Bulk Power System Dynamics and Control - VIII, August 1-6, 2010, Buzios, Rio de Janeiro, Brazil

New angles for monitoring areas

Ian Dobson
ECE Department
University of Wisconsin
Madison, WI 53706 USA
dobson@engr.wisc.edu

Abstract

We define the voltage angle across an area of a power system to measure the area stress. The voltage angle across the area is a weighted combination of the voltage angles at all the buses along the border of an area. The area angle can be divided into two parts; one part is internal stress caused by power injections inside the area and the other part is external stress caused by power flows from other areas. The area angle can be monitored using voltage synchrophasor measurements at the border buses. If the currents or powers flowing into the area at the border buses are also measured, then the internal stress angle can also be monitored. The internal stress angle changes when lines inside the area outage or when there is redispatch of power within the area. The internal stress angle does not change when lines outage outside the area or power is redispatched outside the area. This makes the internal stress angle useful for detecting and monitoring changes inside the area. If it is known which line inside the area is outaged, the angle across the line after the outage can be computed from the change in the internal stress angle. The analysis uses a DC load flow model of the power system and exploits the recently discovered cutset angle.

1 Introduction

Synchronized phasor measurements [7, 5, 3] are becoming more widespread and are opening further opportunities for power transmission system monitoring and control. Here we show how to combine phasor measurements at the border buses of an area of the power system to measure the area stress with three new angles. The total area stress can be divided into an internal stress due to power injections inside the area and an external stress due to power flows from other areas. The new angles obey circuit laws and offer specific information about the chosen area. We expect that the new angles will be easy to implement and will help provide actionable information for engineers and operators.

Cutset area angles are another new way to combine together phasor angle measurements to extract more specific information about power grid stress inside a particular area of the power system. Cutset area angles were invented in 2009, developed from first principles in [1], and illustrated in [2]. Here we significantly extend this previous work and generalize the angles across cutset areas to angles across more general areas of the power system that need not be cutset areas.

The new angles are informally explained and illustrated in Section 2. Section 3 summarizes previous literature. Section 4 derives the new angles and section 5 analyzes how the angles change when lines outage or power is redispatched. Section 6 concludes the paper.

In order to define and analyze the new angles, we assume a DC load flow model of the power system throughout the paper.

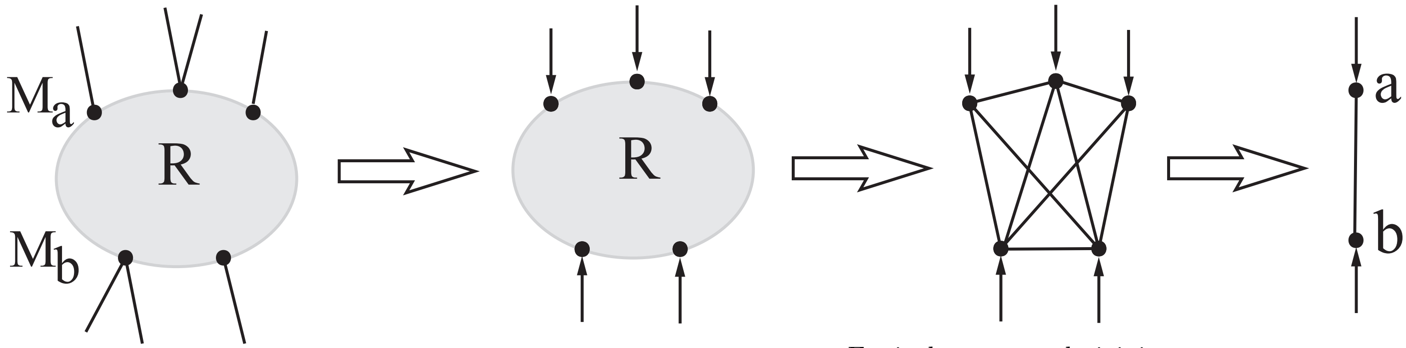
2 Area angles overview

This section informally introduces and illustrates the new area angles.

2.1 Reduction of the area

Consider the area of the power system R shown in Figure 1. The buses at the border of R are shown as black dots in Figure 1. These border buses are at the ends of all tie lines joining area R to the rest of the network. That is, removing the border buses would island the area R. The border buses are divided into buses in M_a and buses in M_b. The goal is to define an area angle $\hat{\theta}_{ab}$, area susceptance b_{ab}, and area power flow P_{ab} between M_a and M_b. This is done by reducing the area R to a single line equivalent as shown in Figure 1. Then the area quantities $\hat{\theta}_{ab}$, b_{ab}, and P_{ab} are the corresponding quantities for the single line equivalent shown in Figure 2.

The reduction process shown in Figure 1 starts by replacing the tie lines joining area R to the rest of the network by the power flows $P^{\rm into}$ along these tie lines. Then a standard network reduction removes all the buses inside area R and replaces them with equivalent lines joining the border buses to each other. All the power injections inside R are replaced by equivalent power injections $P^{\rm R}$ at the border buses. The total power injections at border buses are now $P^{\rm into}+P^{\rm R}$. $P^{\rm into}$ accounts for the external powers flowing into area R from other areas and $P^{\rm R}$ accounts for the internal powers



Area R with border buses M_a and M_b

Rest of network replaced with power injections P^{into} at border buses

Equivalent network joining border buses, and with power injections $P^{\text{into}} + P^{\text{R}}$

Single line equivalent

Fig. 1: Reduction of area R to a single line equivalent that describes the electrical characteristics between the set of border buses M_a and the set of border buses M_b. Angle across area R from M_a to M_b is the angle $\hat{\theta}_{ab}$ across the single line from a to b. Power flow through area R from M_a to M_b is the single line power flow P_{ab}. Area R susceptance is the single line susceptance b_{ab}.

injections inside R. The reduced network is the same as the area R with respect to the border buses, so there is no change in the electrical properties.

In the reduced network, the border buses M_a are connected to the border buses M_b by equivalent lines. Since removing these equivalent lines separates the buses M_a from the M_b, these equivalent lines form a cutset of the reduced network, and we can apply the cutset angle concept of [1] to define the angle $\hat{\theta}_{ab}$ across the cutset. The cutset angle $\hat{\theta}_{ab}$ obeys circuit laws and can be understood as the angle across an equivalent single line joining bus a to bus b. Bus a corresponds to the set of buses M_a and bus b corresponds to the set of buses M_b. Following through this reduction process in Figure 1, we see that the angle $\hat{\theta}_{ab}$ is the angle across the area R from the M_a buses to the M_b buses. We can also use the equivalent quantities of the single line to define the susceptance b_{ab} of the area R and the equivalent power flow P_{ab} through the area R. Since circuit laws apply throughout the reduction, we have

$$P_{ab} = b_{ab}\hat{\theta}_{ab}. \tag{1}$$

Therefore the area angle $\hat{\theta}_{ab}$ is proportional to the effective power passing through the area R and to the area susceptance b_{ab}. The area angle $\hat{\theta}_{ab}$ gives stress information specific to area R.

We also have

$$P_{ab} = P_a^{\text{into}} + P_a^{\text{R}} = -P_b^{\text{into}} - P_b^{\text{R}} \tag{2}$$

where $P_a^{\text{into}} + P_a^{\text{R}}$ is the total equivalent power injected into the buses M_a and $P_b^{\text{into}} + P_b^{\text{R}}$ is the total equivalent power injected into the buses M_b. Here P_a^{into} is the total power entering into the area R along the external tie lines attached to the border buses in M_a.

It follows from (2) that

$$\begin{aligned} P_{ab} &= \tfrac{1}{2}\left[(P_a^{\text{into}} + P_a^{\text{R}}) - (P_b^{\text{into}} + P_b^{\text{R}})\right] && (3)\\ &= \tfrac{1}{2}(P_a^{\text{into}} - P_b^{\text{into}}) + \tfrac{1}{2}(P_a^{\text{R}} - P_b^{\text{R}}) && (4) \end{aligned}$$

Equation (3) relates P_{ab} to the difference of the power flows injected at buses M_a and M_b. Equation (4) splits P_{ab} into two parts. One part is related to the difference of the external power flows injected at buses M_a and M_b. The other part is related to the difference of the power flows equivalent to the power flows internal to area R.

