IMMEDIATE CHANGE IN STABILITY AND VOLTAGE COLLAPSE WHEN POWER SYSTEM LIMITS ARE ENCOUNTERED

by

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Abstract

Voltage collapse is a loss of stability in large scale electric power systems which causes blackout when voltages decrease catastrophically. Voltage collapse is generally associated with bifurcation of the nonlinear power system equations; that is, the disappearance as parameters vary of the stable equilibrium at which the power system is normally operated. System limits such as generator reactive power limits and tapchanging transformer limits are thought to be important in voltage collapse. This thesis studies the statics and dynamics of power system by example and by the generic theory of saddle node and transcritical bifurcations.

When a generator of a heavily loaded electric power system reaches a reactive power limit, the system can become immediately unstable and a dynamic voltage collapse leading to blackout may follow. Load power margin calculations can be misleading if the immediate instability phenomenon is neglected. The dynamics of voltage collapse were illustrated using an example power system. But when locking a tapchaging transformer, the system can become more stable.

All the power system computations have been done using the numeric and symbolic capabilities of the Mathematica computer algebra package.

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Introduction

Voltage collapse is an instability of heavily loaded electric power systems which leads to declining voltages and blackout. It is associated with bifurcation and reactive power limitations of the power system. Power systems are expected to become more heavily loaded in the next decade as the demand for electric power rises while economic and environmental concerns limit the construction of new transmission and generation capacity. Heavily loaded power systems are closer to their stability limits and voltage collapse blackouts will occur if suitable monitoring and control measures are not taken. It is important to understand mechanisms of voltage collapse so that voltage collapse blackouts may be effectively prevented. Most of the current approaches to analyzing voltage collapse are represented in [6].

Voltage collapses are infrequent but catastrophic. One of the examples is the power system failure which occurred on July 23, 1987 in Tokyo, Japan. As the power demand increased, voltage began to fall in spite of the shunt capacitors, resulting in the voltage collapse which eventually caused the major power failure affecting a wide area. The voltage collapse led to the power outage of 8,168 MW and affected about 2.8 million customers in the area.

Voltage collapse is associated with bifurcation of the equations representing the power system. These equations are nonlinear and have multiple solutions which vary as system parameters such as load demand vary. As these parameters are varied, changes may occur in the qualitative structure of the solutions for certain parameter values. These changes are called bifurcations and the parameter values are called bifurcation values. If one of these solutions corresponded to the stable operating point of the power system, the bifurcation causes the system to lose stability and a dynamic voltage collapse follows.

Some explanations of voltage collapse involve generator reactive power output and tap changing transformers reaching limits and the research needs to address the limits which become applicable as voltage collapse is approached.

One aspect is that power systems become more vulnerable to voltage collapse when generator reactive power limits are encountered [7 - 13]. The effect of a generator reactive power limit is to immediately change the system equations. For example, the effect of a generator excitation current limit may be simply modelled by replacing the equation describing a constant output voltage magnitude by an equation describing a constant excitation current. Although the system state is unchanged, the immediate change in the system equations causes a discontinuous change in the stability margin of the system. The case in which the stability margin decreases when the reactive power limit is encountered but the system remains stable is familiar [7,10,14,12,11]. We study the case in which the system becomes immediately unstable when the reactive power limit is encountered. This possibility was mentioned by Borremans et al [9] but otherwise appears to have been overlooked. The immediate instability of the system can lead to voltage collapse and the main purpose of this thesis is to study the statics and dynamics of this mechanism for voltage collapse.

We analyze a power system example and demonstrate that at lesser loadings, encountering the reactive power limit is expected to decrease, but not destroy stability. At sufficiently high loadings, encountering the reactive power limit will immediately destabilize the system and can precipitate a voltage collapse along a trajectory which is one part of the unstable manifold of an unstable equilibrium. The movement along this trajectory is a new model for the dynamics of voltage collapse. Moreover we can argue using the theory of saddle node and transcritical bifurcations that the immediate instability and subsequent dynamic voltage collapse are likely to be typical for a general, heavily loaded power system. The results have important implications for correctly measuring the proximity to voltage collapse using load power margins; it seems that the load power margin to bifurcation can be a misleading indication of system stability unless the possibility of immediate voltage collapse is taken into account.

Another aspect is that power systems become more stable when we lock tapchanger ratios. The effect of the tapchanger limit may be simply modelled by replacing tapchanger ratio n by a constant n^{lim} in system equations. The system changes structure by losing one degree of freedom. We give an example in which locking the tapchanger makes the system more stable.

Some of the material in this thesis will appear in [1]. Other research related to this thesis included how system loads, parameters and controls contribute to voltage collapse. The knowledge of the normal vector to a hypersurface yields the best direction in parameter space to move away from the hypersurface and thus an optimal increase in the system security. We can use the formula for the normal vector to derive algorithms for computing the critical voltage collapse loadings closest to the operating load powers [3]. A direct method and an iterative method to compute the load powers at which bifurcation occurs and are locally closest to the current operating load powers have been developed and tested on a 5 bus power system [2, 4]. We have also discussed another author's paper about the calculation of the extreme loading condition of a system [5].

Chapter 1

Basic Principles of Invariant Manifolds and Bifurcations

This section states some basic concepts about the theory of generic bifurcation and invariant manifolds for nonlinear dynamical systems. The knowledge of this section is covered in [21, 22].

