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A cutset area concept for phasor monitoring

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***Abstract*—We show how to combine together voltage angle phasor measurements at several buses to measure the angle stress across an area of the power system that is called a cutset area. The angle across the cutset area is a weighted average of the angles measured at buses at the borders of the cutset area. The angle across the cutset area is based on circuit theory and it responds to power flow through the area and to line trips inside the cutset area. The angle across the cutset area gives stress information that is specific to the cutset area and is a generalization of the angle difference between two buses. The concepts are illustrated with several choices of cutset areas in a 225 bus model of the Western North American power system.**

***Index Terms*—power system monitoring, circuit analysis**

I. Introduction

Synchronized phasor measurements [1], [4], [6] are becoming more widespread and are opening further opportunities for power transmission system monitoring and control. Cutset area angle monitoring is a new way to combine together phasor angle measurements to extract more specific information about power grid stress. Cutset area angles arise from the development from first principles in [3]. Here our objective is to explain and illustrate concepts to help to formulate the practical applications of cutset area angles.

Previous work on monitoring power system stress with phasor measurements has focused on the difference between voltage phasor angles at pairs of buses. A large angle difference between a pair of buses indicates, in some general sense, a stressed power system with large power flows or increased impedance between the areas. Simulations of the grid before the August 2003 Northeastern blackout show increasing angle differences between Cleveland and West Michigan, suggesting that large angle differences could be a blackout risk precursor [2]. A recent simulation study [7] of potential phasor measurements on the 39 bus New England test system shows that, of several phasor measurements, angle differences were the best in discriminating alert and emergency states. The angle difference between two buses can detect power system stress, but it is typically affected by changes throughout the entire grid. It turns out that a cutset area angle is a generalization of the angle difference between two buses and gives more specific information about the grid related to the chosen cutset area

There has been some previous work that combines phasor measurements at several buses. A weighted average of voltage magnitudes or reactive powers derived from WECC phasor measurements is discussed in [8]. The weighted averages provide robust control signals that are the basis for wide area control schemes for transient and voltage stability. The weights are established by location and sensitivity considerations. Reference [8] also discusses weighting phasor voltage angles to calculate a center of inertia angle for an area. Wide area nomograms involving linear combinations of phasor angles have been suggested for monitoring of security boundaries [5]. The cutset area angle described in this paper amounts to a specific suggestion of weights in a weighted average of phasor angle measurements so that the weighted average has a specific interpretation in terms of area stress.

In [3], we develop from scratch a concept of angle across a cutset of lines and extend the concept to a cutset area angle by considering a reduced network.[1] The new concepts are derived as a non-standard instance of general circuit theory. Although the cutset angle and cutset area angle concepts are simple, we have not yet found any previous literature defining or using these concepts. The development in [3] establishes the circuit theory basis for this paper.

Section II introduces cutset areas and their border buses that are assumed to have the phasor measurements. Section III explains cutset area angle and susceptance and shows how the cutset area angles may be obtained by combining the phasor measurements at the border buses. Section IV shows how the cutset angles depend on power flows across the cutset area and line trips inside the cutset area and section V shows how to compute and monitor cutset angles. Most of the paper assumes a DC load flow model for the computations, except that in section VI we test the results with an AC load flow. Section VII shows that the angle difference between two buses is a special case of a cutset area angle and Section VIII shows how other network quantities such as cutset area complex voltages and complex admittances can be defined. Section IX explains how to choose cutset areas and Section X summarizes the results and concludes the paper.

Throughout the paper we illustrate the concepts using the model of the WECC system shown in Figure 1. The model has 225 buses and is a reduced representation of the higher

[1] [3] starts by defining an angle across a cutset of lines, assuming phasors measured at every bus. To generalize to phasor measurements at only some buses, the network is reduced to an equivalent network, and the cutset and cutset angle in this reduced network correspond to the cutset area and cutset area angle in the original network. The term “cutset area” is not used in [3], but a cutset area is indicated in Figure 3 of [3] by the thick and thin dashed transmission lines and its properties are discussed.

