Estimating propagation and distribution of load shed in simulations of cascading blackouts

Janghoon Kim, Kevin R. Wierzbicki, Ian Dobson, Rodney C. Hardiman

***Abstract*—We estimate with branching process models the propagation of load shed and the probability distribution of total load shed in simulated cascading blackouts of electric power systems. The average propagation of the simulated load shed data is estimated and then the initial load shed is propagated with two different branching process models of cascading failure to estimate the probability distribution of total load shed. The first model discretizes the load shed and then applies a Galton-Watson branching process. The second model is a continuous state branching process. We initially test the estimated distributions of total load shed using load shed data generated by the OPA and TRELSS cascading outage simulations. We discuss for the first model the effectiveness of the estimator in terms of how many cascades need to be simulated to predict the distribution of total load shed accurately.**

***Index Terms*—power system reliability, risk analysis, cascading failure, stochastic processes, branching process**

I. Introduction

Large blackouts are rarer than small blackouts, but are costly to society and have substantial risk [1]. Large blackouts generally become widespread by a cascading process of successive failures [2]–[4]. It is useful to study mechanisms of cascading failure so that blackout risk may be better quantified and mitigated. The electric power infrastructure is vital in maintaining our society, and maintaining high reliability is especially important as the electric power infrastructure is being transformed in response to changes in energy sources, loads, technologies, markets and climate.

There are many and diverse mechanisms in power systems by which components tripping or failures cause further components tripping [1]–[5]. These include line overloads, failures in protection, communication, maintenance or software, various types of instability, and errors in coordination, situational awareness, and planning or operations. It is infeasible to analyze a full range of these mechanisms with one simulation, so cascading failure simulations model and analyze a selected subset of these mechanisms [5]. In this paper we analyze load shed data produced by the OPA simulation of cascading line overloads and the TRELSS simulation of multiple cascading mechanisms. Each simulated cascade has successive generations in which transmission lines are tripped and load is shed, and the total amount of load shed is a measure of the size of the blackout.

In the OPA simulation [6], the power system is represented with a standard DC load flow approximation. Starting from a solved base case, blackouts are initiated by random line outages. Whenever a line is outaged, the generation and load is redispatched using standard linear programming methods. The cost function is weighted to ensure that load shedding is avoided where possible. If any lines were overloaded during the optimization, then these lines are outaged with a specified probability. The process of redispatch and testing for outages is iterated until there are no more outages. Then the total load shed is the power lost in the blackout. The OPA model neglects many of the cascading processes in blackouts and the timing of events. However, the OPA model does represent in a simplified way a dynamical process of cascading overloads and outages that is consistent with some basic network and operational constraints. This paper uses a restricted form of the OPA model in which the power grid is fixed and does not evolve or upgrade; in other work the OPA model also represents the complex dynamics of an evolving grid [1], [7], [8].

TRELSS (Transmission Reliability Evaluation of Large Scale Systems) is a commercially available tool for reliability assessment of composite generation and transmission systems developed by EPRI in cooperation with Southern Company Services [9], [10]. TRELSS uses enumeration of generation and transmission contingencies to evaluate power system reliability. System failure criteria include circuit overloads, voltage violations, capacity deficiency, islanding, and area interchange failures. Here we use the TRELSS Simulation Module (TSM) to simulate cascading failure.

Branching processes have long been used in a variety of applications to model cascading processes [11], [12], but their application to the risk of cascading failure is recent [13], [14]. In particular, Galton-Watson branching processes give a high-level and tractable probabilistic model of cascading failure. There is some initial evidence that Galton-Watson branching processes can capture some general features of simulated and observed cascading line trips [14]–[18] and can approximate other probabilistic models of cascading failure [13], [19], [20]. The branching process gives a simple probabilistic description of the cascading process as an initial disturbance followed by an average tendency for the cascade to propagate in stages

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until the cascade dies out or all the components fail.

In previous work [15], [17], [21], we obtained cascading failure data from the OPA simulation with 118 and 300 bus IEEE standard test systems, estimated the initial number of lines tripped and average propagation of line trips from this data, and then used the branching process to predict the probability distribution of the total number of lines tripped. This predicted distribution was then shown to match well with the empirical distribution produced by exhaustively running the OPA simulation in most of the cases tested. It is useful to predict the distribution of total number of lines tripped via the branching process because this can be done with significantly fewer simulated cascades. The total number of lines tripped is a measure of blackout size of interest to utilities, whereas load shed is a measure of blackout size of much more direct interest to all users of electricity. In this paper we test estimating the propagation and probability distribution of load shed.

In contrast with the case of number of lines tripped, which are nonnegative integers, the amounts of load shed are nonnegative real numbers. We estimate the initial distribution of load shed and the average propagation λ from the simulated load shed data. Then we discretize the continuous initial distribution of load shed and use this discrete distribution as the initial distribution of a Galton-Watson branching process with average propagation λ to estimate a discretized distribution of the total load shed.

We also use an alternative approach [21]–[23] using a continuous state branching process model [24]–[26] to estimate the distribution of the total load shed. The offspring distribution is assumed to be a gamma distribution, with mean λ and variance estimated from the data. Then computer algebra is used to manipulate cumulant generating functions to compute the distribution of total load shed.

We assume some detailed explanations in previous papers. The OPA simulation is explained in [6], the TRELSS simulation is explained in [9], [10], and a variety of cascading failure methods and simulations are explained and referenced in [1], [5]. The branching process model and parameter estimation are explained in [17] and branching processes are explained in [11], [12], [24]–[26]. Initial versions of parts of this paper appeared in the conferences [22], [27].

II. Estimating propagation and distribution of load shed with branching processes

This section describes the procedures for estimating the propagation and probability distribution of load shed with a branching process.