Suppose a nonlinear dynamical system is defined by the differential equation:

$$\dot{x} = f(x, \lambda) \qquad\qquad x \in \mathbf{R}^n, \qquad \lambda \in \mathbf{R}^m \tag{1.1}$$

where x is a vector of state variables and λ is a vector of parameters. A point x_0 is called an equilibrium point of (1.1) if $f(x_0, \lambda_0) = 0$. The eigenvalues of the Jacobian matrix evaluated at an equilibrium point usually determine the dynamic behavior in the neighborhood of the equilibrium. The equilibrium is called hyperbolic or nondegenerate when the Jacobian has no eigenvalue with zero real part.

An invariant manifold W of a system $\dot{x} = f(x)$ is a manifold in the state space with the property that for any initial vector x in W, the associated trajectory $\phi(t, x)$ stays in W for all t; that is, an invariant manifold is composed of flow lines.

The unstable manifold $W^u(x_0)$ of an equilibrium point x_0 is the manifold in the state space from which trajectories converge to x_0 as $t \to -\infty$ and which is tangent at x_0 to the subspace spanned by the eigenvectors associated with eigenvalues with positive real parts.

The stable manifold $W^s(x_0)$ of an equilibrium point x_0 is the manifold in the state space from which trajectories converge to x_0 as $t \to \infty$ and which is tangent at x_0 to the subspace spanned by the eigenvectors associated with eigenvalues with negative real parts.

There exists another invariant manifold, called the center manifold $W^c(x_0)$, which is tangent to the subspace spanned by the eigenvectors associated with the eigenvalues on the imaginary axis.

The system $\dot{x} = f(x, \lambda)$ depend on the parameter λ. Solutions now depend both on the independent variable t and on λ. Consequently, equilibrium points, Jacobian matrices at the equilibrium points, and the eigenvalues μ depend on λ. Upon varying the parameter λ, the position and the qualitative features of a equilibrium point can vary. For example, we choose an eigenvalue μ and imagine a stable focus $Re[\mu(\lambda)] < 0$ for some values of λ. When λ passes some critical value λ_*, $Re[\mu(\lambda)]$ may change sign and the equilibrium point may turn into an unstable focus. Often, qualitative changes such as a loss of stability are encountered when a degenerate case is passed since the eigenvalues are continuous function of parameters.

A commonly encountered nonlinear phenomena is bifurcation in which several equilibria interact. If a system is described by differential equations $\dot{x} = f(x, \lambda)$, the necessary conditions. for the bifurcation at (x_*, λ_*) are $f(x_*, \lambda_*) = 0$ and the Jacobian $Df|_{(x_*, \lambda_*)}$ has a zero eigenvalue. (If the Jacobian is nonsingular, then the implicit function theorem implies that there is no bifurcation in which several equilibria interact.) We describe three types of bifurcation: Saddle node bifurcation, transcritical bifurcation and pitchfork bifurcation.

(1) Saddle node bifurcation:

Saddle node bifurcation occurs when the parameters vary so that two of the multiple solutions of nonlinear equations approach each other, coalesce, and finally disappear.

A typical saddle node bifurcation has transversality conditions: $\frac{\partial^2 f}{\partial x^2}\big|_{x^*} \neq 0$ and $\frac{\partial f}{\partial \lambda}\big|_{x^*} \neq 0$. Usually, a saddle node bifurcation is generic which means the set of one parameter systems with a typical saddle node bifurcation is dense and open. Density means that we can perturb $f(x, \lambda)$ to be a typical saddle node, and openness means that if we have a typical saddle node and we perturb it sufficiently slightly, the system also has a typical saddle node.

To understand the general theory of the saddle node bifurcation, we present the simplest example of $x \in \mathbf{R}, \lambda \in \mathbf{R}$. Assume an equilibrium at zero. We expand system function by using a Taylor series at $x = 0$:

$$f(x, \lambda) = f(0,0) + xf_x + \lambda f_\lambda + \frac{1}{2}x^2 f_{xx} + x\lambda f_{x\lambda} + \frac{1}{2}\lambda^2 f_{\lambda\lambda} + h.o.t. \quad (1.2)$$

Using conditions for a saddle node bifurcation and neglecting high order terms by assuming small x, λ we have

$$f(x, \lambda) = \lambda f_\lambda + \frac{1}{2}x^2 f_{xx}$$

which can be written as

$$f(x, \lambda) = ax^2 + b\lambda = 0$$

The transversality condition ensure that a and b are nonzero. The bifurcation diagrams near $(0, 0)$ are given by $\lambda = -\frac{a}{b}x^2$ shown as figure(1). A saddle node bifurcation contains parameter values for which there are no equilibria near the point of bifurcation (two solutions disappear after bifurcation).

Figure 1 : saddle node bifurcation diagram

(2) Transcritical bifurcation:

Transcritical bifurcation occurs when the parameters vary so that two of the solutions approach each other, coalesce, and then separate with an exchange of stability.

Saddle node bifurcation are typical in a generic system. However if we assume a system satisfies $f(x^*, \lambda^*) = 0$ and $f(x^*, \lambda) = 0$ for all λ, then $x = x^*$ is a equilibrium for all parameter values. This is totally different from a saddle-node bifurcation. If we impose the conditions: $\frac{\partial^2 f}{\partial x \partial \lambda}\big|_{x^*, \lambda^*} \neq 0$ and $\frac{\partial^2 f}{\partial x^2}\big|_{x^*} \neq 0$, we have a transcritical bifurcation. The expanded system function (3.2) at $x = 0$ can be written as

$$f(x, \lambda) = \frac{1}{2} x^2 f_{xx} + x \lambda f_{x\lambda} = a x^2 + b x \lambda$$

The solutions are $x = 0$ and $x = -\frac{a}{b}\lambda$. The transcritical bifurcation diagram is shown in figure(2).