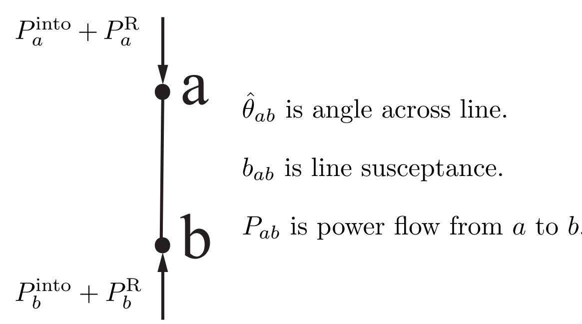


Fig. 2: Single line quantities.

2.2 Angles across the area

Suppose there are nb border buses. Write the phasor angles at the border buses in the vector

$$\theta_m = \begin{pmatrix} \theta_{m1} \\ \theta_{m2} \\ . \\ . \\ \theta_{m\text{nb}} \end{pmatrix} \tag{5}$$

Table 1: Area angles and power flows

	$\hat\theta_{ab}$	$\hat\theta_{ab}^{\rm into}$	$\hat\theta_{ab}^{\rm R}$	$P_a^{\rm into}$	$P_a^{\rm R}$	P_{ab}	$P_b^{\rm into}$	$P_b^{\rm R}$	from bus	to bus
base case	10.85	9.776	1.070	5257	-3523	1734	2131	-3865		
transfer 100 MW inside R	11.10	9.765	1.330	5256	-3482	1774	2133	-3907	SYLMARS	IMPRLVLY
transfer 100 MW outside R	11.45	10.38	1.070	5353	-3523	1830	2035	-3865	TRACY	NAVAJO
line outage inside R	13.36	9.660	3.702	5239	-3102	2137	2150	-4286	DEVERS	VALLEY
line outage outside R	11.23	10.16	1.070	5319	-3523	1796	2070	-3865	LOSBANOS	MIDWAY

angles in degrees powers in MW

$b_{ab} = 91.62$ per unit on 100 MW base

The area angle $\hat\theta_{ab}$ is a weighted linear combination of the border bus angles θ_m:

$$\hat\theta_{ab} = w\,\theta_m = w_1\theta_{m1} + w_2\theta_{m2} + \cdots + w_{\rm nb}\theta_{m{\rm nb}}. \tag{6}$$

The weights in the row vector w are determined by the susceptances of lines in service in R using a DC load flow model of R (see (48)). Therefore, if we place phasor angle measurements at all the border buses of R and know the status of lines in R, we can easily determine the angle $\hat\theta_{ab}$ across the area R. The area susceptance b_{ab} is also easy to evaluate given the DC load flow model of R and the status of the lines.

Combining (1) and (3) gives

$$\hat\theta_{ab} = \frac{1}{2b_{ab}}\left[(P_a^{\rm into} + P_a^{\rm R}) - (P_b^{\rm into} + P_b^{\rm R})\right] \tag{7}$$

That is, the area angle $\hat\theta_{ab}$ results from the difference in the powers injected at the M_a border buses and the powers injected at the M_b border buses. In terms of the single line equivalent shown in Figure 2, $\hat\theta_{ab}$ arises from the difference in the powers injected at the a bus and the powers injected at the b bus.

Combining (1) and (4) gives

$$\begin{aligned}\hat\theta_{ab} &= \frac{P_a^{\rm into} - P_b^{\rm into}}{2b_{ab}} + \frac{P_a^{\rm R} - P_b^{\rm R}}{2b_{ab}} && (8)\\ &= \hat\theta_{ab}^{\rm into} + \hat\theta_{ab}^{\rm R} && (9)\end{aligned}$$

where

$$\hat\theta_{ab}^{\rm into} = \frac{P_a^{\rm into} - P_b^{\rm into}}{2b_{ab}} \tag{10}$$

and

$$\hat\theta_{ab}^{\rm R} = \frac{P_a^{\rm R} - P_b^{\rm R}}{2b_{ab}}. \tag{11}$$

The angle $\hat\theta_{ab}^{\rm into}$ is caused by differences in the powers entering into the area at M_a and M_b and measures the external stress on area R. The angle $\hat\theta_{ab}^{\rm R}$ is caused by differences at M_a and M_b of the powers equivalent to the powers generated or consumed inside area R and measures the internal stress on area R. In particular, $\hat\theta_{ab}^{\rm R}$ only depends on the power injections inside area R and the lines in service inside R.

Now we discuss how the internal stress angle $\hat\theta_{ab}^{\rm R}$ can be obtained from phasor measurements. We assume that the lines in service inside R are known so that the area susceptance b_{ab} can be calculated. The power flowing into a bus can be determined from phasor measurements if the currents in the tie lines and the bus voltage are both measured. If the external tie line power entering each bus in M_a is known from these measurements, these can be summed to obtain the total power $P_a^{\rm into}$ entering into the area R through M_a. The total power $P_b^{\rm into}$ entering into the area R through M_b can be obtained similarly. Then we can determine $\hat\theta_{ab}^{\rm into}$ using (10) and then determine $\hat\theta_{ab}^{\rm R}$ using

$$\hat\theta_{ab}^{\rm R} = \hat\theta_{ab} - \hat\theta_{ab}^{\rm into}. \tag{12}$$

Changes $\Delta\hat\theta_{ab}^{\rm R}$ in the internal stress angle can be related to changes inside the area R such as line outages. In particular, $\Delta\hat\theta_{ab}^{\rm R}$ is proportional to the pre-outage power flow and angle on the line. If it is known by other methods which line inside R has outaged, then the angle across the line *after* the line is outaged is given by

$$\hat\theta_k^{\rm after} = \frac{b_{ab}}{b_k\,\rho_{abrs}^{\rm R}}\Delta\hat\theta_{ab}^{\rm R}. \tag{13}$$

Here b_k is the susceptance of the outaged line and $\rho_{abrs}^{\rm R}$ is a power transfer outage distribution factor computed from the DC load flow model of area R.

We note why it is better to measure stress with angles rather than power flows. Consider the easy case of only two buses a and b joined by two equal lines. In this case the reduction to a single line is clear: $\hat\theta_{ab} = \theta_a - \theta_b$, b_{ab} is the sum of the line susceptances and P_{ab} is the sum of the line power flows. It can be argued that $\hat\theta_{ab}$ measures the line stress better than P_{ab}, because if one of the double line outages, then the power flow P_{ab} (now only on the remaining line remaining in service) does not change but the line susceptance halves and $\hat\theta_{ab}$ doubles.

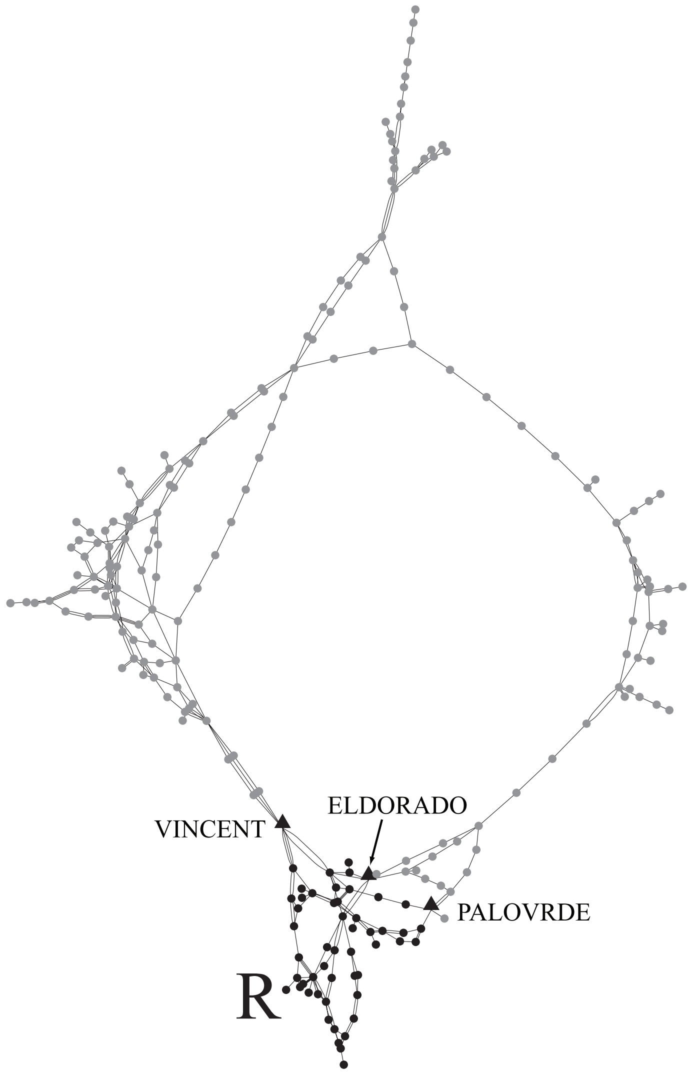


Fig. 3: Area R shown as the black buses for a 225 bus model of the Western USA. Gray buses are outside area R. The border buses with phasor measurements are the labelled black triangles.