A. Processing load shed data

For each simulated cascade, the total load shed as well as the load shed at each intermediate generation of the cascade is recorded. The first step is to round very small load shed amounts that are considered negligible (less than 0.5% of total load) to zero. Then the data is modified so that each cascade starts with a nonzero amount of shed. In particular, the first step is that cascades with no load shed are discarded. The remaining K cascades are those with some non-negligible load shed. Therefore the computed statistics, such as the probability distributions of initial and total load shed, are conditioned on the cascade starting with some non-negligible amount of load shed. The second step is that for the cascades with no load shed in initial generations and non-negligible load shed in subsequent generations, we discard the initial generations with no load shed so that generation zero always starts with a positive amount of load shed.

Now the data has K cascades that start with non-negligible load shed. Letting X_n^i denote the load shed at generation n of cascade i, the data looks like this:

	gen. 0	gen. 1	gen. 2	$\cdots$
cascade 1	$X_0^{(1)}$	$X_1^{(1)}$	$X_2^{(1)}$	$\cdots$
cascade 2	$X_0^{(2)}$	$X_1^{(2)}$	$X_2^{(2)}$	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
cascade K	$X_0^{(K)}$	$X_1^{(K)}$	$X_2^{(K)}$	$\cdots$

The total load shed in cascade i is

$$Y^{(i)} = X_0^{(i)} + X_1^{(i)} + \ldots$$

The estimator for the average propagation λ is the standard Harris estimator [12], [24], [31], [32]:

$$\widehat{\lambda} = \frac{\sum_{k=1}^{K} \left(X_1^{(k)} + X_2^{(k)} + ... \right)}{\sum_{k=1}^{K} \left(X_0^{(k)} + X_1^{(k)} + ... \right)} \tag{1}$$

The Harris estimator (1) is an asymptotically unbiased maximum likelihood estimator [12], [32]. Our cascading process is assumed to be subcritical ($\lambda < 1$) and saturation effects are neglected. (In supercritical or saturating cases, other estimators for λ are appropriate as discussed in [17].)

The load shed amounts $X_0^{(1)}, X_0^{(2)}, \cdots, X_0^{(K)}$ are samples from the probability distribution of initial load shed, assuming that some non-negligible load is shed. The average initial load shed is estimated as

$$\hat{\theta} = \frac{1}{K} \sum_{k=1}^{K} X_0^{(k)}. \tag{2}$$

B. Discretization Method

To estimate the probability distribution of total load shed from the initial load shed and the estimated propagation $\widehat{\lambda}$, we discretize the samples of the initial load distribution and assume they are propagated by a Galton-Watson branching process with a Poisson offspring distribution of mean $\widehat{\lambda}$.

There are general arguments suggesting that the choice of a Poisson offspring distribution is appropriate. A Poisson approximation is a good approximation when there are a large number of other load shed decrements that have small probability and are independent [28]. Moreover, the assumption of strict independence between the load decrements can be relaxed by allowing some dependence between load decrements on the same or a small number of neighboring buses [29] or even also some small dependence on load decrements at other buses [30].

We choose an amount of load shed Δ as the unit of discretization. Then each initial load shed sample X_0^k is discretized to an integer multiple of Δ:

$$Z_0^k = \text{int}\left[\frac{X_0^k}{\Delta} + 0.5\right], \tag{3}$$

where $\text{int}[x]$ = integer part of x.[1] Write Z_0 for the initial load shed expressed in integer multiples of Δ. Then the empirical probability distribution of Z_0 is

$$P[Z_0 = z_0] = \frac{1}{K}\sum_{k=1}^{K} I[Z_0^k = z_0]. \tag{4}$$

It is known in branching processes that the total number of individuals starting from z_0 parents in a branching process with Poisson offspring distribution of mean $\hat{\lambda}$ is distributed according to the Borel-Tanner distribution:

$$P[r \text{ total failures}] = z_0\hat{\lambda}(r\hat{\lambda})^{r-z_0-1}\frac{e^{-r\hat{\lambda}}}{(r-z_0)!}, \quad r \geq z_0 \tag{5}$$

Hence, given the probability distribution (4) of the initial distribution Z_0 and the average propagation estimated from (1), the discretized total load shed is distributed according to a mixture of Borel-Tanner distributions:

$$P[Y = r\Delta] = \sum_{z_0=1}^{r} P[Z_0 = z_0] z_0\hat{\lambda}(r\hat{\lambda})^{r-z_0-1}\frac{e^{-r\hat{\lambda}}}{(r-z_0)!} \tag{6}$$

The sum in (6) runs from $z_0 = 1$ to r since these are the possible numbers of initial multiples of Δ shed for $r\Delta$ total load shed.

C. Continuous state branching process

This subsection explains continuous state branching processes informally and states formulas to compute the distribution of the total amount of load shed. Kallenburg [25] and Seneta and Vere-Jones [26] give useful and systematic accounts of continuous state branching processes, and can be consulted for a more formal background to this subsection.

The branching process starts with an initial amount of load shed X_0 in stage 0 and proceeds to generate a sequence of load shed amounts X_1, X_2, X_3, ... in stages 1, 2, 3, ... respectively. X_0, X_1, X_2, X_3, ... are nonnegative real numbers. The offspring distribution $H(x)$ is defined to be the probability density function (PDF) of load shed in any stage if the load shed in the preceding stage is 1. We write X for a random variable with PDF $H(x)$. The expected value of X is λ.

We will first assume that the initial load shed X_0 is a constant. The load X_1 shed in stage 1 is a random variable determined by the offspring distribution $H(x)$ in the following way: In the special case of $X_0 = 1$, X_1 has PDF $H(x)$. In general, X_1 has PDF $(H(x))^{*X_0}$ where $(H(x))^{*X_0}$ is the convolution of $H(x)$ with itself X_0 times and the PDF of the sum of X_0 independent copies of X. (The computation of $(H(x))^{*X_0}$ using Laplace transforms when X_0 is a noninteger positive real number is discussed below.) X_1 is realized by sampling from $(H(x))^{*X_0}$. Then the load X_2 shed in stage 2 has PDF $(H(x))^{*X_1}$ that is the PDF of the sum of X_1 independent copies of X. X_2 is realized by sampling from $(H(x))^{*X_1}$. Then the load X_3 shed in stage 3 has PDF $(H(x))^{*X_2}$, and so on.