Figure 2 : Transcritical bifurcation diagram

(3) Pitchfork bifurcation:

Pitchfork bifurcation occurs in system with a symmetry when the parameters vary so that one of the solutions changes stability and a new pair of equilibria (related by symmetry) appear to one side of the point of bifurcation in parameter space.

If a system satisfies $f(x^*, \lambda^*) = 0$ and $f(x^*, \lambda) = 0$ for all λ, and also it is an odd function, a transcritical bifurcation cannot occur because $\frac{\partial^2 f}{\partial x^2}\big|_{x^*} = 0$. But another condition may be satisfied: $\frac{\partial^3 f}{\partial x^3}\big|_{x^*} \neq 0$. Then the expanded system function at $x = 0$ can be written as

$$\begin{aligned} f(x,\lambda) =& f(0,0) + x f_x + \lambda f_\lambda + \frac{1}{2}x^2 f_{xx} + x\lambda f_{x\lambda} + \frac{1}{2}\lambda^2 f_{\lambda\lambda} \\ &+ \frac{1}{6}x^3 f_{xxx} + \frac{1}{2}x^2\lambda f_{xx\lambda} + \frac{1}{2}x\lambda^2 f_{x\lambda\lambda} + \frac{1}{6}\lambda^3 f_{\lambda\lambda\lambda} + h.o.t. \end{aligned}$$

By imposing the conditions above, we have

$$f(x,\lambda) = x\lambda f_{x\lambda} + \frac{1}{6}x^3 f_{xxx} = ax^3 + bx\lambda$$

the solutions are $x = 0$ and $x^2 = -\frac{b}{a}\lambda$. This is a pitchfork bifurcation. The bifurcation diagram is shown in figure 3 (a) & (b). If $\frac{b}{a} < 0$, it is a supercritical bifurcation like (a). If $\frac{b}{a} > 0$, it is a subcritical bifurcation like (b).

Fiure 3(a) : supercritical pitchfork bifurcation

Fiure 3(b) : subcritical pitchfork bifurcation

Another kind of bifurcation is Hopf bifurcation. In the generic Hopf bifurcation, the equilibrium changes stability by interacting with a limit cycle. The Jacobian has a single, simple pair of imaginary eigenvalues at the bifurcation. We will not discuss the detail of this kind of bifurcation. More knowledge about it can be obtained from [21, 22].

In power systems, when we discuss the bifurcation with respect to the voltage collapse, usually we do not expect that pitchfork bifurcation or transcritical bifurcation occurs because power system models are usually not symmetric and equilibria are not usually fixed but always change with the parameters.

Chapter 2

Power System Model

This section summarizes the 3 bus power system example of [15,16] (see figure 4) which consists of two generators (one is slack bus), a dynamic load with capacitative support and a tapchanging transformer. The model is that of [15] with the addition of the tapchanging transformer. We assume the tap ratio n of the tapchanging transformer is unity except in section 6 where the tapchanging transformer dynamics and limits are considered. The parameter values used in this paper are identical to those of [15,16] except that the generator damping D has been increased. This eliminates the Hopf bifurcations and other oscillatory phenomena discovered in [17,18] at high loadings. A well designed power system stabilizer would suppress these oscillations. Note that the value of D has no effect on the loading at which the saddle node bifurcation occurs.

The load model includes a dynamic induction motor model with a constant PQ load in parallel. The combined model for the motor and the PQ load is

$$P_l = P_0 + P_1 + K_{pw}\dot{\delta} + K_{pv}(V + T\dot{V}) \tag{2.1}$$

$$Q_l = Q_0 + Q + K_{qw}\dot{\delta} + K_{qv}V + K_{qv2}V^2 \tag{2.2}$$

Q is a parameter which varies with the load reactive power demand. The capacitor is accounted for by adjusting E_0, Y_0 and θ_0 to E_0', Y_0' and θ_0' to give the Thevenin equivalent of the circuit with the capacitor.

The system can be described by following differential equations [15]

$$\dot{\delta}_m = \omega \tag{2.3}$$

$$M\dot{\omega} = -D\omega + P_m + \frac{E_m Y_m V}{n}\sin(\delta - \delta_m - \theta_m) + E_m^2 Y_m \sin\theta_m \tag{2.4}$$

$$K_{qw}\dot{\delta} = -K_{qv2}V^2 - K_{qv}V + Q_l - Q_0 - Q \tag{2.5}$$

$$\begin{aligned} TK_{qw}K_{pv}\dot{V} =& K_{pw}P_{qv2}V^2 + (K_{pw}K_{qv} - K_{qw}K_{pv})V \\ &+ K_{qw}(P_l - P_0 - P_1) - K_{pw}(Q_l - Q_0 - Q) \end{aligned} \tag{2.6}$$