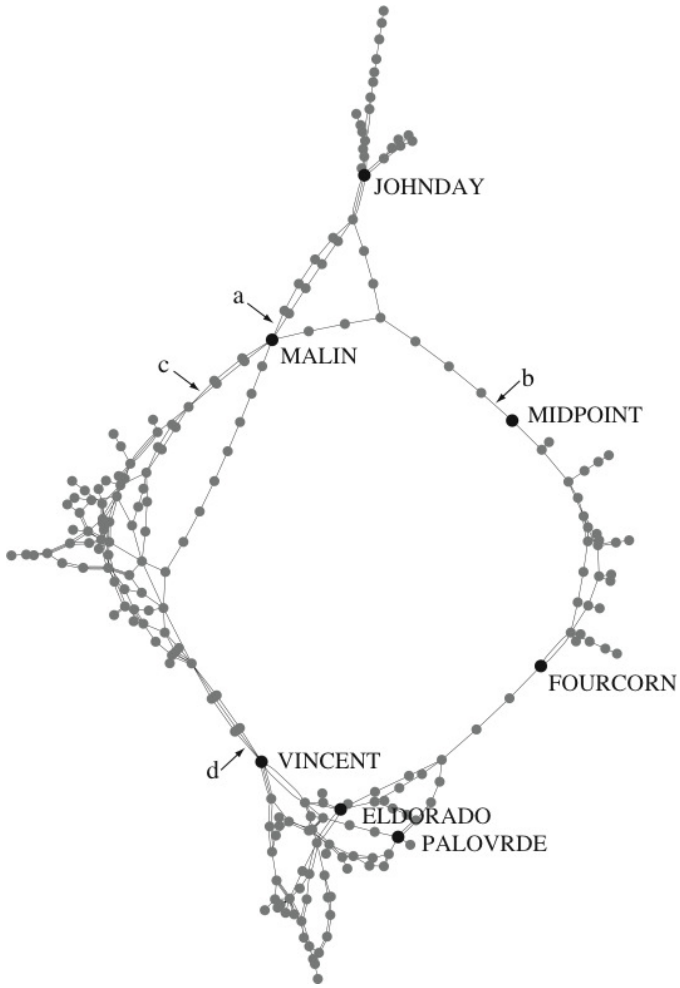

Fig. 1. Reduced model of WECC system with 225 buses. The network layout is roughly geographic so that Canadian buses are at the top, Southern California is at the bottom left and New Mexico is at the lower right. The seven labeled buses shown by black dots are assumed to have phasor measurements. Lines labeled a,b,c,d are also shown.

voltage WECC transmission system. All results use the same base case. For the purpose of illustration, we assume phasor measurements at the seven labeled buses indicated by black dots in Figure 1.

II. Cutset areas and border buses

This section defines and explains cutset areas and the buses that border the cutset area. To start, we recall a standard definition[2] of a cutset of transmission lines:

> A *cutset* of lines is a set of lines that cuts the network into separate networks when that set of lines is removed from the network.

More generally, we define a concept of cutset area as shown in Figure 2:

> A *cutset area* is an area of the network consisting of lines and buses that cuts the rest of the network into separate areas when the cutset area of the network is removed from the network.

[2]Some authors define a cutset to be a *minimal* set of lines that separate the network, but we do not require this here.

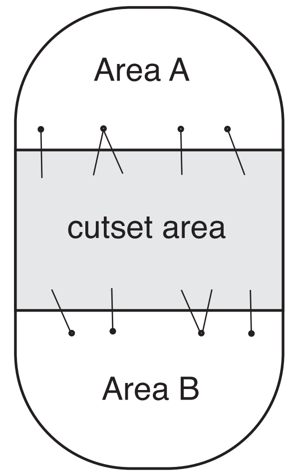

Fig. 2. The gray cutset area separates the rest of the power system into Area A and Area B. The border buses are shown as dots inside Areas A and B. The border buses have phasor measurements.

The areas of the network that are separated are labeled Area A and Area B. One way to make the cutset area separate Area A from Area B is to make the cutset area extend all the way across the power system.

There are buses in Areas A and B that border the cutset area:

> *Border buses* are the buses in Area A or B that have lines connecting them to buses in the cutset area.

We assume that the cutset area is chosen so that the borders are monitored:

> *There are phasor measurements of the voltage phasor angles at all of the border buses of the cutset area.*

Figures 3–6 show examples of cutset areas and border buses in a 225 bus model of WECC power system. All the border buses chosen in these examples are assumed to have phasor measurements. In Figure 5, Area A is the single bus FOURCORN. In Figure 6, Area A is two disjoint regions combined together.

III. Cutset area angle and susceptance

We first make some simple statements about a single transmission line before making some similar claims about cutset areas. Consider a single, lossless transmission line joining bus a to bus b. In the DC load flow for this transmission line we have

$$P_{ab} = b\,\theta_{ab} \tag{1}$$

where

$$\theta_{ab} = \theta_a - \theta_b = \text{angle across the line,}$$
$$b = \text{line susceptance=1/(line reactance),}$$
$$P_{ab} = P_a - P_b = \text{power flow from } a \text{ to } b.$$

TABLE I
WEIGHTING FACTORS FOR CALCULATING CUTSET AREA ANGLES

	JOHNDAY	MALIN	MIDPOINT	VINCENT	ELDORADO	PALOVRDE	FOURCORN
cutset area 1	0	0.9274	0.07255	–0.9274	0	0	–0.07255
cutset area 2	1.000	–0.8968	–0.1032	0	0	0	0
cutset area 3	0	0	–0.1611	0	–0.5917	–0.2473	1.000
cutset area 4	0.4555	0	0.1497	–0.6052	–0.2784	–0.1164	0.3948