The computation of these PDFs is simplified by working in terms of their cumulant generating functions (CGFs). The CGF $h(s)$ of the offspring distribution is the negative logarithm of the Laplace transform of $H(x)$:

$$h(s) = -\ln\int_0^\infty e^{-sx}H(x)dx = -\ln Ee^{-sX}$$

The Laplace transform of $(H(x))^{*X_0}$ is the Laplace transform of $H(x)$ to the power X_0:

$$Ee^{-sX_1} = (Ee^{-sX})^{X_0}$$

Hence the CGF of the load X_1 shed in stage 1 is

$$h_1(s) = -\ln Ee^{-sX_1} = -\ln\left((Ee^{-sX})^{X_0}\right) = X_0 h(s)$$

The CGF of the load X_2 shed in stage 2 is

$$\begin{aligned} h_2(s) &= -\ln Ee^{-sX_2} \\ &= -\ln E[E[(e^{-sX})^{X_1}|X_1]] \\ &= -\ln E[e^{-h(s)X_1}] \\ &= h_1(h(s)) = X_0 h(h(s)) \end{aligned} \tag{7}$$

Similar reasoning shows that the CGF of X_3 is $X_0 h(h(h(s)))$ and that the CGF of X_n is

$$h_n(s) = -\ln Ee^{-sX_n} = X_0 h^{(n)}(s) \tag{8}$$

where $h^{(n)}$ is the n-fold functional composition of h.

As the cascade proceeds, the load shed accumulates and the running total of the load shed at stage n is given by

$$Y_n = X_0 + X_1 + \ldots + X_n.$$

If $\lambda < 1$, the cascade will die out and Y_n converges to the total load shed or blackout size

$$Y = \lim_{n\to\infty} Y_n.$$

Assuming the subcritical case $\lambda < 1$, the distribution of the total load shed Y can be computed from the offspring distribution. First consider the case of $X_0 = 1$ and let $k_\bullet(s)$ be the CGF of Y when $X_0 = 1$. Then $k_\bullet(s)$ satisfies the implicit equation

$$k_\bullet(s) = s + h(k_\bullet(s)). \tag{9}$$

If we assume the subcritical case of $\lambda < 1$, (9) can be solved by the Lagrange inversion method [35]:

$$k_\bullet(s) = s + \sum_{a=1}^{\infty}\frac{1}{a!}\frac{d^{a-1}}{ds^{a-1}}(h(s))^a \tag{10}$$

In practice we use 15 terms of the infinite sum in (10) to obtain a good approximation for $k_\bullet(s)$.

Equation (9) can be understood as follows. Consider $Y - 1 = Y - X_0 = X_1 + X_2 + X_3 + \ldots$. If $X_1 = 1$, then the CGF of $Y - 1$ is $k_\bullet(s)$. If X_1 is constant, then the CGF of $Y - 1$ is

[1] Adding 0.5 before taking the integer part ensures that the expectations of Z_0^k and X_0^k/Δ are the same.

$X_1 k_\bullet(s)$. This follows from the independence of the branching process generated by different portions of X_1. For example, if $X_1 = 2$, $Y-1$ can be regarded as being produced by the sum of two independent branching processes with $X_1 = 1$ so that the CGF of $Y-1$ is $2k_\bullet(s)$. If X_1 has CGF $h(s)$, as it does when $X_0 = 1$, then the CGF of $Y-1$ is

$$\begin{aligned} -\ln E e^{-s(Y-1)} &= -\ln E[E[(e^{-s(Y-1)}|X_1]] \\ &= -\ln E[e^{X_1 k_\bullet(s)}] \\ &= h(k_\bullet(s)) \end{aligned} \tag{11}$$

Now (9) follows since the CGF of $Y-1$ is also

$$-\ln E e^{-s(Y-1)} = -\ln E e^{-sY} - s = k_\bullet(s) - s. \tag{12}$$

This subsection has so far assumed that the initial load shed X_0 is a constant whereas the branching process model of this paper assumes that X_0 is a random variable with CGF $m(s)$. That is, stage 0 of the branching process is generated using $m(s)$ and all subsequent stages are generated using $h(s)$. When X_0 has CGF $m(s)$, the CGF of X_n in (8) becomes

$$h_n(s) = m(h^{(n)}(s)) \tag{13}$$

Let $k(s)$ be the CGF of Y when X_0 has CGF $m(s)$. Then

$$k(s) = m(k_\bullet(s)) \tag{14}$$

The expected value of X_0 is θ and the expected value of the offspring distribution X is λ. The expected value of load shed in stage n can be evaluated by differentiating (13) and setting $s = 1$ to obtain

$$E X_n = \theta \lambda^n \tag{15}$$

Once $k(s)$ has been obtained as an explicit function of s using (14), the PDF $K(x)$ of the total load shed Y is obtained as the inverse Laplace transform of $e^{-k(s)}$ using the Post-Widder method [36]:

$$K(x) = \lim_{a\to\infty} \frac{(-1)^a}{a!} \left(\frac{a}{x}\right)^{a+1} \left(\frac{d^a}{ds^a} e^{-k(s)} \bigg|_{s=a/x} \right) \tag{16}$$

In practice we use $a = 15$ in (16) to obtain a good approximation for $K(x)$.

The general procedure for estimating the blackout size PDF $K(x)$ is

1) Assume a parametrized form for the initial load shed CGF $m(s)$ and offspring CGF $h(s)$.
2) Estimate the parameters of $m(s)$ and $h(s)$ from the data.
3) Compute the blackout size CGF $k(s)$ from $m(s)$ and $h(s)$ using (9) and (14)
4) Compute the inverse Laplace transform of $e^{-k(s)}$ to obtain the blackout size PDF $K(x)$ using (16).

The procedure estimates parameters of an explicit form of $m(s)$ and $h(s)$ so that the computation of $k(s)$ and the Laplace inversion can be done using computer algebra.