2.3 A simple example of an area and its angles

We illustrate the concepts using the model of the WECC system shown in Figure 3. The model has 225 buses and is a reduced representation of the higher voltage WECC transmission system. The network layout is roughly geographic so that Canadian buses are at the top, Southern California is at the bottom left and New Mexico is at the lower right.

We choose the area R that includes the black buses shown in Figure 3 as a simple example. The three border buses labelled by black triangles are $M_a = \{\text{ELDORADO, PALOVRDE}\}$ and $M_b = \{\text{VINCENT}\}$. For the purpose of illustration, we assume phasor measurements at the three border buses.

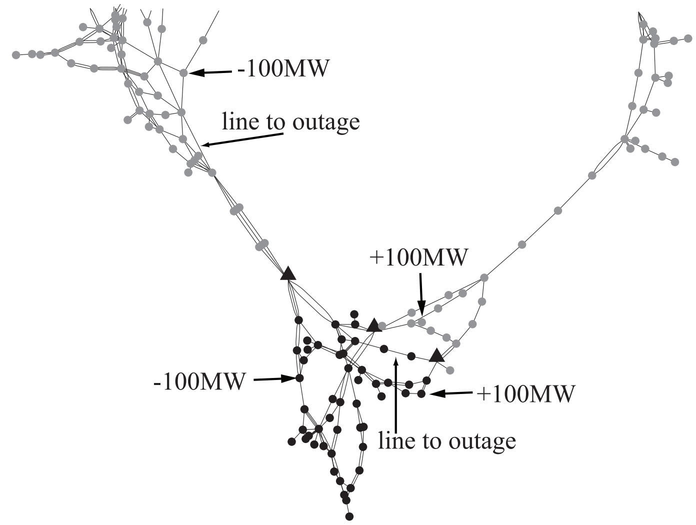


Fig. 4: Area R showing the power transfer and the line to outage inside R (in the black buses) and the power transfer and the line to outage outside R (in the gray buses).

The angle $\hat{\theta}_{ab}$ across the area R is defined as a weighted combination of the phasor angles at the border buses:

$$\hat{\theta}_{ab} = 0.79875\,\theta_{\text{ELDORADO}} + 0.20125\,\theta_{\text{PALOVRDE}} - \theta_{\text{VINCENT}} \tag{14}$$

Various angles and flows are shown in Table 1 for the base case and for some examples of transfers and line outages. The transfers and line outages are shown in Figure 4. The base case area stress is $\hat{\theta}_{ab} = 10.85$ degree and the equivalent power flow west through the area is $P_{ab} = 1734$ MW. The area susceptance is $b_{ab} = 91.62$ per unit on a 100 MW base. The transfer inside area R increases the westward power flow and increases the stress $\hat{\theta}_{ab}$. The transfer outside area R also increases the westward power flow and the stress $\hat{\theta}_{ab}$. Both of the line outages also increase the overall stress on area R.

The area stress can be decomposed into external and internal stress as $\hat{\theta}_{ab} = \hat{\theta}_{ab}^{\text{into}} + \hat{\theta}_{ab}^{\text{R}}$. All the changes affect the power flows into area R and the external stress $\hat{\theta}_{ab}^{\text{into}}$. However, it can be seen from Table 1 that the internal stress $\hat{\theta}_{ab}^{\text{R}}$ only changes when the transfer or line outage is inside area R.

When the line inside area R outages, the internal stress angle $\hat{\theta}_{ab}^{\text{R}}$ increases from 1.070 degrees to 3.702 degrees, so that $\Delta\hat{\theta}_{ab}^{\text{R}} = 2.632$ degrees. Then evaluating (13) shows that the angle across the outaged line after the outage is $\hat{\theta}_{k}^{\text{after}} = 21.49$ degrees. This information is useful in determining whether the line can be safely reclosed.

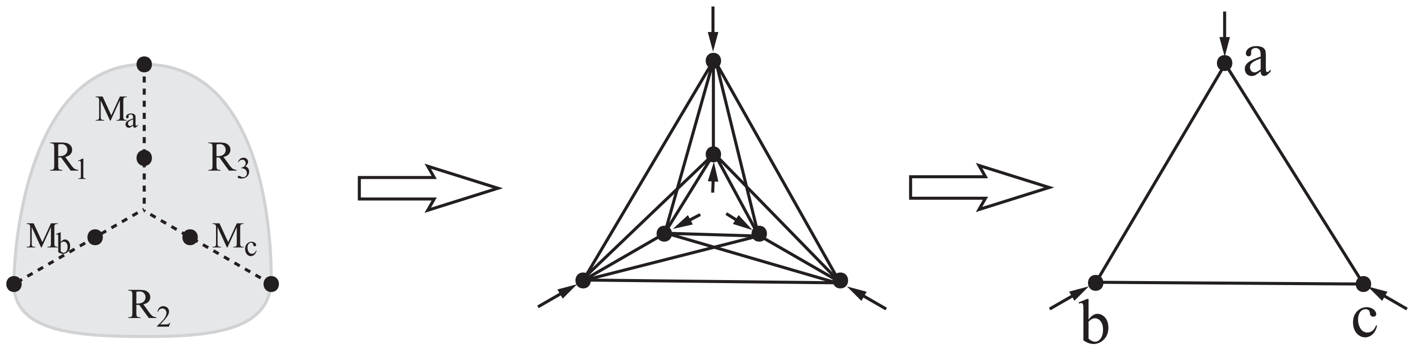


Fig. 5: Reduction of a three area power system to a three node equivalent

2.4 Multiple areas

Area angles can be applied to several areas in a power system. For example, consider the power system with the three areas R_1, R_2, R_3 shown in the left hand of Figure 5. Border buses are shown as black dots. Areas R_1 and R_2 have border buses M_b in common, areas R_2 and R_3 have border buses M_c in common, and areas R_3 and R_1 have border buses M_a in common. Although two border buses are shown in Figure 5 for each common border, there could be any number of border buses on each common border.

We can proceed with the network reduction process for all three areas at once. The first step is to replace each area by an equivalent network joining the border buses and with additional power injections at each border bus that are equivalent to the power injections inside the area. The result of the first step is shown in the middle of Figure 5. The second step is to apply the cutset angle concept to each of the equivalent subnetworks to define the area angles $\hat{\theta}_{ab}$, $\hat{\theta}_{bc}$, $\hat{\theta}_{ca}$, the area susceptances b_{ab}, b_{bc}, b_{ca}, and the power flows P_{ab}, P_{bc}, P_{ca}. The effect is to reduce the three area power system to the three bus power system shown in the right hand of Figure 5.

Area angles are applied to multiple cutset areas of a power system in [2].

3 Previous work

In [1], we develop from scratch a concept of angle across a cutset of lines and extend the concept to a cutset area angle by considering a reduced network. The new concepts are derived as a non-standard instance of general circuit theory. In [2], we further develop and illustrate the cutset area and the monitoring of the cutset area angle. Although the cutset angle and cutset area angle concepts are simple, we have not yet found any previous literature defining or using these concepts. This paper significantly generalizes the cutset area of [1, 2], particularly in not requiring the area to separate the power system.

