigma\Delta$ modulator.

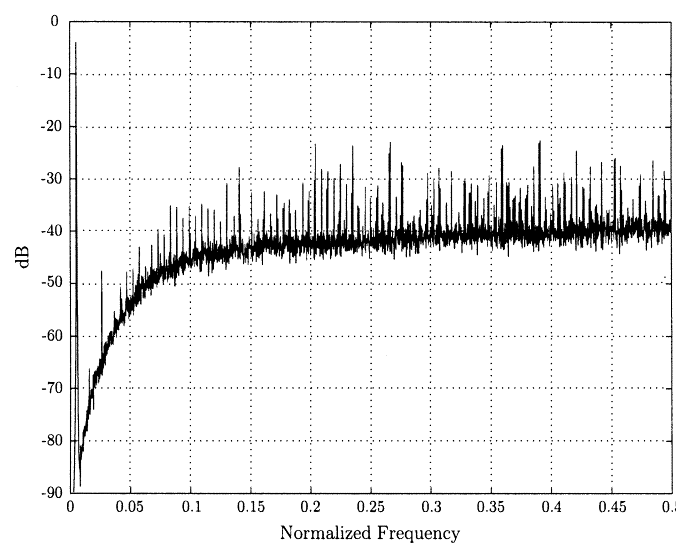

Fig. 12. Spectrum of double-loop hexagonal $\Sigma\Delta$ modulator.

This double-loop hexagonal $\Sigma\Delta$ modulator is an adaptation of the modulator proposed by Candy in [4] and is superior to the single-loop hexagonal $\Sigma\Delta$ modulator because it only requires a moderate increase in circuit complexity, and yet it achieves a 15 dB/octave tradeoff between SNR and OSR, whereas the single-loop hexagonal $\Sigma\Delta$ modulator achieves only 9 dB/octave. Furthermore, both modulators have the same stable input range (dynamic range).

A typical output line-neutral waveform (reference superimposed) and spectrum for the double-loop hexagonal $\Sigma\Delta$ modulator is shown in Figs. 11 and 12, respectively. The OSR is 64 and the input amplitude is 80% of the full-scale linear range. Note that in contrast to the single-loop case, the double-loop hexagonal $\Sigma\Delta$ modulator produces more nonadjacent transitions.

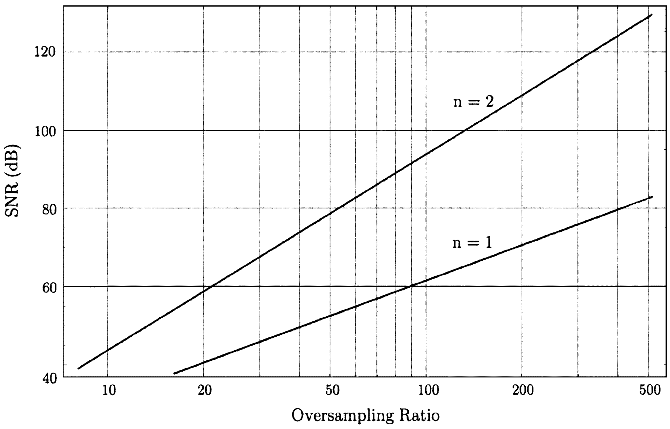

Fig. 13. SNR as a function of OSR for single-loop and double-loop hexagonal $\Sigma\Delta$ modulators.

B. Modulation Noise Analysis

Recall that if L is a multivariate linear filter with transfer function matrix $B(f)$ and $y(t) = L(x(t))$, then the spectral density matrix functions of the input x and output y are related by [49]

$$S_y(f) = B(f)S_x(f)B(f)^* \tag{10}$$

The modulation noise vector, n is the quantization error vector, e filtered by the NTF matrix. For the single-loop hexagonal $\Sigma\Delta$ modulator, the spectral density of the modulation noise is

$$\begin{aligned} S_n(f) &= (I + G_1(e^{i2\pi f/f_s}))^{-1}S_e(I + G_1^*(e^{i2\pi f/f_s}))^{-1} \\ &= 4\sin^2(\pi f/f_s)S_e. \end{aligned} \tag{11}$$

Then the modulation noise power for the single-loop hexagonal $\Sigma\Delta$ modulator in the baseband $0 \le f < f_0$ is

$$\sigma_n^2 = \int_0^{f_0} S_n(f)df \approx \frac{5\pi^2}{216}\mathrm{OSR}^{-3}P, \quad f_s^2 \gg f_0^2. \tag{12}$$