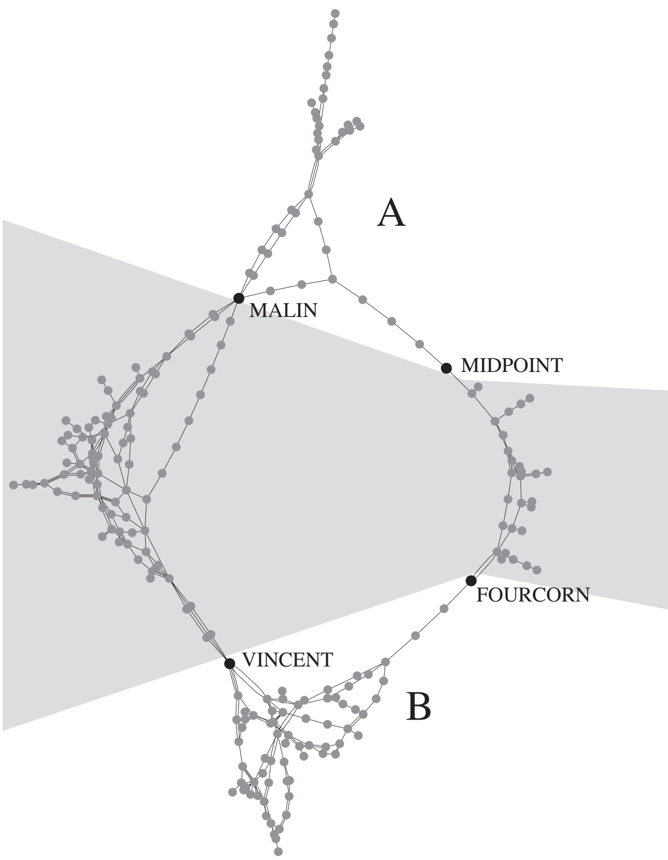

Fig. 3. Cutset area 1 for the 225 bus WECC model. The gray cutset area 1 separates Area A, a northern portion of WECC (OR, WA, Canada) from Area B, a southern portion of WECC (Southern CA, AZ, NM). The border buses with phasor measurements are the labeled black dots.

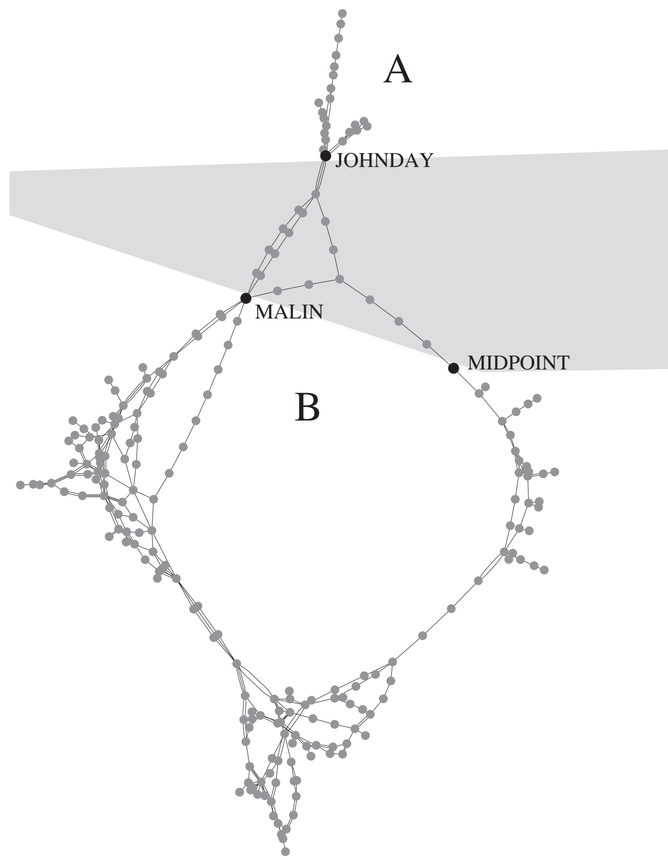

Fig. 4. Cutset area 2. The gray cutset area 2 (mostly OR) separates Area A, which is WA and Canada from Area B, which is CA, ID and points south. The border buses with phasor measurements are the labeled black dots.

The line is stressed if the angle θ_{ab} across the line is too large.

Based on [3], for a cutset area we can similarly define

$$\theta_{AB} = \text{angle across the cutset area,}$$
$$b_c = \text{cutset area susceptance,}$$
$$P_{AB} = \text{power flow through the cutset area.}$$

Moreover, Ohm's law applies to these quantities so that

$$P_{AB} = b_c \theta_{AB} \tag{2}$$

The cutset area angle θ_{AB} is readily computed:

The cutset area angle θ_{AB} is a weighted average of the phasor angles at the border buses.

The weights are obtained using a DC load flow grid model and the calculation of the weights is illustrated in section V. Table I shows the computed weights for the border buses phasor angles. For example, Table I shows that the cutset area 1 angle is computed from the angles measured at the cutset area 1 border buses as

$$\theta_{AB1} = 0.9274\,\theta_{\text{MALIN}} + 0.0725\,\theta_{\text{MIDPOINT}} - 0.9274\,\theta_{\text{VINCENT}} - 0.0725\,\theta_{\text{FOURCORN}} \tag{3}$$

The angles at the Area A border buses have positive weights and the angles at the Area B border buses have negative weights. In (3), the larger coefficient 0.9274 shows that θ_{AB1} depends mainly on the angle difference between MALIN and VINCENT. This is expected since the western path transferring power south through cutset area 1 has lower impedance than the eastern path. The DC load flow portion of Table II shows the base case cutset area angles computed with these weights.