Tate and Overbye [9] give an algorithm for detecting which lines have outaged using phasor measurements. Their analysis also exploits, among other methods, the standard network reduction of DC load flow and participation factors to represent line outages. Tate and Overbye [9] should also be read for more information about the signal processing of the phasor measurements and the practical availability of the DC load flow models since these useful topics are not treated in this paper.

Previous work by others on monitoring power system stress with phasor measurements has focused on the angle difference between two buses. Simulations of the grid before the August 2003 Northeastern blackout show increasing angle differences between Cleveland and West Michigan, suggesting that large angle differences could be a blackout risk precursor [4]. A recent simulation study [8] of potential phasor measurements on the 39 bus New England test system shows that, of several phasor measurements, angle differences were the best in discriminating alert and emergency states. A large angle difference between two buses does indicate, in some general sense, a stressed power system. However, this angle difference it is generally affected by changes throughout the entire grid, and it is difficult to interpret the reason for changes in the angle difference or set thresholds. The angle across an area is a generalization of the angle difference between two buses (see subsection 5.5) and gives specific information about the area.

There has been some previous work that combines phasor measurements at several buses. A weighted average of voltage magnitudes or reactive powers derived from WECC phasor measurements is discussed in [10]. The weighted averages provide robust control signals that are the basis for wide area control schemes for transient and voltage stability. The weights are established by location and sensitivity considerations. Reference [10] also discusses weighting phasor voltage angles to calculate a center of inertia angle for an area. Wide area nomograms involving linear combinations of phasor angles have been suggested for monitoring of security boundaries [6].

4 Area angles

This section derives the new area angles.

4.1 Notation and definitions

This subsection specifies notation and definitions for some basic network quantities.

We recall standard definitions[1] of cutsets of buses and transmission lines:

> A *nodal cutset* is a set of buses that cuts the network into separate networks when that set of buses is removed from the network.

> A *cutset* of lines is a set of lines that cuts the network into separate networks when that set of lines is removed from the network.

Let θ be the vector of bus angles and P be the vector of bus power injections. The DC load flow equations of the base case grid are

$$P = B\theta \tag{15}$$

where

$$B = A\Lambda A^T \tag{16}$$

and Λ is the diagonal matrix of line susceptances

$$\Lambda = \mathrm{diag}\{b_1, b_2, \cdots, b_{\mathrm{nline}}\}$$

and A is the incidence matrix

$$A_{ij} = \begin{cases} 1 & \text{bus } i \text{ is sending bus of line } j \\ -1 & \text{bus } i \text{ is receiving bus of line } j \\ 0 & \text{otherwise.} \end{cases} \tag{17}$$

The transpose of the incidence matrix A relates the bus angles θ to the line angle differences $\hat{\theta}$:

$$\hat{\theta} = A^T\theta \tag{18}$$

To streamline the notation, we often do not indicate the length of a row or column vector and this length must be deduced from the context.

4.2 Angle across a cutset

We derive the angle across a cutset in an example to briefly explain the concept and establish notation. A more thorough explanation of cutset angle can be found in [1].

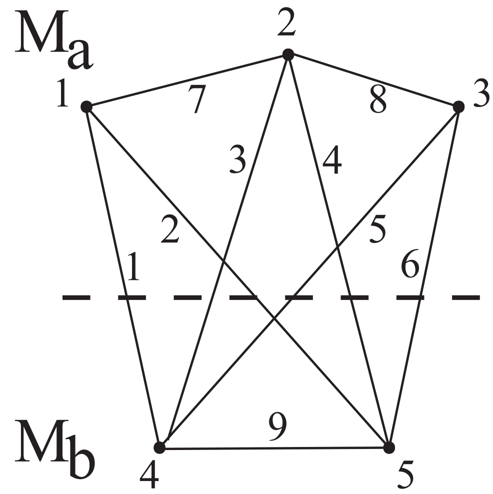


Fig. 6: Cutset angle example: Buses 1,2,3 in M_a are separated from buses 4,5 in M_b by the cutset of lines 1,2,3,4,5,6. The cutset is indicated by the dashed line.

Consider the example network in Figure 6 with buses 1,2,3 in M_a and buses 4,5 in M_b. The buses in M_a are separated from the buses in M_b by the cutset of lines 1,2,3,4,5,6 that is indicated by the dashed line. Write θ_i for the voltage angle at bus number i and $\hat{\theta}_j$ for the angle difference across line number j. The susceptance of line number j is b_j.

The power flowing from M_a to M_b along line j of the cutset is $b_j\hat{\theta}_j$. For this illustration, it is convenient to assume that the angle difference $\hat{\theta}_j$ on line j is defined so that $\hat{\theta}_j$ is positive for positive power flowing on line j from M_a to M_b. The power P_{ab} flowing through the cutset is the sum of the powers flowing in each line of the cutset:

$$P_{ab} = b_1\hat{\theta}_1 + b_2\hat{\theta}_2 + b_3\hat{\theta}_3 + b_4\hat{\theta}_4 + b_5\hat{\theta}_5 + b_6\hat{\theta}_6. \tag{19}$$

The cutset susceptance is

$$b_{ab} = b_1 + b_2 + b_3 + b_4 + b_5 + b_6. \tag{20}$$

Following [1], we define the angle across the cutset as

$$\hat{\theta}_{ab} = \frac{b_1}{b_{ab}}\hat{\theta}_1 + \frac{b_2}{b_{ab}}\hat{\theta}_2 + \frac{b_3}{b_{ab}}\hat{\theta}_3 + \frac{b_4}{b_{ab}}\hat{\theta}_4 + \frac{b_5}{b_{ab}}\hat{\theta}_5 + \frac{b_6}{b_{ab}}\hat{\theta}_6, \tag{21}$$

which is a linear combination of the cutset line angle differences, weighted according to the line susceptances. Then (19), (20), and (21) imply that

$$P_{ab} = b_{ab}\hat{\theta}_{ab}, \tag{22}$$

which expresses the power flowing through the cutset as the product of the cutset susceptance and the angle across the cutset. Equation (21) can be rewritten in terms of the bus angles as

$$\begin{aligned}\hat{\theta}_{ab} = {} & \frac{b_1+b_2}{b_{ab}}\theta_1 + \frac{b_3+b_4}{b_{ab}}\theta_2 + \frac{b_5+b_6}{b_{ab}}\theta_3 \\ & - \frac{b_1+b_3+b_5}{b_{ab}}\theta_4 - \frac{b_2+b_4+b_6}{b_{ab}}\theta_5 \end{aligned} \tag{23}$$

[1] Some authors define a cutset to be a *minimal* set of buses or lines that separate the network, but we do not require this here.

For practical calculations, it is better to use vector and matrix notation. The buses 1,2,3 in M_a are defined by the ones in

$$\sigma_a = (1, 1, 1, 0, 0) \tag{24}$$

The bus angles are

$$\theta_m = (\theta_1, \theta_2, \theta_3, \theta_4, \theta_5)^T \tag{25}$$

Let B^{eq}_{mm} be the 5×5 susceptance matrix of the network in Figure 6. The entries of the first 3 rows of B^{eq}_{mm} are

$$\begin{pmatrix} b_1 + b_2 + b_7 & -b_7 & 0 & -b_1 & -b_2 \\ -b_7 & b_3 + b_4 + b_7 + b_8 & -b_8 & -b_3 & -b_4 \\ 0 & -b_8 & b_5 + b_6 + b_8 & -b_5 & -b_6 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

Now it can be easily verified that (20) can be written as

$$b_{ab} = \sigma_a B^{\text{eq}}_{mm} \sigma_a^T \tag{26}$$

and (23) can be written as

$$\hat{\theta}_{ab} = \frac{\sigma_a B^{\text{eq}}_{mm} \theta_m}{b_{ab}}. \tag{27}$$

4.3 Angle between sets of buses and across an area

We explain how to obtain the angle $\hat{\theta}_{ab}$ between a set of buses M_a and set of buses M_b and across an area or region R of the network. We will also define the power flow P_{ab} from M_a to M_b through the area and the susceptance b_{ab} of the area from M_a to M_b. The sets M_a and M_b are assumed to have no buses in common. We write $M = M_a \bigcup M_b$. M will be the border buses of area R.