Similarly, for the double-loop hexagonal $\Sigma\Delta$ modulator, the modulation baseband noise power can be shown to be

$$\sigma_n^2 \approx \frac{\pi^4}{72}\mathrm{OSR}^{-5}P, \quad f_s^4 \gg f_0^4. \tag{13}$$

Fig. 13 shows SNR curves for the single-loop and double-loop $\Sigma\Delta$ modulator versus OSR. A sine wave input of full-scale linear range amplitude is used. These graphs are derived from (12) and (13) and demonstrate that as modulator order increases the lines become steeper; this implies that the double-loop filter realizes higher resolution from oversampling. Although the impact on filter design is complex, the lower noise floor of the double-loop $\Sigma\Delta$ modulator tends to reduce size of the output filter magnetics.

VI. Dynamic Range

The trackable input signal range for the single-loop and double-loop hexagonal $\Sigma\Delta$ modulators is the shaded hexagon which passes through the outer six output space vectors of Fig. 14. When the input signal exceeds this range, the modulator's integrators wind-up. However, overmodulation causes the input signal to exceed the stable range. To address this problem, limiters may be placed on the input signal to ensure it remains in the shaded region. This stabilization technique ensures that the output voltage waveforms gradually degrade into six-step mode as the input signal is increased beyond the stable input signal range. This technique may also be applied to higher-order (> 2) modulators which always have the potential for oscillation when the input exceeds the stable input range [32].

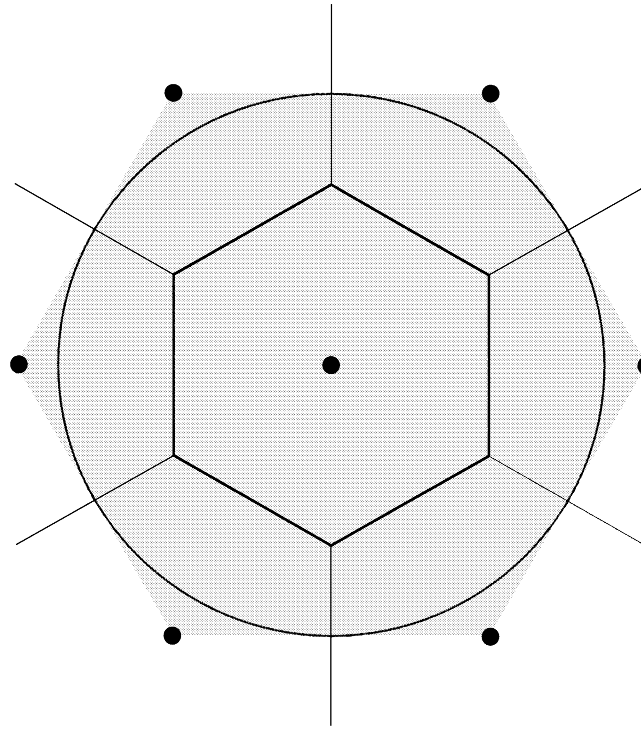

Fig. 14. Stable input signal range and boundary of the linear range for the hexagonal $\Sigma\Delta$ modulator.

A parameter of interest in power electronics is the range within which the $\Sigma\Delta$ modulator is linear for sinusoidal inputs. For the hexagonal $\Sigma\Delta$ modulator, the boundary of the linear range is given by the inscribed circle of the shaded hexagon in Fig. 14. This boundary coincides with the linear range of space vector PWM which is $2/\sqrt{3}$ larger than the linear range of sine-triangle PWM and the threefold-scalar $\Sigma\Delta$ modulator [5], [24], [25].

VII. Simulation of the Hexagonal $\Sigma\Delta$ Modulator

The following results were obtained from an adaptation of Richard Schreier's MATLAB Delta-Sigma toolbox [50]. The SNR versus input amplitude plot for a double-loop hexagonal $\Sigma\Delta$ modulator with an OSR of 32 is shown in Fig. 15. The procedure to generate this graph is: First, simulate modulator output sequences of length 8192 for a set of input tones. Second, Hann-windowed FFT's are computed for each input tone. Finally, the SNR is calculated as the the ratio of the sine wave power to the power in all in-band bins other than those associated with the input tone.