The cutset area susceptance b_c is calculated from the sus-

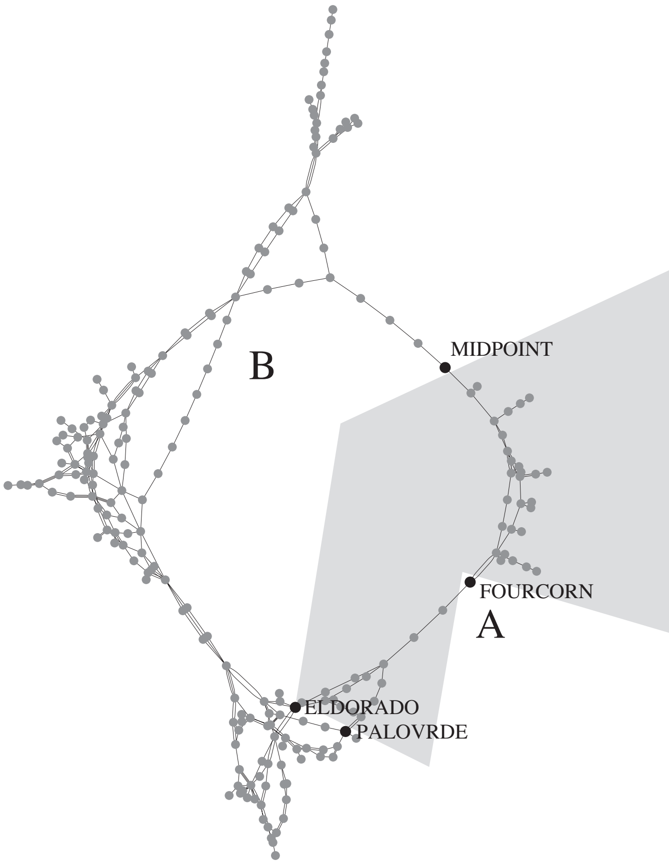

Fig. 5. Cutset area 3. The gray cutset area 3 transfers power west into LA and also north and separates Four Corners from Southern CA and ID. Area A is the bus FOURCORN only. Area B is the white area of the network except for FOURCORN. The border buses with phasor measurements are the labeled black dots.

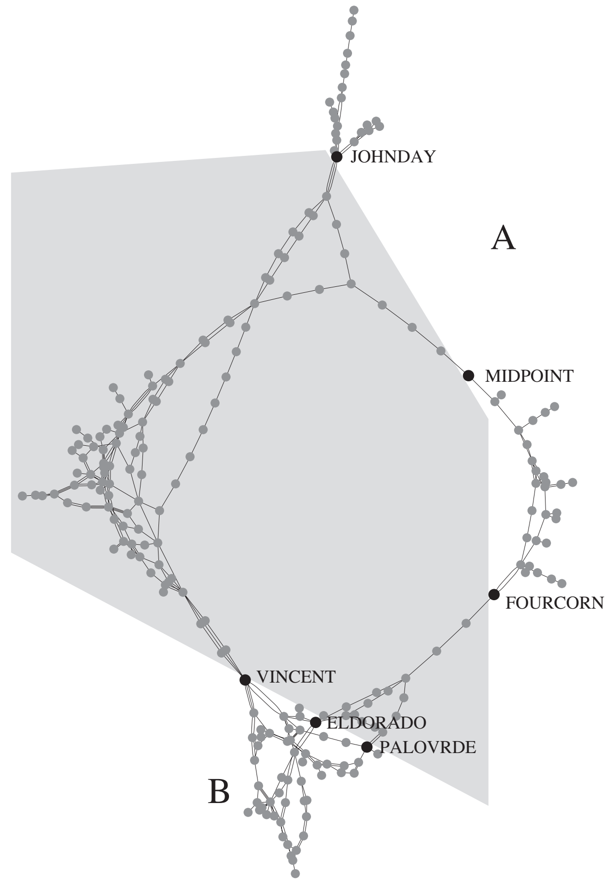

Fig. 6. Cutset area 4. The gray cutset area 4 separates large generation in Canada, ID and NM from the largest load area in LA and San Diego. Area A is the two (disjoint) areas in Canada and in the East. Area B is the Southern area containing LA and San Diego. The border buses with phasor measurements are the labeled black dots.

ceptances of lines in the cutset area as explained in section V. b_c does not depend on the susceptance of lines outside the cutset area.

The power flow P_{AB} can be computed from the network power injections. In the special case that there are no power injections in the cutset area, P_{AB} is the sum of the power flows on the lines joining Area A to the cutset area, and is the sum of the power flows on the lines joining the cutset area to Area B, and is also the total power injected in Area A minus the total power injected in Area B.

TABLE II
BASE CASE CUTSET AREA ANGLES, SUSCEPTANCES, POWER FLOWS

DC load flow	θ_{AB}	b_c	P_{AB}
cutset area 1	20.22	69.21	2443
cutset area 2	11.96	89.69	1872
cutset area 3	21.35	31.18	1162
cutset area 4	32.22	66.26	3726

AC load flow	θ_{AB}
cutset area 1	20.95
cutset area 2	11.54
cutset area 3	21.24
cutset area 4	32.72

θ_{AB} in degrees; P_{AB} in MW;
b_c in per unit on 100 MW base

IV. Properties of cutset area angle

The cutset area angle θ_{AB} responds to changes in the power flowing through the cutset area from area A to area B. For example, the changes in the cutset area angles when an additional 100 MW is generated in Canada and consumed in Southern California are shown in the DC load flow portion of Table III.