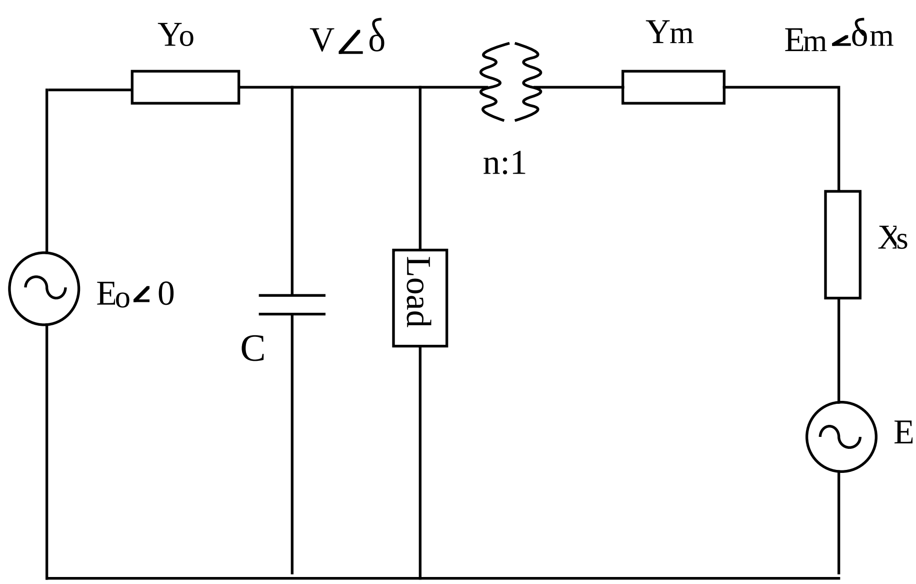
where the real and reactive powers supplied to the load by the network are

$$
\begin{aligned}
P_l = & - E_0' Y_0' V \sin(\delta + \theta_0') - \frac{E_m Y_m V}{n} \sin(\delta - \delta_m + \theta_m) \\
& + (Y_0' \sin\theta_0' + \frac{Y_m \sin\theta_m}{n^2}) V^2 \qquad (1.7)
\end{aligned}
$$

$$
\begin{aligned}
Q_l = & E_0' Y_0' V \cos(\delta + \theta_0') + \frac{E_m Y_m V}{n} \cos(\delta - \delta_m + \theta_m) \\
& - (Y_0' \cos\theta_0' + \frac{Y_m \cos\theta_m}{n^2}) V^2 \qquad (1.8)
\end{aligned}
$$

E_m is the terminal voltage of the generator and E is the internal voltage of the generator.

The state vector is $x = (\delta_m, \omega, \delta, V)$ and M, D and P_m are the generator inertia, damping and mechanical power respectively. The load parameter values are $K_{pw} = 0.4$, $K_{pv} = 0.3$, $K_{qw} = -0.03$, $K_{qv} = -2.8$, $K_{qv2} = 2.1$, $T = 8.5$, $P_0 = 0.6$, $Q_0 = 1.3$, $P_1 = 0.0$ and the network and generator parameter values are $Y_0 = 20.0$, $\theta_0 = -5.0$, $E_0 = 1.0$, $C = 12.0$, $Y_0' = 8.0$, $\theta_0' = -12.0$, $E_0' = 2.5$, $Y_m = 5.0$, $\theta_m = -5.0$, $E_m = 1.0$, $P_m = 1.0$, $D = 0.12$, $M = 0.3$, $X_s = 0.15$.



Fiure 4 : 3 – bus example power system

Chapter 3

Model of the Generator Q-Limit

The two main causes of the reactive power output Q_g reaching limit in a generator are the excitation current limit and the stator thermal limit [19, 20], shown in figure 5.

Figure 5 : Generator reactive power limit curve

The two limits have similar overall effects on the system and we only consider the excitation current limit (rotor limit). For any round rotor generator connected to an infinite bus in shown figure 6(a), we can draw a phasor diagram like figure 6(b).

Figure 6(a) : Generator connected to an infinite bus

Figure 6(b) : Phasor diagram

Here E_m is the terminal voltage of the generator, E is the internal voltage of the generator, δ is the angle between E and E_m, ϕ is the angle between I and E_m, x_s is synchronous reactance and I is current.

From figure 6(b):

$$IX_s \cos\phi = E \sin\delta$$

$$\Rightarrow P_g = E_m I \cos\phi = \frac{EE_m}{X_s} \sin\delta$$

$$IX_s \sin\phi + E_m = E \cos\delta$$

$$\Rightarrow Q_g = E_m I \sin\phi = \frac{EE_m}{X_s} \cos\delta - \frac{E_m^2}{X_s}$$

Because the generator internal voltage E is proportional to the excitation current, the excitation current limit may be modelled by E encountering a limit E^{lim}. Before the limit is encountered, $E < E^{lim}$ and E_m is controlled so that $E_m = E_m^{imp}$, the terminal voltage imposed by the regulator [10]. When the limit is encountered, $E = E^{lim}$ and E_m varies with $E_m < E_m^{imp}$. Encountering the limit may be thought of as changing from the constraint of constant E_m to the constraint of constant E. The effect of the limit on the system differential equations (2.3)-(2.6) is to replace the constant E_m by an expression involving the system state and E^{lim}. We now derive the necessary equations for this replacement.