We assume that the border buses M are a nodal cutset.

This assumption formalizes the concept that the M is all the buses along the border of the area R to be chosen below. The assumption that M is a nodal cutset holds throughout the paper, except that section 5.5 briefly considers the case that M is not a nodal cutset. Since M is a nodal cutset, if the buses in M are removed from network, then the remaining network has several connected components, each of which is a proper subset of the network. One of these connected components N is chosen and augmented with the buses in M to form the area or subnetwork $R = M \bigcup N$. The buses in M are called the border buses for the area R, and the buses in N are called the interior buses of R. The quantities $\hat{\theta}_{ab}$, P_{ab}, and b_{ab} will be defined across or through the area R from the border buses M_a to the border buses M_b.

For a given area R with border buses M, there are usually multiple ways to divide the border buses into the buses M_a and M_b. Each choice of M_a and M_b gives different quantities θ_{ab}, P_{ab}, and b_{ab}. For example, it is reasonable that a "North to South" angle across an area is different than an "East to West" angle across the area.

We will assume phasor measurements at all the border buses M. Some of the buses in N may also have phasor measurements, but we will not make use of them.

The row vector σ_r defines the buses in R by

$$\sigma_{ri} = \begin{cases} 1 & \text{bus } i \text{ in R} \\ 0 & \text{bus } i \text{ not in R.} \end{cases} \tag{28}$$

Then the lines that join R to the rest of the network are indicated by the row vector

$$c_r = \sigma_r A \tag{29}$$

Since R is a proper subnetwork, the lines joining R to the rest of the network form a cutset. And

$$c_{rj} = \begin{cases} 1 & \text{line } j \text{ in cutset has sending bus in R} \\ -1 & \text{line } j \text{ in cutset has receiving bus in R} \\ 0 & \text{line } j \text{ not in cutset.} \end{cases} \tag{30}$$

Write A_m for the rows of A corresponding to the buses in M. Define the matrix τ by

$$\tau_{mr} = A_m \otimes |c_r| \tag{31}$$

where $\otimes$ is element by element multiplication of each row of A_m with the row vector $|c_r|$. $|c_r|$ is the row vector of absolute values of the components of c_r.

Then

$$\tau_{mrij} = \begin{cases} 1 & \text{bus } i \text{ in } M \text{ is the sending bus of line } j \\ & \text{joining bus } i \text{ to a bus not in R} \\ -1 & \text{bus } i \text{ in } M \text{ is the receiving bus of line } j \\ & \text{joining bus } i \text{ to a bus not in R} \\ 0 & \text{bus } i \text{ in } M \text{ not joined by line } j \\ & \text{to a bus not in R} \end{cases} \tag{32}$$

Each bus i that is a border bus in M has lines not in R entering bus i. We write P_i^{into} for the total power entering into bus i along all the lines not in R. We write $P_m^{\text{into}} = \{P_i^{\text{into}}, i \in M\}$ for the column vector of powers entering into buses in M along the lines not in R. Then

$$P_m^{\text{into}} = -\tau_{mr} \Lambda A^T \theta. \tag{33}$$

We write P_m for the vector of powers injected at the buses in M due to load or generation at these buses. Then the combined effect of the lines outside R and any load or generation at the border buses in M is given by power injections of $P_m^{\text{into}} + P_m$ at the border buses.

It is now convenient to focus on the subnetwork R. The power injections P_m^{into} at the border buses account for the effect of the rest of the network on area R.

The next step is to reduce area R to an equivalent network. The border buses M are retained and the interior buses N are equivalenced out. Write column vectors θ_m and P_m for the angle and power injected at the border buses and θ_n and P_n for the angle and power injected at buses N inside the area R. Order the buses in R so that the border buses come first. Then

$$\theta_r = \begin{pmatrix} \theta_m \\ \theta_n \end{pmatrix} \quad \text{and} \quad P_r = \begin{pmatrix} P_m^{\text{into}} + P_m \\ P_n \end{pmatrix}.$$

Then the DC load flow equations for area R are

$$\begin{pmatrix} P_m^{\text{into}} + P_m \\ P_n \end{pmatrix} = B^{\text{R}} \begin{pmatrix} \theta_m \\ \theta_n \end{pmatrix} = \begin{pmatrix} B_{mm}^{\text{R}} & B_{mn} \\ B_{nm} & B_{nn} \end{pmatrix} \begin{pmatrix} \theta_m \\ \theta_n \end{pmatrix}. \tag{34}$$

Here B^{R} is the susceptance matrix for the subnetwork R. (Note that B^{R} is not identical to the submatrix B_{rr} of the susceptance matrix B for the entire network. B_{mm}^{R} is the submatrix of B^{R} corresponding to the buses in M. The diagonal entries of B_{mm}^{R} are different than the submatrix B_{mm} of the susceptance matrix B for the entire network, because they do not include the susceptances of the lines outside R that are joined to buses in M.)

Eliminating θ_n from (34) in the usual way gives

$$P_m^{\text{into}} + P_m - B_{mn} B_{nn}^{-1} P_n = (B_{mm}^{\text{R}} - B_{mn} B_{nn}^{-1} B_{nm})\, \theta_m$$

and, letting

$$P_m^{\text{eq}} = P_m^{\text{into}} + P_m - B_{mn} B_{nn}^{-1} P_n \tag{35}$$

and

$$B_{mm}^{\text{eq}} = B_{mm}^{\text{R}} - B_{mn} B_{nn}^{-1} B_{nm}, \tag{36}$$

we obtain a reduced grid electrically equivalent to R with the DC load flow equations

$$P_m^{\text{eq}} = B_{mm}^{\text{eq}} \theta_m. \tag{37}$$

The reduced grid electrically equivalent to R is the border buses M joined by equivalent transmission lines and with additional power injections.

From (35), the equivalent power injections P_m^{eq} at buses in M have two parts

$$P_m^{\text{eq}} = P_m^{\text{into}} + P_m^R, \tag{38}$$

where P_m^{into} is the external powers entering into buses in M from lines outside R and

$$P_m^R = P_m - B_{mn} B_{nn}^{-1} P_n \tag{39}$$

is the internal powers entering into buses in M that account for power injections in buses at R.

The reduced network has the border buses M_a separated from the border buses M_b by a cutset of equivalent lines. The angle across the cutset will be the area angle, the susceptance of the cutset will be the area susceptance and the power flow through the cutset will be the power flow through the area from M_a to M_b. This applies the cutset angle concept indicated in section 4.2.

The row vector σ_a defines the buses in M_a by

$$\sigma_{ai} = \begin{cases} 1 & \text{bus } i \text{ in } M_a \\ 0 & \text{otherwise} \end{cases} \tag{40}$$

Define the equivalent power flow P_{ab} through area R from M_a to M_b as the cutset power flow

$$P_{ab} = \sigma_a P_m^{\text{eq}} = \sigma_a (P_m^{\text{into}} + P_m^R) = P_a^{\text{into}} + P_a^R, \tag{41}$$

where the total power flowing into M_a from lines outside R is

$$P_a^{\text{into}} = \sigma_a P_m^{\text{into}} \tag{42}$$

and the total power injected into M_a that accounts for power injections in buses of R is

$$P_a^{\text{R}} = \sigma_a P_m^{\text{R}} \tag{43}$$

Define the susceptance b_{ab} of area R from M_a to M_b as the cutset susceptance

$$b_{ab} = \sigma_a B_{mm}^{\text{eq}} \sigma_a^T \tag{44}$$

Define the angle $\hat{\theta}_{ab}$ across area R from M_a to M_b as the the cutset angle

$$\hat{\theta}_{ab} = \frac{\sigma_a B_{mm}^{\text{eq}}\, \theta_m}{\sigma_a B_{mm}^{\text{eq}} \sigma_a^T} \tag{45}$$

$$= \frac{\sigma_a B_{mm}^{\text{eq}}\, \theta_m}{b_{ab}} \tag{46}$$

$$= w\, \theta_m, \tag{47}$$

where the row vector of weights is

$$w = \frac{\sigma_a B_{mm}^{\text{eq}}}{b_{ab}}. \tag{48}$$

Then

$$P_{ab} = \sigma_a P_m^{\text{eq}} = \sigma_a B_{mm}^{\text{eq}} \theta_m = b_{ab} \frac{\sigma_a B_{mm}^{\text{eq}}\, \theta_m}{b_{ab}} = b_{ab} \hat{\theta}_{ab} \tag{49}$$

It is important to note that B_{nn} only depends on the network in the interior of R. $B_{nm} = B_{mn}^T$ only depends on the lines connecting M to the interior of R. Therefore B_{mm}^{eq} only depends on lines in R. It follows that b_{ab} only depends on the lines in service in R and that $\hat{\theta}_{ab}$ only depends on the lines in service in R and the border bus angles θ_m.