The angles across cutset areas 1, 2 and 4 increase because of the additional 100 MW flowing through these cutset areas. According to (2), the angle increases are proportional to the increase in power flow through the cutset area and the constant of proportionality is the cutset area reactance $1/b_c$. The angle across cutset area 3 does not change because the additional 100 MW does not flow through cutset area 3 and is a transfer of power entirely within area B. This is also consistent with (2): The transfer of power within area B does not affect the total power injected into area B or the total power injected in area A. Therefore the power P_{AB3} through cutset area 3 is

TABLE III
CHANGES IN CUTSET AREA ANGLES WITH 100 MW POWER TRANSFER

DC load flow

$\Delta\theta_{AB1}$	$\Delta\theta_{AB2}$	$\Delta\theta_{AB3}$	$\Delta\theta_{AB4}$
0.8278	0.6388	0.0	0.8647

AC load flow

$\Delta\theta_{AB1}$	$\Delta\theta_{AB2}$	$\Delta\theta_{AB3}$	$\Delta\theta_{AB4}$
1.0053	0.5996	0.0100	0.9709

all angles in degrees

unchanged. Since the cutset susceptance b_{c3} is also constant, (2) implies that the cutset area 3 angle θ_{AB3} does not change, and so $\Delta\theta_{AB3} = 0$. More generally, the cutset area angle is not sensitive to any change in power injections in Area A that does not change the total export from Area A, such as generator redispatch entirely within Area A. Similarly, the cutset area angle is not sensitive to any change in power injections in Area B that does not change the total export from Area B.

The cutset area angle θ_{AB} responds to line tripping in the cutset area even when the power generation and loads remain the same. The reason is that tripping a line in the cutset area changes the cutset area susceptance, and then the same power flow through the cutset area gives a different cutset area angle. However, with one exception, the cutset area angle is not sensitive to line trips in the interior of Area A or Area B. The exception is that lines in the interior of Area A or Area B that island the network when tripped can change the power injections in Area A or Area B and the power flowing through the cutset area and hence change the cutset angle.

TABLE IV
CHANGES IN MONITORED CUTSET AREA ANGLES WHEN SELECTED LINES ARE TRIPPED

DC load flow

line	$\Delta\theta_{AB1}$	$\Delta\theta_{AB2}$	$\Delta\theta_{AB3}$	$\Delta\theta_{AB4}$
a GRIZZLY6→MALIN	0	2.728	0	1.585
b BURNS1→MIDPOINT	0	-15.05	0	21.84
c ROUNDMT→MALIN6	0.3754	0	0	0.2450
d VINCENT→MIDWAY4	1.415	0	0	0.9232

AC load flow

line	$\Delta\theta_{AB1}$	$\Delta\theta_{AB2}$	$\Delta\theta_{AB3}$	$\Delta\theta_{AB4}$
a GRIZZLY6→MALIN	0.4471	2.861	0.0183	1.992
b BURNS1→MIDPOINT	0.9667	-17.22	-1.351	24.44
c ROUNDMT→MALIN6	1.521	0.6378	0.0169	1.337
d VINCENT→MIDWAY4	1.499	-0.0037	0.0483	0.9963

all angles in degrees

As discussed further in section V, for the purpose of monitoring a cutset area, cutset area angles are computed as a weighted average of phasor angle measurements, and the weights are obtained from a base case DC load flow assuming the susceptances of the base case DC load flow. Therefore, if the power system susceptances are the same as the base case susceptances, the monitored cutset area angles are the same as the cutset area angles. For example, the monitored cutset angles are the same as the cutset area angles for the 100 MW power transfer considered above. However, when a line trips, the power system susceptances change from the base case susceptances and the monitored cutset area angle differs from the cutset area angle. It remains the case that the monitored cutset area angle responds to line tripping in the cutset area and does not respond to line tripping outside the cutset area. However, the monitored cutset area angle does not satisfy Ohm's law.

Examples of changes in monitored cutset angles when lines are tripped are shown in the DC load flow portion of Table IV. The lines a,b,c,d are identified in Figure 1. Lines a and b are in cutset areas 2 and 4, so tripping line a or line b changes the cutset angles θ_{AB2} and θ_{AB4}. Lines a and b are not in cutset areas 1 and 3, so tripping line a or line b has no effect on cutset angles θ_{AB1} and θ_{AB3}. Lines c and d are in cutset areas 1 and 4, so tripping them changes θ_{AB1} and θ_{AB4}.

V. Calculating the Cutset Area Angle and Susceptance

The calculation of cutset area angle and susceptance works by reducing the network to an equivalent network that replaces the cutset area by the border buses and lines directly connecting the border buses. Other buses in areas A or B can also be eliminated in the reduction. The cutset area becomes a cutset of lines between the border buses in the reduced network. The cutset area angle and susceptance are then the cutset angle and susceptance in the reduced network. Cutset angle and susceptance are introduced and derived from first principles in [3], so in this section we can simply illustrate them by example.