We assume, as discussed at the end of the subsection, that the offspring distribution is a gamma distribution with mean λ and shape κ. Then the CGF of the offspring distribution is

$$h(s) = \kappa \ln\left(1 + s\frac{\lambda}{\kappa}\right). \tag{17}$$

The pdf of the initial load shed is parameterized as a weighted sum of the pdfs of two gamma distributions with respective means θ_1 and θ_2 and respective shapes κ_1 and κ_2.

$$\begin{aligned} f_{X_0}(x) = c_1 \frac{(\kappa_1/\theta_1)^{\kappa_1}}{\Gamma(\kappa_1)}\, x^{\kappa_1 - 1}\, e^{-x\kappa_1/\theta_1} + \\ (1-c_1)\frac{(\kappa_2/\theta_2)^{\kappa_2}}{\Gamma(\kappa_2)}\, x^{\kappa_2 - 1}\, e^{-x\kappa_2/\theta_2} \end{aligned} \tag{18}$$

Then the Laplace transform of the distribution of initial load shed is

$$e^{-m(s)} = c_1 \left(1 + s\frac{\theta_1}{\kappa_1}\right)^{-\kappa_1} + (1-c_1)\left(1 + s\frac{\theta_2}{\kappa_2}\right)^{-\kappa_2} \tag{19}$$

The parameters of the initial distribution $\theta_1, \theta_2, \kappa_1, \kappa_2$ are obtained by numerically finding a good fit of (18) to the empirical pdf of the initial load shed X_0.

The offspring mean λ is estimated from the data as described in the previous subsection. The offspring shape κ seems harder to estimate and our initial approach used here is to apply the method of moments to X_1. The second moment of X_1 is

$$\begin{aligned} E X_1^2 &= \frac{d^2}{ds^2} e^{-m(h(s))}\big|_{s=0} \\ &= \lambda^2 \left[c_1 \left(\frac{\theta_1}{\kappa} + \frac{\theta_1^2(\kappa_1+1)}{\kappa_1} \right) + \right. \\ &\qquad \left. (1-c_1)\left(\frac{\theta_2}{\kappa} + \frac{\theta_2^2(\kappa_2+1)}{\kappa_2} \right) \right]. \end{aligned}$$

Then the estimated offspring shape $\hat{\kappa}$ is found by solving

$$\begin{aligned} \frac{1}{J}\sum_{i=1}^{J} (X_1^{(i)})^2 = \hat{\lambda}_{\text{init}}^2 \left[c_1 \left(\frac{\theta_1}{\hat{\kappa}} + \frac{\theta_1^2(\kappa_1+1)}{\kappa_1} \right) + \right. \\ \left. (1-c_1)\left(\frac{\theta_2}{\hat{\kappa}} + \frac{\theta_2^2(\kappa_2+1)}{\kappa_2} \right) \right], \end{aligned}$$

where

$$\hat{\lambda}_{\text{init}} = \frac{\sum_{i=1}^{K} X_1^{(i)}}{\sum_{i=1}^{K} X_0^{(i)}}.$$

Here K is the number of cascades with a non-trivial amount of load shed.

We now discuss the choice of the form of the offspring distribution. Any parametrized, nonnegative distribution that is infinitely divisible can be a candidate to describe the offspring distribution. There should be a minimal number of parameters and they should be easy to estimate from data. Moreover, it is advantageous for the computations for the distribution to have a tractable CGF. This makes it more difficult to work with some seemingly more natural choices such as the log-normal distribution. Also, estimating the offspring distribution empirically from data has some challenges. We have not so far found any single choice of offspring distribution that has tractable CGF, fits well with data in a range of cases, or has general arguments supporting it. To give a first demonstration of the method in this paper, we choose the gamma distribution because it has a tractable CGF for which the computations of the distribution of the total load shed can be carried out by computer algebra. We hope that future work may produce a better, or better justified choice of offspring distribution based on some combination of further insights and advances in estimation procedures and computational approaches.

III. Results

A. Discretization method applied to results from OPA

The cascading failure data in this subsection is produced by the OPA simulation on the IEEE 300 bus standard test system [37]. Three load levels are considered: 1.0, 1.05 and 1.1 times the base case load. 20 000 cascades were simulated for each load level. The number of cascades K with non-negligible load shed is shown in Table I for each load level. The probability of a cascade with non-negligible load shed is $K/20000$.

For the IEEE 300 bus system the load shed discretization Δ is chosen to be 952 MW, which is 4% of the base case load of 23 800 MW. This value of Δ is chosen by experimenting with a range of values. (As a possible point of reference, the power system contains 409 lines as discrete elements and each line comprises 0.24% of the total number of lines.) Too small a value of Δ does not allow sufficient samples within each discretization bin to get a good estimate of the frequency of blackouts in that discretization bin. Too large a value of Δ gives insufficient resolution in the load shed. In the cases tested we find that varying Δ by a factor of 2 has not much effect on the results. The choice of Δ does affect the way that the branching process models the cascading load, and we hope that future work will establish more systematic methods for the choice of discretization.

The average propagation λ is estimated using (1) for each load level and is shown in Table I. The average initial load shed θ estimated using (2) for each load level is also shown in Table I.

TABLE I
Average propagation $\hat{\lambda}$ and average initial load shed $\hat{\theta}$

load level	$\hat{\lambda}$	$\hat{\theta}$ (GW)	K
1.0	0.09	3.72	4137
1.05	0.21	3.57	8568
1.1	0.42	3.29	9381

Case: OPA on IEEE 300 bus

For the base case load level, the probability distribution of total load shed estimated via the branching process is compared to the empirical distribution of total load shed in Figure 1. Although both probability distributions are discretized in load, the distribution of total load shed estimated via the branching process has its points joined by a line so it can be clearly distinguished. The match is good, but this is expected in this case since the average propagation $\hat{\lambda} = 0.09$ is small and the cascading effect is small, so that the distribution of total load is close to the initial distribution of load.