Write the powers delivered to the network by the generator as:

$$P_g = EE_mY_s \sin\delta_s \tag{3.1}$$

$$Q_g = EE_mY_s \cos\delta_s - E_m^2 Y_s \tag{3.2}$$

where δ_s is the difference of the angle between E and E_m and Y_s is $1/X_s$. Squaring and adding (3.1) and (3.2) yields

$$\left(Q_g + E_m^2 Y_s\right)^2 + P_g^2 = (EE_mY_s)^2 \tag{3.3}$$

Real and reactive power balance at the generator gives

$$P_g = -E_m^2 Y_m \sin\theta_m + E_m Y_m V \sin(\delta_m - \delta + \theta_m) \tag{3.4}$$

$$Q_g = E_m^2 Y_m \cos\theta_m - E_m Y_m V \cos(\delta_m - \delta + \theta_m) \tag{3.5}$$

A quadratic equation in E_m can be obtained from (3.3) to (3.5):

$$E_m^2 (Y_m \cos\theta_m + Y_s)^2 - 2E_m Y_m V (Y_m \cos\theta_m + Y_s) \cos(\delta_m - \delta + \theta_m)$$

$$+Y_m^2 V^2 \cos^2(\delta_m - \delta + \theta_m) + E_m^2 Y_m^2 \sin^2\theta_m - 2E_m Y_m^2 V \sin\theta_m \sin(\delta_m - \delta + \theta_m)$$

$$+Y_m^2 V^2 \sin^2(\delta_m - \delta + \theta_m) - E^2 Y_s^2 = 0 \tag{3.6}$$

The limited system equations are obtained by solving (3.6) with E replaced by E^{lim} for the positive solution of E_m and substituting it into (2.3)-(2.6). It remains to compute the value of E^{lim}. Suppose Q_g reaches its limit at $Q = Q^{lim}$. Then E^{lim} is computed from equations (3.3)-(3.5) where δ_m, δ, V are equilibrium solutions of (2.3)-(2.6) when $Q = Q^{lim}$.

Chapter 4

Immediate Voltage Collapse when a Limit is Encountered

We present two cases of the example power system encountering an excitation current limit. It is convenient to write J for the Jacobian of the unlimited system evaluated at the operating equilibrium x_0 and J^{lim} for the Jacobian of the limited system evaluated at x_0. The first case is well known and the bifurcation diagrams of the unlimited and limited systems are shown in figure 7(a). The limit is encountered at $Q = Q^{lim} = 11.0$. When the limit is encountered the system changes structure and J changes to J^{lim}. The eigenvalues of J^{lim} differ from the eigenvalues of J but their real parts remain negative (see table 1, case (a)). The equilibrium at x_0 of the limited system has reduced stability but remains stable and the system reactive power margin $Q^* - Q^{lim}$ is reduced. Note that the upper portion of each of the bifurcation diagrams is stable and the lower portion is unstable; in this case x_0 is on the upper and stable portion of both the unlimited and limited system bifurcation curves.

In the second case, the limit is encountered at the higher system loading $Q = Q^{lim} = 11.4$. The system changes so that J^{lim} has an eigenvalue with positive real part (see table 1, case(b)). In this case x_0 is on the upper and stable portion of unlimited system bifurcation curve and the lower and unstable portion of limited system bifurcation curve (see figure 7(b)). The operating equilibrium x_0 becomes immediately unstable when the limit is encountered and the system dynamics will move the system state away from x_0. Now we describe the dynamic consequences of the instability; see [21,22] for background in dynamical systems and bifurcations.

Figure 7(a). Limit encountered at a lesser loading

Figure 7(b). Immediate instability when limit is encountered

Table 1

The unstable equilibrium x_0 has a one dimensional unstable manifold W^u which consists of two trajectories W^u_- and W^u_+ leaving x_0 and x_0 itself (see the idealized sketch of figure 8). W^u is a smooth curve passing through x_0 which is tangent at x_0 to the eigenvector v of J^{lim} associated with the positive eigenvalue. x_0 also has a three dimensional stable manifold W^s which divides the state space near x_0 into two parts. Note that the stable manifolds sketched in figure 8 have an additional dimension which is not shown.

Figure 8. Idealized sketch of limited system dynamics

The system state is initially at x_0 but cannot remain there because of the inevitable small perturbations on the state. If the state is perturbed from x_0 to one side of W^s, it will be attracted towards W^u_- and we can simply approximate the dynamics by motion along W^u_-. Similarly, if the state is perturbed from x_0 to the other side of W^s, it will be attracted towards W^u_- and we can simply approximate the dynamics by motion along W^u_-. In short, the dynamical consequences of the immediate instability are motion along the trajectory W^u_- or the trajectory W^u_+. Either outcome is possible and there seems no reason to regard W^u_- or W^u_+ as more likely.

We integrated the differential equations of the limited system starting near x_0 to find the outcome of motion along W^u_- or W^u_+. (The initial conditions were chosen to be $x_0 \pm 0.001v$ and the Gear (stiff) ordinary differential equation solver in NAG Fortran Library Routines [23] was used since differential equations are stiff near a saddle node bifurcation.) The corresponding time histories of V are shown in figure 9. The trajectory W^u_- tends to the nearby stable equilibrium x_1^{lim} and the initial portion of the slow, oscillatory convergence of V to x_1^{lim} is shown in the upper graph of figure 9. (This may not occur in practice because the voltage control system could prevent the voltage from rising [14].) The trajectory W^u_+ diverges so that V decreases and the motion along W^u_+ shown in the lower graph of figure 9 is a voltage collapse. Thus we describe a model of a new mechanism for voltage collapse:

The operating equilibrium x_0 becomes immediately unstable when a reactive power limit is encountered and one of the possible dynamical consequences is voltage collapse along part of the unstable manifold of x_0.

This model of immediate voltage collapse has some similarities with the center manifold collapse model [15]; the voltage collapse dynamics can be modelled by movement along a particular trajectory. However, the trajectory is part of an unstable manifold instead of the unstable part of a center manifold and there is also the possibility of convergence to a nearby stable equilibrium along the other part of the unstable manifold.