Let θ be the vector of bus angles and P be the vector of bus power injections. The DC load flow equations of the 225 bus WECC system are

$$P = B\theta \tag{4}$$

Write θ_m and P_m for the angle and power injected at the border buses with phasor measurements and (optionally) other buses in Areas A and B. Write $\theta_{\overline{m}}$ and $P_{\overline{m}}$ for the angle and power injected at the other buses. All the buses in the cutset area are included in the $\overline{m}$ buses. Order the buses so that the m come first. Then the DC load flow equations (4) may be rewritten as

$$\begin{pmatrix} P_m \\ P_{\overline{m}} \end{pmatrix} = \begin{pmatrix} B_{mm} & B_{m\overline{m}} \\ B_{\overline{m}m} & B_{\overline{m}\overline{m}} \end{pmatrix} \begin{pmatrix} \theta_m \\ \theta_{\overline{m}} \end{pmatrix}. \tag{5}$$

Now we apply a standard reduction. Define

$$P_{eq} = P_m - B_{m\overline{m}} B_{\overline{m}\overline{m}}^{-1} P_{\overline{m}} \tag{6}$$

$$B_{eq} = B_{mm} - B_{m\overline{m}} B_{\overline{m}\overline{m}}^{-1} B_{\overline{m}m} \tag{7}$$

The reduced but equivalent grid contains the border buses and replaces the cutset area by a cutset of lines connecting the border buses. The reduced grid has the DC load flow equations

$$P_{eq} = B_{eq}\theta_m \tag{8}$$

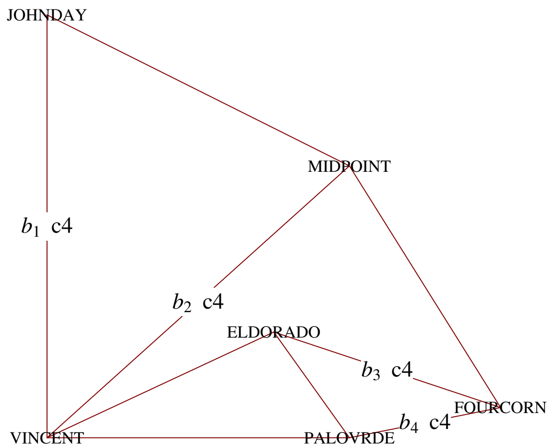

Fig. 7. Reduction of the 225 bus system to the 6 border buses of cutset area 4. Cutset area 4 becomes in the reduced system the cutset of lines marked c4. b_1, b_2, b_3, b_4 are the respective susceptances of the lines in cutset c4.

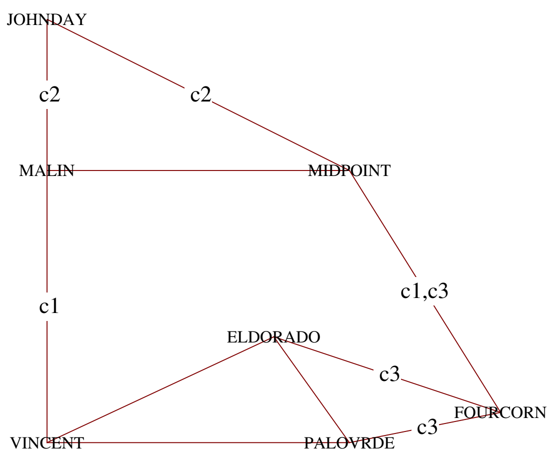

Fig. 8. Reduction of the 225 bus system to 7 border buses. Cutset areas 1, 2, and 3 become in the reduced system the cutsets of lines marked c1, c2, and c3 respectively.

We can determine from the B_{eq} matrix which border buses are joined by lines in the reduced network and the susceptances of those lines. The reduced network has the same angles θ_m at the border buses as the 225 bus system and the power injections P_{eq} at the border buses in the reduced network are related to the power injections of the 225 bus system by (6).

For the case of cutset area 4 with 6 border buses, we can reduce the 225 node system to the equivalent system of 6 border buses shown in Figure 7. For the cases of cutset areas 1, 2, and 3, we can reduce the 225 node system to the equivalent system of 7 buses shown in Figure 8. (The same reduction can be used for cutset areas 1, 2, and 3.)

Take the case of cutset area 4, which corresponds in the reduced system to the cutset c4 of 4 lines shown in Figure 7. Suppose that the 4 lines in the cutset c4 have susceptances b_1, b_2, b_3, b_4 as shown in Figure 7. It is important to note that the susceptances b_1, b_2, b_3, b_4 in the reduced system depend only on the susceptances of lines in cutset area 4 of the 225 bus system. (This result emerges from explanations in [3], based on the way the cutset area and border buses are defined and on the resulting structure of the B_{eq} matrix.) Then, according to [3], we can define the cutset susceptance

$$b_{c4} = b_1 + b_2 + b_3 + b_4 \tag{9}$$

and the cutset angle

$$\begin{aligned}\theta_{AB4} = \\ &\frac{b_1}{b_{c4}}(\theta_{\text{JOHNDAY}} - \theta_{\text{VINCENT}}) + \frac{b_2}{b_{c4}}(\theta_{\text{MIDPOINT}} - \theta_{\text{VINCENT}}) + \\ &\frac{b_3}{b_{c4}}(\theta_{\text{FOURCORN}} - \theta_{\text{ELDORADO}}) + \frac{b_4}{b_{c4}}(\theta_{\text{FOURCORN}} - \theta_{\text{PALOVRDE}})\end{aligned} \tag{10}$$

$$\begin{aligned}= &\frac{b_1}{b_{c4}}\theta_{\text{JOHNDAY}} + \frac{b_2}{b_{c4}}\theta_{\text{MIDPOINT}} + \frac{b_3+b_4}{b_{c4}}\theta_{\text{FOURCORN}} \\ &- \frac{b_1+b_2}{b_{c4}}\theta_{\text{VINCENT}} - \frac{b_3}{b_{c4}}\theta_{\text{ELDORADO}} - \frac{b_4}{b_{c4}}\theta_{\text{PALOVRDE}}\end{aligned} \tag{11}$$