For the higher load level 1.05 times the base case, the probability distribution of total load shed estimated via the branching process is compared to the empirical distribution of total load shed in Figure 3. The average propagation

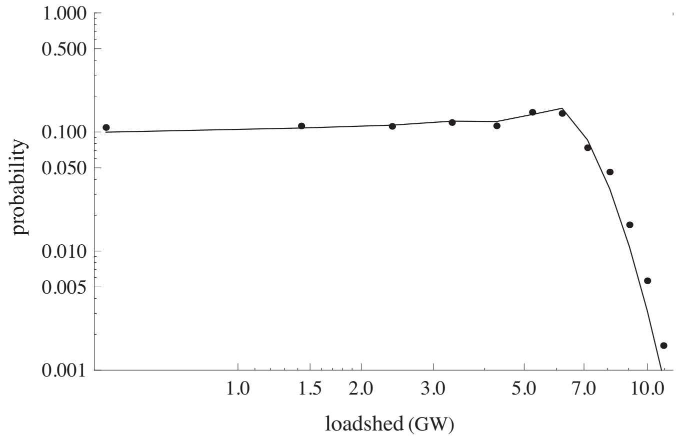

Fig. 1. Probability distributions of total load shed by OPA on IEEE 300 bus system at the base case load level. Dots are the empirical distribution; line is estimated with the branching process.

$\hat{\lambda} = 0.21$ and the match is good. The empirical initial load shed distribution is shown in Figure 2. The cascading has the effect of changing the initial distribution of load shed into a distribution of total load shed with larger blackouts.

For the higher load level 1.1 times the base case, the probability distribution of total load shed estimated via the branching process is compared to the empirical distribution of total load shed in Figure 5. The average propagation $\hat{\lambda} = 0.42$ and the match is good except for the sharply dropping portion of the tail. The empirical initial load shed distribution is shown in Figure 4.

B. Discretization method applied to results from TRELSS

The cascading data in this subsection is generated by the TRELSS simulation. The case used in the TRELSS analysis contains approximately 6250 buses, 9850 branches, and 1200 plants. In TRELSS, a subset of the case called the study area is selected and analyzed. The study area comprises 4100 buses, 4750 branches, 600 transformers, and 325 plants. The study area is modeled at anticipated summer peak conditions. The study area load is almost 48 GW and the online system generation is approximately 51 GW.

In the results, the number of non-negligible cascades is $K = 305$. The application of the discretization method to this TRELSS data is the same as for the OPA data considered in subsection III-A, except that an alternative method of computing the load shed discretization Δ is used. The load shed discretization of $\Delta = 12$ MW is computed as the sum of total load shed divided by the sum of total number of outaged lines. For this system, it is better not to estimate Δ using the total system load because most cascades are confined in extent and do not typically involve a large fraction of the system load.

The average propagation λ is estimated using (1) and is shown in Table II. The average initial load shed θ estimated using (2) is also shown in Table II.

The empirical distribution of initial load shed is shown in Fig. 6. For larger amounts of initial load shed, there are

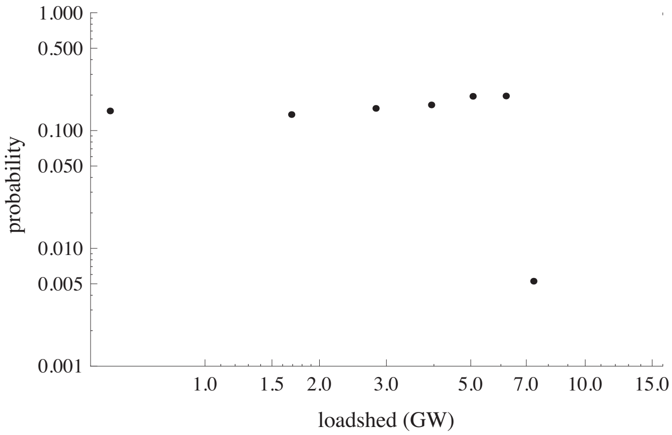

Fig. 2. Probability distribution of initial load shed by OPA on IEEE 300 bus system at load level 1.05 times the base case.

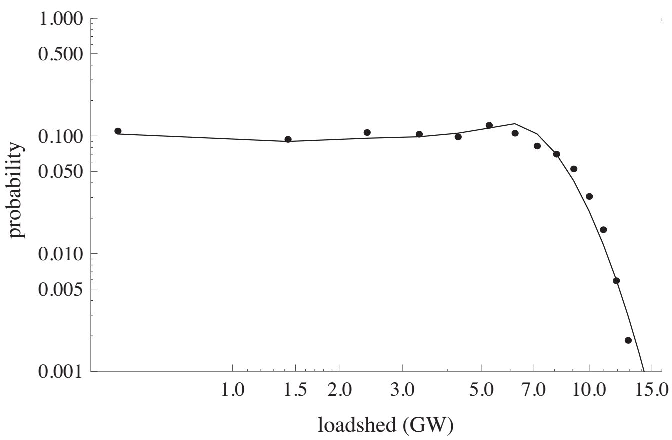

Fig. 3. Probability distributions of total load shed by OPA on IEEE 300 bus system at load level 1.05 times the base case. Dots are the empirical distribution; line is predicted with the discretized branching process.

TABLE II
AVERAGE PROPAGATION $\hat{\lambda}$ AND AVERAGE INITIAL LOAD SHED $\hat{\theta}$

$\hat{\lambda}$	$\hat{\theta}$ (MW)	K
0.33	140.	305

Case: TRELSS

few samples in each discretization bin and the empirical distribution is noisy and discretized to multiples of $1/K$. The distribution of total load shed estimated using the discretized branching process is compared to the empirical distribution of of total load shed in Fig. 7. In Fig. 7 the empirical distribution of total load shed for the larger amounts of load shed is computed for larger bins in order to maintain at least ten samples per bin.

These cases using two different simulations show how discretizing the load shed and then applying a Galton-Watson branching process can be used to estimate the distribution of total load shed given the average propagation and the distribution of initial load shed.