Figure 9. Possible dynamics caused by immediate instability

Chapter 5

Approximate analysis showing that immediate voltage collapse can occur for high loading

We expect the immediate instability of our example to be typical for sufficiently high loadings. This can be demonstrated analytically by approximating the equations of the example system as follows. First consider the relationship between V, E and E_m. Write $\alpha = \delta_m - \delta$, and approximate the line impedance to be purely reactive so that $\theta_m = 0$. Equation (3.6) becomes

$$E_m^2(Y_m + Y_s)^2 - 2E_m Y_m V(Y_m + Y_s)\cos\alpha + Y_m^2 V^2 - E^2 Y_s^2 = 0 \tag{5.1}$$

Assuming that α is small ($\leq 20°$), we approximate $\cos\alpha \approx 1$, and write $Y_m' = Y_m + Y_s$ to obtain the reduced equation:

$$E_m = \frac{EY_s}{Y_m'} + \frac{Y_m V}{Y_m'} \tag{5.2}$$

We substitute (5.2) into system differential equations (2.5) and get a decoupled dynamical equation for the limited system with the left hand side set to zero:

$$\begin{aligned} Q_0 + Q - \left(-K_{qv} + E_0' Y_0' \cos\delta + \frac{EY_m Y_s}{Y_m'}\cos\alpha\right) V \\ + \left(K_{qv2} + Y_0' + Y_m - \frac{Y_m^2}{Y_m'}\cos\alpha\right) V^2 = 0 \end{aligned} \tag{5.3}$$

Using the approximations of [16], we suppose all angles are small:

$$Q_0 + Q - \left(-K_{qv} + E_0' Y_0' + \frac{EY_m Y_s}{Y_m'}\right) V + \left(K_{qv2} + Y_0' + Y_m - \frac{Y_m^2}{Y_m'}\right) V^2 = 0 \tag{5.4}$$

Equation (5.4) describes the bifurcation curve (the relation between V and Q) of the limited system as shown in figure 7(b). The gradient of (5.4) is

$$\frac{dQ}{dV} = -K_{qv} + E_0'Y_0' + \frac{EY_mY_s}{Y_m'} - 2V\left(K_{qv2} + Y_0' + Y_m - \frac{Y_m^2}{Y_m'}\right) \tag{5.5}$$

For comparison, we write the equation of the bifurcation curve for the unlimited system as [16]:

$$Q_0 + Q - (-K_{qv} + E_0'Y_0' + E_mY_m)V + (K_{qv2} + Y_0' + Y_m)V^2 = 0 \tag{5.6}$$

The gradient of (5.6) is

$$\frac{dQ}{dV} = -K_{qv} + E_0'Y_0' + E_mY_m - 2V(K_{qv2} + Y_0' + Y_m) \tag{5.7}$$

At the intersection of two curves, we can substitute (5.2) into (5.5). Then (5.5) becomes:

$$\frac{dQ}{dV} = -K_{qv} + E_0'Y_0' + E_mY_m - 2V(K_{qv2} + Y_0' + Y_m) + V\frac{Y_m^2}{Y_m'} \tag{5.8}$$

Comparing (5.7) and (5.8), it is clear that the gradient of the bifurcation curve of the unlimited system is always larger than the gradient of the bifurcation curve of the limited system at their intersection since VY_m^2/Y_m' is always positive.

Now suppose the limit is encountered at the saddle node bifurcation of the unlimited system so that $Q^{lim} = Q^*$, the value of Q at the saddle node bifurcation. For the unlimited system we have the gradient $\frac{dQ}{dV}|_{V_*} = 0$ at the saddle node bifurcation point where V_* is the voltage at the saddle node bifurcation. For the limited system, it follows that $\frac{dQ}{dV}|_{V_*} > 0$ and hence $\frac{dV}{dQ}|_{Q^*} > 0$. Therefore the two curves cross each other at the bifurcation point as shown in figure 10. Because the lower portion of the curve is unstable, the equilibrium x_0 becomes unstable and J^{lim} has a positive eigenvalue when the limit $Q^{lim} = Q^*$ is encountered.

Now we suppose that the limit is encountered at a loading Q^{lim} less than and close to Q^*. Since the eigenvalue of J^{lim} is a continuous function of Q^{lim}, we conclude that the equilibrium x_0 will become immediately unstable when the limit is encountered for Q^{lim} sufficiently close to Q^*.

Figure 10. Immediate instability at a saddle node bifurcation

Chapter 6

Immediate instability and voltage collapse in a general power system model

The particular example presented above shows that a sufficiently heavily loaded but stable system can become immediately unstable when a reactive power limit is encountered. The dynamical consequences of this instability are either collapse along the unstable manifold trajectory W^u_+ or convergence to a nearby stable equilibrium along the unstable manifold trajectory W^u_-. Now we argue that this description is expected to apply in the general case. Our main assumptions are that applying a reactive power limit does not increase the margin of system stability, the phenomena occurring are generic and a simplification that only one bifurcation occurs.

Consider a general power system modelled by smooth parameterized differential equations $\dot{x} = f(x, \lambda)$, where $x \in \mathbf{R}^n$ is the system state and $\lambda \in \mathbf{R}^m$ are slowly changing system parameters such as real and reactive load powers [15]. We suppose the system is operated at a stable equilibrium x_0 when the parameters are λ_0. When a reactive power limit is encountered the system equations immediately change to $\dot{x} = f^{lim}(x, \lambda)$ but the position of the equilibrium x_0 is unchanged. That is, $0 = f(x_0, \lambda_0) = f^{lim}(x_0, \lambda_0)$.