Equation (10) expresses the cutset angle θ_{AB4} as a weighted average of angle differences across the 4 lines in the cutset c4 with the weights proportional to the line susceptances. Equation (11) shows that the cutset angle θ_{AB4} is a weighted average of angles at the 5 border buses for cutset area 4. The expressions for the cutset angles θ_{AB1}, θ_{AB2}, θ_{AB3} are similar weighted averages of the angle differences across lines in their respective cutsets $c1$, $c2$, $c3$ and also weighted averages of angles at their respective border buses. For practical calculations, it is better to use the matrix formulas in [3] that are more systematic forms of (11), but require more circuit theory to derive.

It is straightforward to show that the definitions of cutset susceptance and cutset angle in (9) and (10) yield Ohm's law (2): Multiplying (10) by (9) gives

$$\begin{aligned}b_{c4}\theta_{AB4} &= P_{\text{JOHNDAY}\to\text{VINCENT}} + P_{\text{MIDPOINT}\to\text{VINCENT}} + \\ &\quad P_{\text{FOURCORN}\to\text{ELDORADO}} + P_{\text{FOURCORN}\to\text{PALOVRDE}} \\ &= \text{total power through cutset of reduced system} \\ &= P_{AB4}\end{aligned}$$

We now discuss how (11) is used to monitor cutset area 4. The angles at the 6 border buses are obtained from phasor measurements. The weights are functions of the susceptances b_1, b_2, b_3, b_4 of lines in the cutset c4 of the reduced system. The susceptances b_1, b_2, b_3, b_4 are computed by reducing the base case 225 bus DC network model to the 6 border buses using (7). It should be noted that if the 225 bus network changes, for example, by a line tripping in cutset area 4, then the susceptances b_1, b_2, b_3, b_4 and the corresponding weights would change, but that for the purpose of monitoring the cutset area angle, we retain the susceptances b_1, b_2, b_3, b_4 and weights computed for the base case 225 DC network. It remains the case that lines tripping in cutset area 4 will cause the monitored cutset area angle to change, and the precise way it changes is described in [3].

VI. Testing with AC load flow

The cutset area angle and its properties are derived assuming a DC load flow model of the power system. In particular, the weighting factors in Table I used to combine the phasor measurements are computed using the DC load flow model. In this section, we redo the cutset area angle examples using the same weighting factors from Table I to combine the angles from an AC load flow of the 225 bus WECC system. In practical application, the measured phasor angles would similarly be combined using the weighting factors computed from a DC load flow. Thus this section is an initial test of whether similar results are obtained when the combined angles are from the AC load flow model.

The AC load flow portions of Tables II, III, and IV show the cutset area angles when they are computed using the voltage angles from the AC load flow. In Table II, the base case AC cutset area angles are within 0.8 degree of the corresponding DC cutset area angles. In Table III, the changes in AC cutset area angles with the 100 MW power transfer are within 0.2 degree of the corresponding changes in DC cutset area angles. Note that the real power injections in the AC case are a bit different than the real power injections in the lossless DC case because in the AC case the slack bus must supply the losses. In Table IV, the changes in monitored AC cutset area angles for the line trips are compared to the corresponding changes in monitored DC cutset area angles. Angle changes which are precisely zero in the DC case become small angles in the AC case. The changes in monitored AC cutset area angles for the line trips are within 3 degree of the corresponding changes in monitored DC cutset area angles. A general baseline for judging the closeness of approximation between the AC and DC load flows is that the base case DC and AC flows have angles that differ by no more than 3 degrees and the standard deviation of the angle difference is approximately 1 degree.

More testing is required for any firm conclusions about the accuracy of cutset area angles, but this initial test seems satisfactory.

VII. Angle between two buses is a special case of a cutset area angle

This section explains how the cutset area angle reduces in a special case to the angle difference between two buses. Suppose that Area 1 consists of bus 1 only and area 2 consists of bus 2 only as shown in Figure 9. Buses 1 and 2 are assumed to have phasor measurements. Then the cutset area is all of the network except for buses 1 and 2. Then it turns out that the cutset area angle is simply the difference of the two phasor angles. (In the formula for the cutset area angle, bus 1 has weight 1 and bus 2 has weight –1.) This shows that the cutset area angle reduces in this special case to the current practice of monitoring an angle difference between two buses. The angle difference depends on essentially all of the network and gives a measure of network stress that is non-specific.

We conclude that a cutset area angle is a generalization of the angle difference between two buses. In all cases the cutset area angle gives information about the stress in the cutset area. The cutset area is smaller than the entire power system, except in the special case of the angle difference between two buses.