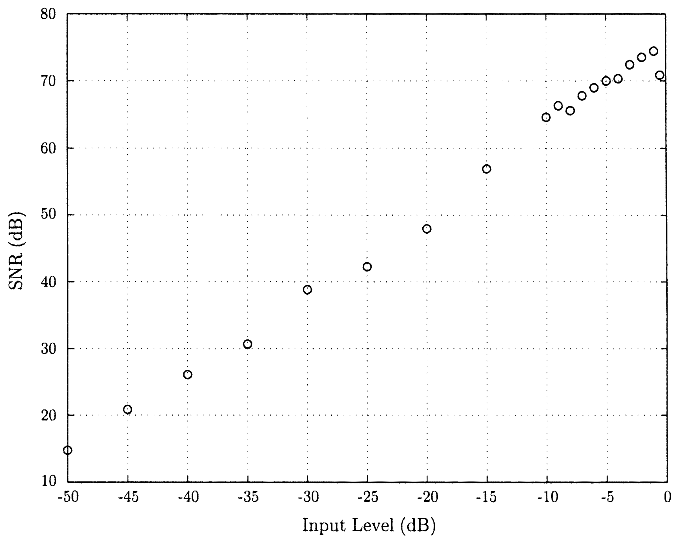

Fig. 15. SNR versus input level for double-loop hexagonal $\Sigma\Delta$ modulator with $\mathrm{OSR} = 32$.

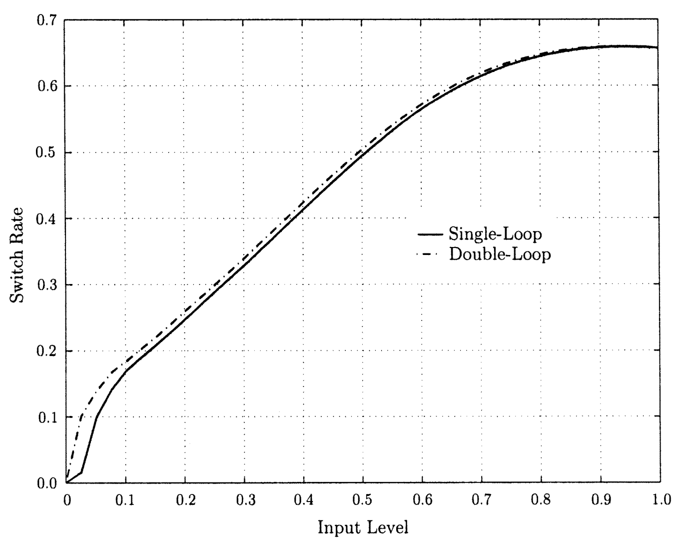

Fig. 16. Switching rate versus input level for single-loop and double-loop hexagonal $\Sigma\Delta$ modulator with OSR of 64.

Switching rate is an important performance measure in power electronic design since device switching loss is directly proportional to switching rate [6]. Fig. 16 shows the simulated average switching rate for sinusoidal inputs of amplitudes within the linear range. The simulation length is 65 536 points and the OSR is 64. In Fig. 16 the lower curve and upper curve are the single-loop and double-loop hexagonal $\Sigma\Delta$ modulators.

VIII. Conclusion

In general there is a history of methods devised in communications such as PWM being effective when adapted and applied to power electronics. In this paper, we think of the power electronic circuit acting as an analog-to-digital converter. In particular, we formulate the problem of reproducing a desired signal with a high frequency power electronic circuit by regarding the circuit as performing quantization in an interpolative $\Sigma\Delta$ modulator. The binary quantizers of conventional scalar $\Sigma\Delta$ modulators generalize easily to the vector quantizers appropriate to power electronic circuit topologies. For example, a nearest neighbor hexagonal quantizer becomes an obvious choice for a VSI.

Viewing high frequency power electronic circuits as performing quantization allows them to be regarded as part of the modulator topology (Fig. 5). Then noise-shaping methods from communications theory can then be applied to shape the quantization noise so that it is pushed to higher frequencies and will be attenuated by the load.

$\Sigma\Delta$ modulation is well established in communications and higher order $\Sigma\Delta$ modulator architectures [28] and stability issues [36], [32] are discussed extensively in the communications literature. We have found that these more complex architectures can similarly be extended to the vector case and in particular to modulators with hexagonal quantization. Thus we propose single-loop and double-loop hexagonal $\Sigma\Delta$ vector modulator designs for a VSI. Analysis and simulation of these designs show that the approach yields significant improvements in spectral performance.

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