Now suppose that the equations $\dot{x} = f(x, \lambda)$ are gradually changed into the equations $\dot{x} = f^{lim}(x, \lambda)$. This can easily be done by combining f and f^{lim} into new equations

$$\dot{x} = g(x, \lambda_0, k) \tag{6.1}$$

with a parameter k so that $g(x, \lambda_0, 0) = f(x, \lambda_0)$ and $g(x, \lambda_0, 1) = f^{lim}(x, \lambda_0)$. We also require that

$$g(x_0, \lambda_0, k) = 0 \qquad \text{for} \qquad k \in [0, 1] \tag{6.2}$$

In short, we construct a homotopy joining f and f^{lim} which preserves the equilibrium at x_0. Note that λ is fixed at λ_0 as the parameter k is varied. k gradually increasing from 0 to 1 has the effect of gradually applying the reactive power limit to the system. This allows the change in structure between f and f^{lim} to be studied using bifurcation theory. However, we do not seek to represent the manner in which the reactive power limit is applied in practice by the gradual increase in k.

(6.1) is a one parameter system of differential equations with the restriction (6.2) of an equilibrium at x_0. If we assume that this is a generic one parameter system of differential equations, then the only bifurcations which can occur are the transcritical bifurcation and the Hopf bifurcation [21]. (In the set of all smooth one parameter differential equations without restrictions or symmetries, the generic bifurcations are the saddle node bifurcation and the Hopf bifurcation. An equilibrium disappears in a saddle node bifurcation and therefore the restriction (6.2) precludes saddle node bifurcations.) In a generic transcritical bifurcation two equilibria approach each other, coalesce and then separate with an exchange of stability. The Jacobian has a single, simple zero eigenvalue at the bifurcation. If one of the equilibria is stable before the bifurcation, then the other is type one unstable. After the bifurcation the equilibria are also stable and type one unstable but each equilibrium has changed its stability. Figure 11 shows a typical bifurcation diagram for the transcritical bifurcation in which the solid line indicates stable equilibria and the dashed line denotes unstable equilibria. In the generic Hopf bifurcation, the equilibrium changes stability by interacting with a limit cycle. The Jacobian has a single, simple pair of imaginary eigenvalues at the bifurcation. Of course it is also generic for there to be no bifurcation for $k \in [0, 1]$.

We first consider the special case of encountering a reactive power limit at the point of voltage collapse; that is, λ_0 is chosen so that $\dot{x} = f(x, \lambda)$ with parameter

λ has a generic saddle node bifurcation at (x_0, λ_0). Then x_0 is a degenerate equilibrium formed by the coalescence of a stable equilibrium and a type one unstable equilibrium and the Jacobian of f evaluated at (x_0, λ) is singular [15]. Since the Jacobian of f evaluated at (x, λ_0) and the Jacobian of g evaluated at $(x, \lambda_0, 0)$ are identical, the Jacobian of g evaluated at $(x, \lambda_0, 0)$ is also singular. Therefore $(x_0, \lambda_0, 0)$ is also a bifurcation point of the system (6.1) with k as parameter. If we consider only generic phenomena, then the bifurcation of (6.1) at $(x_0, \lambda_0, 0)$ is a transcritical bifurcation. The Hopf bifurcation is precluded by the single zero eigenvalue and absence of nonzero imaginary eigenvalues of the Jacobian at x_0 at the generic saddle node bifurcation of $\dot{x} = f(x, \lambda)$ with λ as parameter.

In a generically occuring transcritical bifurcation the bifurcation diagram (suitably reduced to the center manifold of the suspended system [22]) is as shown in figure 11. There are two possible “directions” through the bifurcation as k is increased from zero so that the equilibrium x_0 is either stable or unstable for small positive k. Since reactive power limits are generally observed to limit system performance, it seems likely that partially applying a reactive power limit (imposing small positive k) should destabilize rather than stabilize the system. Thus the more usual case to consider should be x_0 unstable for small positive k. For small positive k there is a type one unstable equilibrium x_1 close to x_0 and part of the unstable manifold W_-^u of x_0 is a trajectory tending to x_1. In this case, if we further assume for simplicity that there are no further bifurcations as k increases from a small positive value to one, then we can conclude that at $k = 1$, the system $\dot{x} = g(x, \lambda_0, 1) = f^{lim}(x, \lambda_0)$ has x_0 unstable and type one. Moreover, there is a stable equilibrium x_1 in the vicinity and the part W_-^u of the unstable manifold of x_0 is a trajectory tending to x_1.

The assumption of genericity of the one parameter differential equations (6.1) implies that the occurence of the transcritical bifurcation is robust to small changes in (6.1). In particular, if we consider a sufficiently heavily loaded but still stable system $f(x, \lambda_0')$ with λ_0' chosen close to λ_0 so that $f(x, \lambda_0')$ has a stable equilibrium x_0', then the corresponding homotopy $g(x, \lambda_0', k)$ will have a transcritical bifurcation as k increases from zero to one and we can extend the conclusions for the case of a reactive power limit encountered at the saddle node bifurcation to the case of a reactive power limit encountered just before the saddle node bifurcation. The only difference is that x_0 is stable for $k = 0$ and the transcritical bifurcation will occur at a positive value of k. Figure 11 shows the bifurcation diagram for the change in stability of the example system in case (b) (the black circles are the data we computed).