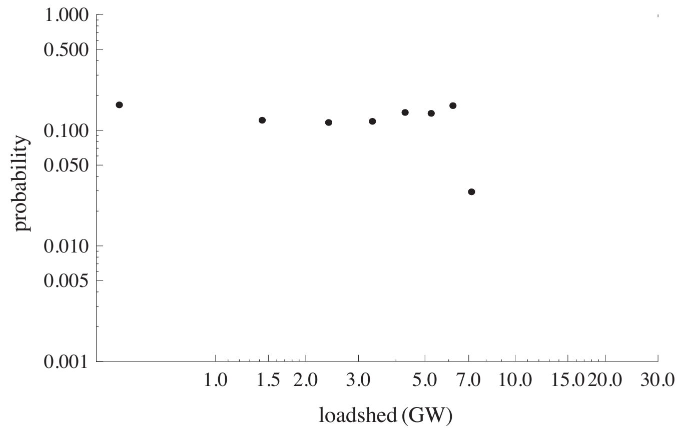

Fig. 4. Probability distribution of initial load shed by OPA on IEEE 300 bus system at load level 1.1 times the base case.

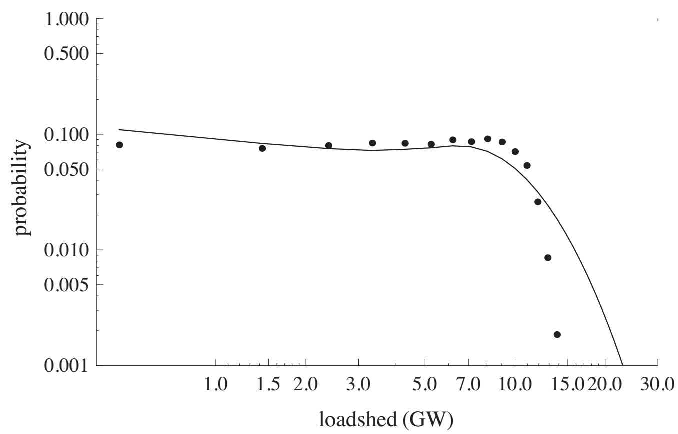

Fig. 5. Probability distributions of total load shed by OPA on IEEE 300 bus system at load level 1.1 times the base case. Dots are the empirical distribution; line is predicted with the discretized branching process.

C. Continuous state branching process method

The method is tested on the IEEE 118 bus test system with data produced by the OPA simulation. $K = 5000$ cascades with nontrivial amounts of load shed are simulated for each of the four loading levels 0.85, 0.90, 0.95, and 1.0 times the base case loading. These loading levels are chosen so as to avoid significant saturation effects.

Ten stages of each cascade are simulated. The load shed is measured as a fraction of the total load so that the maximum possible load shed is 1.0, or total blackout.

The estimated propagation $\hat{\lambda}$, offspring distribution variance σ^2_{off}, and initial load shed distribution parameters are estimated for the OPA data according to the methods described in section II. The empirical PDF of load shed is also obtained.

1) Estimated propagation $\hat{\lambda}$: The estimated propagation at each load level computed from the load shed data is shown in Table III. As expected, $\hat{\lambda}$ increases with loading.

2) Blackout size PDF: Table IV shows the parameters of the initial load shed and offspring distributions estimated from the load shed data. All cases considered are subcritical ($\lambda < 1$)

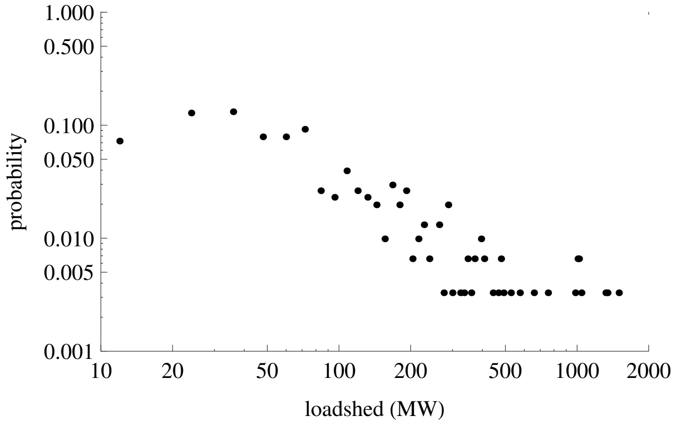

Fig. 6. Distribution of initial load shed. Data produced by TRELSS.

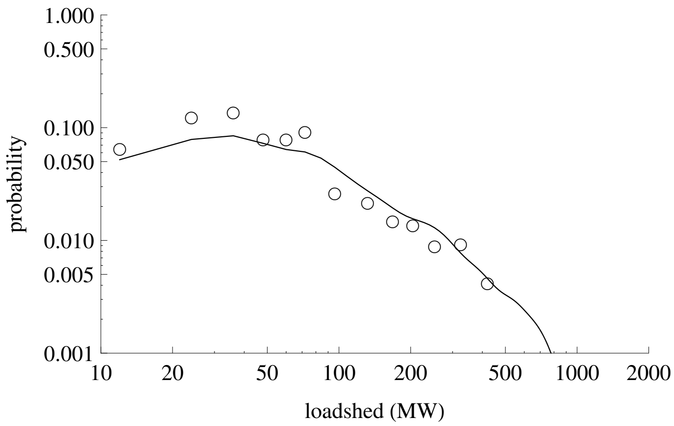

Fig. 7. Distribution of total load shed estimated by the discretized branching process (line) is compared to the binned empirical distribution of total load shed (circles). Data produced by TRELSS.

TABLE III
AVERAGE PROPAGATION $\hat{\lambda}$

load level	$\hat{\lambda}$
0.85	0.128
0.9	0.159
0.95	0.264
1.0	0.429

Case: OPA on IEEE 118 bus

as required for the Lagrange inversion (10).

Figure 8 compares the empirical and estimated PDFs for load level 0.85 times the base case load, and Figure 9 compares the empirical and estimated PDFs for the base case load level. The blackout size is plotted on a log scale over two decades, from a small blackout shedding 0.01 of the total load to a total blackout shedding all of the load.