Given the status of lines within R, *one can compute the susceptance* b_{ab} *of* R. *If the border bus angles* θ_m *are also measured, then one can compute the angle* $\hat{\theta}_{ab}$ *across* R.

4.4 *Internal and external area power flows and angles*

The total power entering into R through the buses in M_a is

$$P_a^{\text{into}} = \sigma_a P_m^{\text{into}} = -\sigma_a \tau_{mr} \Lambda A^T \theta \tag{50}$$

Another way to express the total power P_a^{into} entering into R through the buses in M_a is to define the lines entering R through buses in M_a with a row vector c_a with

$$c_{aj} = \begin{cases} 1 & \text{line } j \text{ has sending bus in } M_a \\ & \text{and receiving bus not in R} \\ -1 & \text{line } j \text{ has receiving bus in } M_a \\ & \text{and sending bus not in R} \\ 0 & \text{line } j \text{ does not have a bus in } M_a \\ & \text{or line } j \text{ has one bus in } N. \end{cases} \tag{51}$$

Then

$$c_a = \sigma_a \tau_{mr} \tag{52}$$

and

$$P_a^{\text{into}} = -c_a \Lambda A^T \theta \tag{53}$$

The row vector σ_b defines the buses in M_b by

$$\sigma_{bi} = \begin{cases} 1 & \text{bus } i \text{ in } M_b \\ 0 & \text{otherwise} \end{cases} \tag{54}$$

Then

$$P_{ba} = \sigma_b P_m^{\text{eq}} \tag{55}$$

Write $\mathbf{1}$ for a row vector of all ones, and with varying length as needed by the context. Then $\sigma_b = \mathbf{1} - \sigma_a$ and

$$P_{ab} + P_{ba} = \mathbf{1} P_m^{\text{eq}} = \mathbf{1} P_m^{\text{into}} + \mathbf{1} P_m^R \tag{56}$$

$\mathbf{1} P_m^{\text{eq}} = 0$ because M is a nodal cutset. This follows since $\mathbf{1} P_m^{\text{eq}}$ is the power entering the reduced area R and there are no sources or sinks of power within the reduced area R. Therefore

$$P_{ba} = -P_{ab} \tag{57}$$

We state several formulas for the total power P^{into} entering into area R. By definition,

$$P^{\text{into}} = \mathbf{1} P_m^{\text{into}} = (\sigma_a + \sigma_b) P_m^{\text{into}} = P_a^{\text{into}} + P_b^{\text{into}} \tag{58}$$

Moreover, since M is a nodal cutset, the total power P^{into} entering into area R is also the total power consumed inside R:

$$P^{\text{into}} = -\mathbf{1} \begin{pmatrix} P_m \\ P_n \end{pmatrix} \tag{59}$$

The standard property $\mathbf{1} B^{\text{R}} = 0$ implies $\mathbf{1} B_{mn} + \mathbf{1} B_{nn} = 0$. Therefore

$$\mathbf{1} P_m^{\text{R}} = \mathbf{1}(P_m - B_{mn} B_{nn}^{-1} P_n) = \mathbf{1} P_m + \mathbf{1} P_n = -P^{\text{into}}. \tag{60}$$

Hence

$$P^{\text{into}} = -\mathbf{1} P_m^{\text{R}} = -(\sigma_a + \sigma_b) P_m^{\text{R}} = -P_a^{\text{R}} - P_b^{\text{R}}. \tag{61}$$

In summary,

$$P^{\text{into}} = P_a^{\text{into}} + P_b^{\text{into}} = -P_a^{\text{R}} - P_b^{\text{R}}. \tag{62}$$

From (41), and using (62),

$$\begin{aligned} P_{ab} &= P_a^{\text{into}} + P_a^R \\ &= \tfrac{1}{2}(P_a^{\text{into}} - P_b^{\text{into}}) + \tfrac{1}{2}(P_a^{\text{into}} + P_b^{\text{into}}) \\ &\quad + \tfrac{1}{2}(P_a^{\text{R}} - P_b^{\text{R}}) + \tfrac{1}{2}(P_a^{\text{R}} + P_b^{\text{R}}) \\ &= \tfrac{1}{2}(P_a^{\text{into}} - P_b^{\text{into}}) + \tfrac{1}{2}(P_a^{\text{R}} - P_b^{\text{R}}) \end{aligned} \tag{63}$$

Defining

$$\hat{\theta}_{ab}^{\text{into}} = \frac{P_a^{\text{into}} - P_b^{\text{into}}}{2b_{ab}} \tag{64}$$

and

$$\hat{\theta}_{ab}^{\text{R}} = \frac{P_a^{\text{R}} - P_b^{\text{R}}}{2b_{ab}}, \tag{65}$$

and dividing (63) by b_{ab}, we obtain the splitting of the area stress angle $\hat{\theta}_{ab}$ into internal and external angles as

$$\hat{\theta}_{ab} = \hat{\theta}_{ab}^{\text{into}} + \hat{\theta}_{ab}^{\text{R}} \tag{66}$$

5 Changes in area angles

This section analyzes how area angles change when lines outage or power is redispatched.

5.1 *Standard line outage and distribution factors*

We recall, based on [11], the modeling of line outages with power injections and the standard line outage distribution factors. It is convenient to write e_k for the column vector of all zeros except that the kth component is one. The length of the vector e_k varies with context.

The power transfer distribution factor ρ_{krs} is the increase in power flow in line k joining u to v due to a unit injection of power at bus r and a unit decrement of power injected at bus s:

$$\begin{aligned} \rho_{krs} &= e_k^T \Lambda A^T B^{-1}(e_r - e_s) \\ &= b_k(e_u^T - e_v^T) B^{-1}(e_r - e_s) \end{aligned} \tag{67}$$

Suppose line k joining bus r to bus s has power flow $P_k^{\rm line}$. Assume that the outage of line k does not island the network. Then the effect of outage of line k on the rest of the network is equivalent to preserving the network structure and injecting power $P_k^{\rm outage}$ at bus r and injecting $-P_k^{\rm outage}$ at bus s, where

$$P_k^{\rm outage} = \frac{P_k^{\rm line}}{1-\rho_{krs}} \tag{68}$$

The power flow on the line k *after* the power injections is $P_k^{\rm line}+\rho_{krs}P_k^{\rm outage} = P_k^{\rm outage}$, and it is this fact that implies that the injections have the same effect on the rest of the network as outaging line k [11]. It also follows that the angle across line k *after* it is outaged is

$$\hat\theta_k^{\rm after} = \frac{P_k^{\rm outage}}{b_k} = \frac{\hat\theta_k}{1-\rho_{krs}}, \tag{69}$$

where $\hat\theta_k$ is the angle across line k before it outaged.

5.2 Effect of power transfers on area angle

Consider the power transfer in which power $P^{\rm inject}$ is injected at bus r and $-P^{\rm inject}$ is injected at bus s. This power transfer may be an actual power transfer or may represent the effect of a line outage.