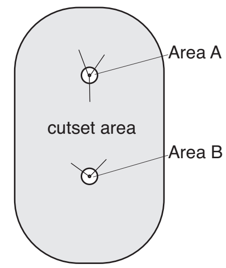

Fig. 9. The gray cutset area is all of the network except bus 1 and bus 2. Area A is bus 1 only and Area B is bus 2 only. In this special case, the cutset area angle θ_{AB} is the angle difference between bus 1 and bus 2.

VIII. Cutset areas with other network variables

The ingredients required to get cutset area angles, power flows, and susceptances to work are an "across" circuit quantity (angle difference), a "through" circuit quantity (power flow) and an admittance-like quantity (susceptance), that are related together by an Ohm's law such as (1). For developing applications of cutset areas, it is important to note that one can substitute into [3] and this paper any three corresponding across, through and admittance network quantities and all the statements remain valid.

For example, let the "across" circuit quantity be the complex phasor voltage difference V, the "through" circuit quantity be complex current I, and the admittance-like quantity be complex admittance Y. The DC load flow equations (4) are rewritten as $I = YV$. Then, in an exactly similar way as (11) we can define the complex voltage phasor V_{AB4} across cutset area 4 as a weighted average of the complex phasor voltages at the border buses:

$$V_{AB4} = \frac{y_1}{y_{c4}} V_{\text{JOHNDAY}} + \frac{y_2}{y_{c4}} V_{\text{MIDPOINT}} + \frac{y_3 + y_4}{y_{c4}} V_{\text{FOURCORN}} - \frac{y_1 + y_2}{y_{c4}} V_{\text{VINCENT}} - \frac{y_3}{y_{c4}} V_{\text{ELDORADO}} - \frac{y_4}{y_{c4}} V_{\text{PALOVRDE}}$$

where y_1, y_2, y_3, y_4 are the complex admittances of the 4 lines in the cutset c4 and the complex admittance of cutset c4 is $y_{c4} = y_1 + y_2 + y_3 + y_4$. Moreover

$$I_{AB4} = y_{c4} V_{AB4} \tag{12}$$

where I_{AB4} is the effective phasor current through cutset c4.

IX. Choosing cutset areas

Once the buses with phasor measurements are known, we need to choose cutset areas that have some of those buses as its border buses and cut the rest of the network into Areas A and B. We informally outline several approaches to choosing the cutset areas.

1) Choose any cutset of transmission lines. Then find all the buses and lines connected to the lines in this cutset by a network path that does not include any buses with phasor measurements. All these buses and lines are the cutset area. This is the procedure indicated in [3].
2) Remove all the buses with phasor measurements from the network. This breaks the network up into disconnected subnetworks. Join together some of the subnetworks by restoring some of the phasor measurement buses to obtain the three subnetworks Area A, Area B, and the cutset area.
3) A bus cutset is defined as a set of buses that, when removed from the network, divides the network into at least two networks not connected to each other. Observe that the border buses in Area A (or Area B) are bus cutsets. Choose two bus cutsets that all have phasor measurements and have no buses in common. These two cutsets divide the network into 3 parts. Choose the middle part as the cutset area.

One can maximize the possible cutset areas that can be chosen and monitored by placing phasor measurements at buses that form bus cutsets.

X. Conclusion

In this paper we define a concept of cutset area that is an area of the power system that separates the power system and has border buses with phasor measurements. The angle across the cutset area is a weighted average of the angles at the border buses, where the weights are computed from the impedances of a DC load flow model with circuit theory developed in [3].

We choose example cutset areas in a 225 bus DC load flow model of the Western area power system and illustrate the following properties of the angle across the cutset area:

1) The cutset area angle changes proportionally to the effective power flow through the cutset area. The cutset area angle does not change when power is redispatched outside the cutset area.
2) The cutset area angle changes when lines are tripped inside the cutset area and does not change (except in cases of islanding) when lines are tripped outside the cutset area.

These properties remain approximately true when the cutset area angle is calculated from the voltage angles from an AC power system model. The properties show how monitoring the cutset area angle gives stress information specific to the cutset area. It should be advantageous for monitoring and control to use cutset area angles that are responsive to system stress in each specific cutset area and largely insensitive to stress outside each cutset area. For example, it should be easier to interpret changes in the cutset area angle due to line trippings inside the cutset area. The fact that the cutset area angle is based on circuit theory makes it likely to be a more meaningful and robust quantity to monitor than an arbitrary combination of angles. The cutset area angle augments the usual notions of power flow between two areas with information about the cutset angle and impedance. As a special case, the cutset area angle reduces to the angle difference between two buses if the cutset area is chosen to be all of the power system except the two buses. Therefore monitoring the cutset area angle generalizes the monitoring of angle differences between pairs of buses in such a way that the monitoring is restricted to a specific area of the power system. It is easy to define other circuit quantities related to cutset areas such as the complex phasor voltage across a cutset area and the complex admittance of a cutset area since the theory is exactly parallel and can be obtained by simply substituting corresponding variables.

We hope that the definitions and explanations in this paper, together with the new circuit theory developed in [3], will move cutset area angle monitoring towards practical applications in power system monitoring and control.

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