3) Initial load shed and offspring distributions: We discuss the choices of the forms of initial load shed and offspring distributions that are assumed in the computations.

The initial load shed gamma distribution parameters $\hat{\theta}$ and $\hat{\sigma}^2_{init}$ shown in Table IV are relatively insensitive to loading changes. For all these cases $\hat{\sigma}^2_{init} \approx \hat{\theta}^2$ and hence the initial load shed is approximately exponentially distributed. Figure 10 shows estimated and empirical initial failure distributions for the base case load level.

Figure 11 shows the estimated offspring distribution PDF for the base case load level. This is a gamma distribution with mean 0.0383 and variance 0.0016 that is approximately a normal distribution. However, the offspring PDF becomes more asymmetrical when the load level is decreased.

IV. NUMBER OF CASCADES FOR ACCURATE ESTIMATES

This section considers the number of cascades needed for accurate estimates with the discretized load shed method. In particular, it roughly estimates how many fewer cascades are needed to estimate propagation and then estimates the probability distribution of discretized load shed with the Galton-Watson branching process compared to direct empirical estimation of the probability distribution of load shed.

In our case of a Poisson offspring distribution, the asymptotic standard deviation of the Harris estimator can be worked out using the methods of [32] to be

$$\sigma(\widehat{\lambda}) \sim \frac{\sqrt{\lambda(1-\lambda)}}{\sqrt{K\theta/\Delta}} \tag{20}$$

Note that $\hat{\theta}/\Delta$ estimates $EX_0/\Delta = EZ_0$, which is the mean number of discretized amounts of initial load shed.

Let $p_{\rm branch}$ be the probability of shedding total load S, computed via estimating λ from $K_{\rm branch}$ simulated cascades with non-negligible load shed and then using the branching process model. $p_{\rm branch}$ is conditioned on a non-negligible amount of load shed. Assume that the initial distribution of load shed is known with high accuracy. Then the standard deviation of $p_{\rm branch}$ is

$$\begin{aligned} \sigma(p_{\rm branch}) &= \left|\frac{dp_{\rm branch}}{d\lambda}\right| \sigma(\widehat{\lambda}) \\ &= \left|\frac{dp_{\rm branch}}{d\lambda}\right| \sqrt{\frac{\lambda(1-\lambda)\Delta}{K_{\rm branch}\theta}} \end{aligned} \tag{21}$$

Let $p_{\rm empiric}$ be the probability of shedding total load S, computed empirically by simulating $K_{\rm empiric}$ cascades with non-negligible load shed. Then the standard deviation of $p_{\rm empiric}$ is

$$\sigma(p_{\rm empiric}) = \sqrt{\frac{p_{\rm empiric}(1-p_{\rm empiric})}{K_{\rm empiric}}} \tag{22}$$

If we require the same standard deviation for both methods, then we can equate (21) and (22) to approximate the ratio of the required number of simulated cascades as

$$\frac{K_{\rm empiric}}{K_{\rm branch}} = \frac{p_{\rm empiric}(1-p_{\rm empiric})\theta}{\lambda(1-\lambda)\Delta}\left(\frac{dp_{\rm branch}}{d\lambda}\right)^{-2} \tag{23}$$

To obtain a rough estimate of the ratio, we evaluate (23) for total load shed $S = 9.52$ GW for each of the three load levels. $dp_{\rm branch}/d\lambda$ is estimated by numerical differencing. We find that $K_{\rm empiric}$ exceeds $K_{\rm branch}$ by an order of magnitude or more. That is, if the initial load shed distribution is known accurately, then accurately estimating the distribution of the total amount of load shed via discretization and the Galton-Watson branching process requires substantially fewer cascades.

TABLE IV
INITIAL LOAD SHED AND OFFSPRING DISTRIBUTION PARAMETERS

load level	$\hat{\lambda}$	$\hat{\theta}$	$\hat{\sigma}^2_{\text{init}}$	$\hat{\sigma}^2_{\text{off}}$
0.85	0.128	0.0520	0.00198	0.00431
0.9	0.159	0.0482	0.00195	0.00568
0.95	0.264	0.0445	0.00182	0.00995
1.0	0.429	0.0383	0.00160	0.01230

Case: OPA on IEEE 118 bus

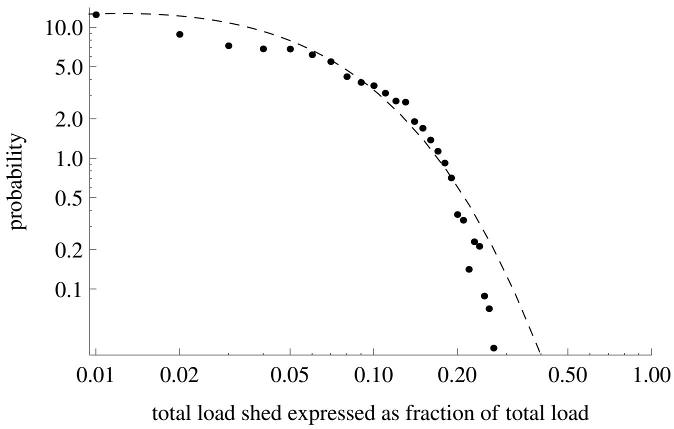

Fig. 8. Probability density function of total load shed on log-log plot. PDF estimated with continuous state branching process (dashed line) is compared to empirical PDF (dots). Empirical data from OPA on IEEE 118 bus system with load level 0.85 times the base case.

V. CONCLUSION

In this paper, we suggest approximating the cascading process of load shed in simulations of cascading blackouts by two methods. The load shed data is the load shed in each generation or stage of each simulated cascade. Both methods preprocess the load shed data in the same way and estimate the average propagation of failures λ using the standard Harris estimator.