Figure 11. Transcritical bifurcation diagram when limit is gradually applied

Chapter 7

Locking the Tap-Changing Transformer can Improve the System Stability

Liu and Vu [13] describe examples in which locking the tap ratios of tapchanging transformers can improve the system stability and avoid voltage collapse. We show how locking a tap ratio n can be described as the dynamic variable n encountering a limit n^{lim} and confirm the results of [13] in our example power system. Note that the tap ratio may also encounter a limit when the maximum or minimum tap setting is reached. A typical range for a tapchanger tap ratio is 10% of the nominal value of n.

To consider dynamics of the tapchanging transformer we include the tap ratio n in the state vector and add the following tapchanging dynamics to the differential equations (2.3)-(2.6) [25]:

$$\dot{n} = \frac{1}{T_s}(V_s - V) \tag{6.1}$$

Since a step in tap position typically changes the voltage in small increments (e.g. 0.625 percent of the nominal voltage [13]), using a continous tapchanging transformer model instead of a discrete model is practically acceptable. In (6.1), T_s is the time constant and V_s is the reference voltage. We use $T_s = 2.0$ and $V_s = 1.0$.

The effect of the n encountering the limit n^{lim} is that n is set to the constant value n^{lim} and equation (6.1) is omitted. Thus the system changes structure by losing one degree of freedom; the state vector dimension and the number of differential equations are reduced by one. The operating equilibrium when the limit is encountered is x_0. We compute the Jacobian J of the unlimited system

(2.3)-(2.6) and (6.1) at x_0 and its four eigenvalues as shown in table 2. We set $n = n^{lim} = 1.1$ in equations (2.3)-(2.6) and compute the Jacobian J^{lim} of the limited system (2.3)-(2.6) at x^{lim} and its three eigenvalues as shown in table 2. Comparing the eigenvalues of the unlimited and limited system shows that the equilibrium x_0 is more stable in the limited system so that locking the tap ratio improves the stability in this example.

Eigenvalues of J			Eigenvalues of J^{lim}	
-106.79	-2.48±1.9j	-6.24	-106.82	-5.58±1.95j

Table 2

Chapter 8

Discussion and Conclusions

The instantaneous change in the system equations when a generator reactive power limit is encountered causes the system dynamics and structure to instantaneously change, although the system state is unchanged. In particular, the Jacobian at the operating equilibrium and the closest unstable equilibrium change discontinuously. It follows that most of the voltage collapse indices proposed in the literature which are functions only of the system *before* a reactive power limit is encountered are *discontinuous* when the reactive power limit is encountered. We emphasize this point because the literature often neglects this somewhat unpalatable discontinuity or misleadingly describes it as a discontinuity in the derivative of the index. Exceptions are indices such as the total generated reactive power index of Begovic and Phadke [11] which are a functions of the system operating point only and hence continuous when a reactive power limit is encountered. Two other exceptions are the energy function index of Overbye and DeMarco [24] and the load power margin index when proper account is taken of the reactive power limits as, for example, in Van Cutsem [14].

One important consequence of a generator reactive power limit causing an immediate instability is that the load power margin can be misinterpreted to give an incorrect measure of system stability. A load power margin measures the load increase that the system can sustain before bifurcation but is only valid when the system is operated at a stable equilibrium. If the operating equilibrium becomes immediately unstable, the load power margin is incorrect because the system is (at least momentarily) at an *unstable* equilibrium. (If the system were operated at the nearby stable equilibrium, then the load power margin would be a valid measure of system stability. We note that the system state might converge to the nearby stable equilibrium as a result of the immediate instability but the possibility of voltage collapse as a result of the immediate instability is at least as likely.)

In case (a) the example power system retains stability when the reactive power limit is encountered and the reactive power margin $Q^* - Q$ does measure the system stability (see figure 7(a)). In the more heavily loaded case (b), the system becomes immediately unstable when the reactive power limit is encountered and the system may immediately collapse whereas the reactive power margin $Q^* - Q$ can be misinterpreted as a positive margin of stability (see figure 7(b)). We conclude that the stability of the limited system equilibrium should be checked when load power margins are computed.

Borremans et al [9] recognized that a generator reactive power limit could precipitate a voltage collapse in the manner of figure 7(b), but regarded this possibility as more theoretical than the case of figure 7(a). We show by an example and general arguments that immediate voltage collapse is likely to be typical for a sufficiently highly loaded system encountering a generator reactive power limit. However, our results have not established that immediate voltage collapse occurs over a significant range of loadings up to the bifurcation. That is, we have not excluded the possibility that the immediate voltage collapse might only exist for a small interval of loadings before bifurcation. (Our power system example is not conclusive in this regard because of its small size and the lack of validation of the load model.) We conclude that immediate instability when a generator reactive power limit is encountered is a plausible cause of voltage collapse whose relative importance is not yet established.

We also study the simplest and most likely dynamical consequences of the immediate instability in a general power system using the theory of generic saddle node and transcritical bifurcations. The dynamical consequences are either convergence to a nearby stable equilibrium or a voltage collapse. The voltage collapse dynamics may be modelled by movement along a specific trajectory which is part of the unstable manifold of the unstable equilibrium. This is a new model for voltage collapse dynamics with some similarities with the center manifold theory of voltage collapse [15]. The dynamics of voltage collapse are easy to simulate by numerical integration along the unstable manifold and were illustrated using the example power system.

We also confirm the results of [13] in our example power system that locking the tap ratio improves the stability since the equilibrium x_0 is more stable in the limited system in this example.

We hope our example and general analysis will encourage more study of the interaction of system limits and voltage collapse. In particular, we note the usefulness of the transcritical bifurcation in understanding how encountering a system limit can affect the system behaviour.

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