It is useful to generalize power transfer distribution factors to the incremental effect of the transfer on the internal power flow $\frac{1}{2}(P_a^{\rm R}-P_b^{\rm R})$ across the area (see (63)). In particular, define the power transfer distribution factor $\rho_{abrs}^{\rm R}$ to be the change in $\frac{1}{2}(P_a^{\rm R}-P_b^{\rm R})$ due to a unit injection of power at bus r and a unit decrement of power injected at bus s. Let $\Delta P_a^{\rm into}$ and $\Delta P_b^{\rm into}$ be the respective changes in $P_a^{\rm into}$ and $P_b^{\rm into}$ due to the transfer. Then

$$\Delta P_{ab} = \tfrac{1}{2}(\Delta P_a^{\rm into}-\Delta P_b^{\rm into}) + \rho_{abrs}^{\rm R}P^{\rm inject} \tag{70}$$

We consider the cases in which either buses r and s are both in R or are both outside R. Then the total power entering into R does not change, so that $\Delta P^{\rm into}=0$. It follows from (62) that

$$\Delta P_a^{\rm into} = -\Delta P_b^{\rm into} \tag{71}$$

and

$$\Delta P_a^{\rm R} = -\Delta P_b^{\rm R}. \tag{72}$$

Then

$$\Delta P_{ab} = \Delta P_a^{\rm into}+\Delta P_a^{\rm R} = \Delta P_a^{\rm into} + \rho_{abrs}^{\rm R}P^{\rm inject} \tag{73}$$

A formula for $\Delta P_a^{\rm into}$ is

$$\Delta P_a^{\rm into} = -c_a\Lambda A^T B^{-1}(e_r-e_s)P^{\rm inject}, \tag{74}$$

but our intention is to determine $\Delta P_a^{\rm into}$ from phasor measurements. Moreover, since $\frac{1}{2}(\Delta P_a^{\rm R}-\Delta P_b^{\rm R}) = \Delta P_a^{\rm R}$, $\rho_{abrs}^{\rm R}$ can be computed as the change in $P_a^{\rm R}$ due to a unit injection of power at bus r and a unit decrement of power injected at bus s. That is, from (43) and (39),

$$\rho_{abrs}^{\rm R} = f(r)-f(s) \tag{75}$$

where

$$f(r) = \begin{cases} -\sigma_a B_{mn}B_{nn}^{-1}e_r, & \text{bus } r \text{ in } N \\ 1 & \text{bus } r \text{ in } M_a \\ 0 & \text{bus } r \text{ in } M_b \\ 0 & \text{bus } r \text{ not in R} \end{cases} \tag{76}$$

$\rho_{abrs}^{\rm R}$ only depends on the area R.

The case of islanding and the case in which one of the buses r and s is in R and the other bus is not in R are not treated here.

5.3 Monitoring line outages

The objective is to monitor line outages by changes in the angles across area R and changes in power $P_a^{\rm into}$ entering into area R. It is necessary that the status of lines in R before the line outage is known. (The line status is required so that $B_{mm}^{\rm eq}$ can be evaluated.)

There is an important choice to be made when using area angles to monitor line outages. When the line outage occurs inside area R, the bus angles and line power flows change throughout the network and the line status changes inside R. The change in line status implies that the definitions of area angles, susceptances, power flows, and power transfer distribution factors all change. That is, when the line outage occurs, both the bus angles and line power flows change and the definitions of the area quantities we are using to monitor the system change. One can either monitor the system after the line outage using the old area quantities that apply before the line outage, or one can update the definitions of the area quantities and monitor the system using the new area quantities after the line outage. A disadvantage of updating the definitions of the area quantities when the line outage occurs is that one needs to know which line outaged to calculate the new definitions. In this paper we choose to monitor the system after the line outage occurs using the old area quantities that were defined before the line outage. (In a more developed application of the area angles, one could make this same choice retaining the old area quantities just after the line outage occurs for the purpose of detecting and quantifying the line outage, and then update the definitions to reflect the line outage after the line outaged is identified in order to be better positioned for further line outages.)

We make phasor measurements at the border buses in M of the angles θ_m and the powers $P_m^{\rm into}$ entering into the border buses. Then, given a DC load flow model of R, we can compute $\hat{\theta}_{ab}$, b_{ab}, $P_a^{\rm into}$, and $P_b^{\rm into}$. We monitor the changes in these quantities and compute the change in the internal area angle

$$\Delta\hat{\theta}_{ab}^{\rm R} = \Delta\hat{\theta}_{ab} - \frac{\Delta P_a^{\rm into} - \Delta P_b^{\rm into}}{2b_{ab}} \tag{77}$$

and monitor the changes $\Delta\hat{\theta}_{ab}^{\rm R}$. In some cases, only one of $\Delta P_a^{\rm into}$ and $\Delta P_b^{\rm into}$ need be monitored.

Suppose that the power system is not islanded by the line outage. In this case, the effect of the outage of line k is equivalent to retaining the current line statuses and injecting powers $\pm P_k^{\rm outage}$ determined by (68) at the ends of the line. Then, if the outaged line has no buses in R, the power injections inside R do not change and

$$\Delta\hat{\theta}_{ab}^{\rm R} = 0. \tag{78}$$

On the other hand, if the outaged line is within R, then

$$\begin{aligned}
\Delta\hat{\theta}_{ab}^{\rm R} &= \frac{\Delta P_a^{\rm R}}{b_{ab}} && (79)\\
&= \frac{\rho_{abrs}^{\rm R} P_k^{\rm outage}}{b_{ab}} && (80)\\
&= \frac{\rho_{abrs}^{\rm R}}{b_{ab}(1-\rho_{krs})} P_k^{\rm line} && (81)\\
&= \frac{b_k\, \rho_{abrs}^{\rm R}}{b_{ab}(1-\rho_{krs})} \hat{\theta}_k && (82)\\
&= \frac{b_k\, \rho_{abrs}^{\rm R}}{b_{ab}} \hat{\theta}_k^{\rm after} && (83)
\end{aligned}$$

Equations (81) and (82) show that $\hat{\theta}_{ab}^{\rm R}$ is proportional to the power flow $P_k^{\rm line}$ through the line before the outage and the angle $\hat{\theta}_k$ across the line before the outage. Equation (83) shows that $\hat{\theta}_{ab}^{\rm R}$ is proportional to angle $\hat{\theta}_k^{\rm after}$ across the line after the outage. Rearranging (83) gives

$$\hat{\theta}_k^{\rm after} = \frac{b_{ab}}{b_k\, \rho_{abrs}^{\rm R}} \Delta\hat{\theta}_{ab}^{\rm R} \tag{84}$$

If it is known which line outaged so that $b_k\, \rho_{abrs}^{\rm R}$ can be calculated, (84) determines the angle $\hat{\theta}_k^{\rm after}$ across the line after the outage from the phasor measurements.

In this section, we analyze the simplest case of line outages in which the line buses are either both in R or both not in R and in which the line outage does not cause islanding. If the line outage islands the power system, any power redispatch caused by the islanding should be taken into account. We expect that the methods of the paper can be extended to this case as long as the redispatch of the power imbalance caused by the islanding is specified. It is not essential to analyze the case of line trips in which one of the line buses is a border bus and the other bus is outside area R. Our proposed monitoring already measures the current or power flowing in such lines and the line outage can be directly detected from these measurements.

5.4 Cutset areas and areas with M_a or M_b a cutset

A useful special case is that the buses in M contain two nodal cutsets and in particular that the buses M_a is one nodal cutset and M_b is another nodal cutset. Then the area between M_a and M_b is called a cutset area [2]. In this subsection, we consider a case slightly more general than a cutset area in which at least one of M_a or M_b is a nodal cutset. Without loss of generality, suppose that M_a is a nodal cutset. We continue to assume that M is a nodal cutset.

If M_a is a nodal cutset, there is a nice simplification for monitoring line outages. Suppose that the line buses are both in R or are both outside R. Then the equal and opposite power injections representing the effect of the line outage do not change total power entering into R, so that $\Delta P^{\rm into} = 0$. Moreover, since M_a is a nodal cutset, the total power entering M_a does not change so that $\Delta P_a^{\rm into} = 0$. Then we can use (62) and $\Delta P^{\rm into} = 0$ to deduce that $\Delta P_b^{\rm into} = 0$. Thus $\Delta P_a^{\rm into} = \Delta P_b^{\rm into} = \Delta P^{\rm into} = 0$ and the change in the external area stress always vanishes. Then the phasor measurements of the current or powers entering the area are not needed to monitor the line outages with changes in the stress angle. Several examples of monitoring a cutset area in this way are given in [2].