In the first method, we discretize the load shed and then use a Galton-Watson branching process. The Galton-Watson branching process model estimates the probability distribution of load shed from the discretized distribution of initial load shed and the estimate of propagation λ. We test this estimation on cascading failure data from the OPA simulation of cascading transmission line outages in the IEEE 300 bus test system and on another case using the TRELSS simulation. The estimated distribution is close to the empirical distribution in most of the cases tested, suggesting that the branching process model with an averaged propagation can capture some aspects of the cascading of load shed, at least for the purpose of estimating the probability distribution of total load shed.

In the second method, we directly apply a continuous state branching process to the preprocessed load shed data. Then the continuous state branching process model estimates the probability distribution of load shed from the distribution of initial load shed and the estimate of λ. We test this estimation on cascading failure data from the OPA simulation of cascading transmission line outages in the IEEE 118 bus test system. The estimated distribution is similar, but not very close, to the empirical distribution in the cases tested. There

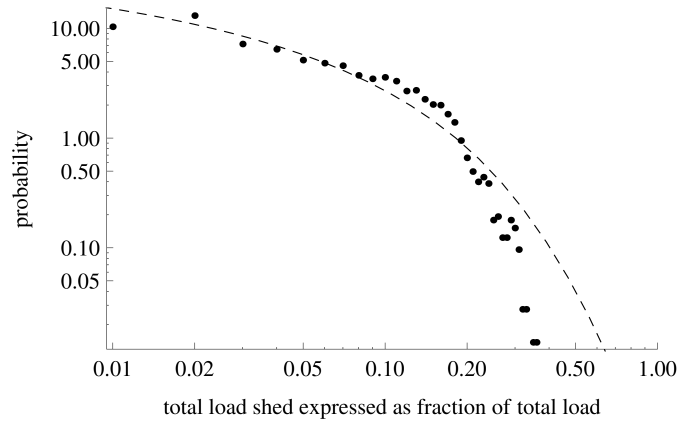

Fig. 9. Probability density function of total load shed on log-log plot. PDF estimated with continuous state branching process (dashed line) is compared to empirical PDF (dots). Empirical data from OPA on IEEE 118 bus system with the base case load level.

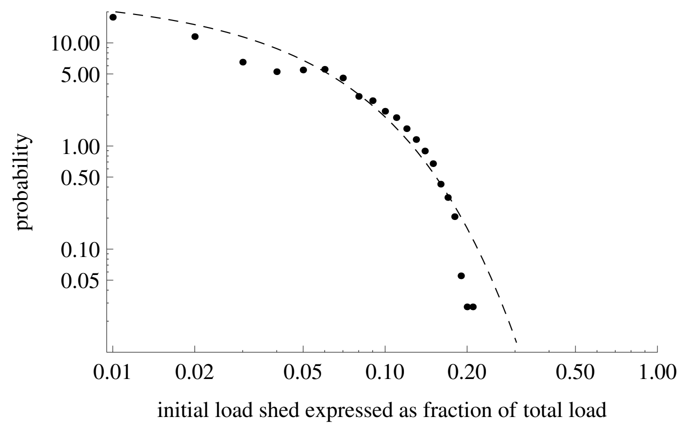

Fig. 10. Probability density function of initial load shed X_0 on log-log plot. Estimated PDF (dashed line) is compared to empirical PDF (dots). Empirical data from OPA on IEEE 118 bus system with the base case load level.

are several ways that the second method could be improved. It is not yet known what form of offspring distribution fits power system cascading data well (here we choose the gamma distribution because it is easy to compute with). Also, there remain challenges in estimating parameters of the offspring distribution other than the mean and in computing the distribution of load shed for other choices of offspring distribution. These challenges for the second method may be met in the future, but at present, the first method appears to be easier and more accurate.

The approach via propagation and the branching process opens opportunities for estimation of the probability distribution of load shed from fewer observed or simulated cascades. We assume that the probability distribution of initial load shed is known accurately. These initial load shed statistics can be estimated by methods of conventional reliability or by observations, since some load is shed much more frequently than there is a large cascading blackout. Given that the probability distribution of initial load shed is known accurately, our initial testing of the estimation via the branching process of the probability distribution of total load shed suggests that an order of magnitude or more fewer cascades are needed for

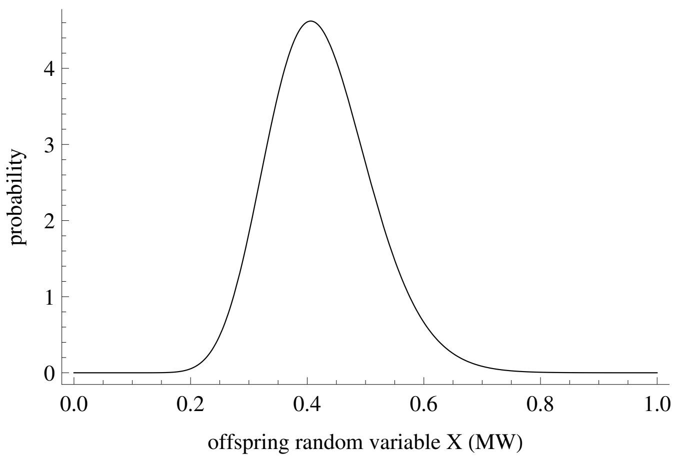

Fig. 11. Probability density function $H(x)$ of offspring distribution that is a gamma distribution with mean $\lambda = 0.429$ and variance $\sigma^2_{\text{off}} = 0.0123$. Parameters computed from data from OPA on IEEE 118 bus system with the base case load level.

this estimation in the tail of the distribution than is needed for direct empirical estimation of the probability distribution of load shed. This is useful in reducing simulation times, which are always burdensome and often prohibitive for cascading failure simulations of large power system models. Obtaining useful results from fewer cascades would also be a crucial attribute in designing practical methods of estimating the probability distribution of load shed from cascades observed in the power system. Empirical methods of accumulating blackout statistics that simply wait for enough cascades to occur take too long to be practical when estimating the rare but important large blackouts in the tail of the distribution.

This paper estimates average propagation and the distribution of load shed using branching process models. These first results are sufficiently promising that further testing with cascading failure data from other power system models and other cascading failure simulations is warranted.

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