5.5 Border buses not a cutset

All of the paper, except this subsection, assumes that the border buses M are a nodal cutset. If the border buses M are not a nodal cutset, then Section 4 applies with minor modifications and in particular one can still define the angle between M_a and M_b across the area R. However, following the procedure in subsection 4.3, the area R is necessarily the entire network. There are no external powers entering the network, so $\hat{\theta}_{ab}^{\rm into} = 0$ and $\hat{\theta}_{ab} = \hat{\theta}_{ab}^{\rm R}$. We speculate that having the area R be the entire network does not seem advantageous for applications to large power grids because the specificity of the area is lost.

One extreme example of both a cutset area (see [2]) and a case with M not a nodal cutset can often arise in the case of M_a a single bus and M_b a single bus. In this case, the angle $\hat{\theta}_{ab}$ across the entire network is simply the angle difference between the two buses.

5.6 Area angles with other network variables

The ingredients required to get area angles, power flows, and susceptances to work are an "across" circuit quantity (angle difference), a "through" circuit quantity (power flow) and an admittance-like quantity (susceptance), that are related together by an Ohm's law such as (1). For developing applications of area angles such as model reduction, it is important to note that one can substitute into the circuit theory derivations of this paper any three corresponding across, through and admittance network quantities and all the statements remain valid.

For example, let the "across" circuit quantity be the complex phasor voltage difference V, the "through" circuit quantity be complex current I, and the admittance-like quantity be complex admittance Y. The DC load flow equations (15) are rewritten as $I = YV$. Then, in an exactly similar way as (14) we can define the complex voltage phasor $\hat{V}_{ab}$ across area R as a weighted average of the complex phasor voltages at the border buses:

$$\hat{V}_{ab} = \frac{y_1}{y_{ab}} V_{\text{ELDORADO}} + \frac{y_2}{y_{ab}} V_{\text{PALOVRDE}} - V_{\text{VINCENT}}$$

where y_1 and y_2 are the complex admittances of the 2 lines in the cutset separating $M_a = \{\text{ELDORADO, PALOVRDE}\}$ from $M_b = \{\text{VINCENT}\}$ in the reduced network of the 3 border buses. The complex admittance of the area is $y_{ab} = y_1 + y_2$. Moreover,

$$I_{ab} = y_{ab} \hat{V}_{ab}, \tag{85}$$

where I_{ab} is the effective phasor current through the area.

6 Conclusions

We define new angles across a power system area to be able to monitor the total stress of the area, as well as the parts of the stress that are related to the power flows internal and external to the area. The new angles can be easily calculated from phasor measurements at all the buses along the border of the area using a DC load flow model of the area and knowledge of which lines in the area are in service. The new angles generalize the cutset area angles of [1, 2] and their calculation uses the new cutset angle concept introduced in [1].

Previous approaches to measuring stress with phasor measurements have used the difference of angles at two buses or searched for patterns in angles from many buses. The new area angles have some advantages over these approaches. The area angles give stress information specific to an area of the power system. This corresponds with the way large power systems are operated, and information that describes specific properties of a specific area of a large power system is more actionable. Since the new area angles obey circuit laws, they are more meaningful than an arbitrary combination of angles and should provide quantities summarizing the power system condition that will behave in ways consistent with the intuition of operators and engineers about power flows.

The area angles may also be useful in summarizing and communicating the state of large power systems when the angles and power flows are known from state estimation. The area angles could also be applied to problems of model reduction. It is easy to define other circuit quantities related to areas such as the complex phasor voltage across an area and the complex admittance of an area since the theory is exactly parallel and can be obtained by simply substituting corresponding variables.

One disadvantage of the area angles is that a DC load flow model of the area and a knowledge or assumption of the line status in the area is required. The DC load flow approximation is expected to be practical, but has only been tested against AC load flow results for the special case of cutset area angles [2]. It is an open question what level of network detail in the DC load flow model is needed. It is possible that measurements at border buses only connected to high impedance or low voltage lines may be neglected, but this has not yet been determined.

The area angles respond to line outages and larger changes in area angles correspond to outages of lines with larger power flows in accordance with circuit laws. The area angles also respond proportionally to power redispatches. It is useful that the internal area stress angle does not respond to line outages and power redispatches outside the area. Changes in the internal stress angle across the area can detect line outages inside the area. If, in addition, it is known which line is outaged, then the angle across the outaged line after it outages can be computed from the change in the internal stress angle. This could be helpful in assessing whether it safe to reclose the line. We suggest that the method of Tate and Overbye [9] could be used to determine which line in the area is outaged. The monitoring of line outages in an area simplifies if the area is a cutset area, because then only the border bus voltages, and not any currents, need to be monitored.

The paper shows the valuable information about an area that can be obtained by placing phasor measurements at all the buses along the border of the area. It is natural to have already placed phasor measurements at major area tie lines, so these could be systematically augmented to obtain phasor measurements at all the buses along the border. More generally, the method of the paper suggests that phasor measurements be placed along nodal cutsets. Instead of fixing the areas and asking where phasor measurements should be added, one can ask what areas are implied by the current or planned phasor measurement locations. All the possible areas can be obtained by considering the network formed by removing all the buses with phasor measurements. Then the possible areas are all the islands (components) of this network, or combinations of these islands.

The new area angles are very promising quantities for monitoring area stress and extracting more value from phasor measurements. We look forward to finding and developing their practical applications.

Acknowledgments

We thank Bob Cummings of NERC for suggesting that the angle across an outaged line be investigated. We thank Chen-Ching Liu of University College Dublin and Nanpeng Yu and Juan Li of Iowa State University for graciously providing the power system model for the example.

The work described in this paper was coordinated by the Consortium for Electric Reliability Technology Solutions with funding provided in part by the California Energy Commission, Public Interest Energy Research Program, under Work for Others Contract No. 500-99-013, BO-99-2006-P. The Lawrence Berkeley National Laboratory is operated under U.S. Department of Energy Contract No. DE-AC02-05CH11231.

References

[1] I. Dobson, M. Parashar, C. Carter, Combining phasor measurements to monitor cutset angles, 43rd Hawaii International Conference on System Sciences, Kauai HI, Jan. 2010.

[2] I. Dobson, M. Parashar, A cutset area concept for phasor monitoring, IEEE PES General Meeting, Minneapolis, MN July 2010.

[3] B. Bhargava, A. Salazar, Synchronized phasor measurement system (SPMS) for monitoring transmission system at SCE, presentation at NASPI Meeting, Carson, CA, May 2007.

[4] R. W. Cummings, Predicting cascading failures, presentation at NSF/EPRI Workshop on Understanding and Preventing Cascading Failures in Power Systems, Westminster CO, October 2005.

[5] Z. Huang, J. Dagle, Synchrophasor measurements: System architecture and performance evaluation in supporting wide-area applications, IEEE Power and Energy Society General Meeting, Pittsburgh PA, July 2008.

[6] M. Parashar, W. Zhou, D. Trudnowski, Y. Makarov, I. Dobson, Phasor technology applications feasibility assessment and research results, DOE/CERTS report, http://certs.lbl.gov/pdf/phasor-feasibility-2008.pdf, June 2008.

[7] A.G. Phadke, J.S. Thorp, Synchronized Phasor Measurements and Their Applications, Springer NY, 2008.

[8] V. Venkatasubramanian, Y. X. Yue, G. Liu, M. Sherwood, Q. Zhang, Wide-area monitoring and control algorithms for large power systems using synchrophasors, IEEE Power Systems Conference and Exposition, Seattle WA, March 2009.

[9] J. E. Tate, T. J. Overbye, Line outage detection using phasor angle measurements, IEEE Trans. Power Systems, vol. 23, no. 4, Nov. 2008, pp. 1644-1652.

[10] C. W. Taylor, D. C. Erickson, K. E. Martin, R. E. Wilson, V. Venkatasubramanian, WACS–Wide-area stability and voltage control system: R&D and online demonstration, Proceedings of the IEEE, vol. 93, no. 5, May 2005, pp. 892-906.

[11] A. Wood and B. Wollenberg, Power Generation, Operation, and Control. New York: Wiley, 1984.