Complex Systems Analysis of Series of Blackouts: Cascading Failure, Criticality, and Self-organization

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Abstract

We give a comprehensive account of a complex systems approach to large blackouts caused by cascading failure. Instead of looking at the details of particular blackouts, we study the statistics, dynamics and risk of series of blackouts with approximate global models. North American blackout data suggests that the frequency of large blackouts is governed by a power law. This result is consistent with the power system being a complex system designed and operated near criticality. The power law makes the risk of large blackouts consequential and implies the need for nonstandard risk analysis.

Power system overall load relative to operating limits is a key factor affecting the risk of cascading failure. Blackout models and an abstract model of cascading failure show that there are critical transitions as load is increased. Power law behavior can be observed at these transitions.

The critical loads at which blackout risk sharply increases are identifiable thresholds for cascading failure and we discuss approaches to computing the proximity to cascading failure using these thresholds. Approximating cascading failure as a branching process suggests ways to compute and monitor criticality by quantifying how much failures propagate.

Inspired by concepts from self-organized criticality, we suggest that power system operating margins evolve slowly to near criticality and confirm this idea using a blackout model. Mitigation of blackout risk should take care to account for counter-intuitive effects in complex self-organized critical systems. For example, suppressing small blackouts could lead the system to be operated closer to the edge and ultimately increase the risk of large blackouts.

1 Introduction

Cascading failure is the usual mechanism for large blackouts of electric power transmission systems. For example, long, intricate cascades of events caused the August 1996 blackout in Northwestern America (NERC [44]) that disconnected 30,390 MW of power to 7.5 million customers [41, 57]). An even more spectacular example is the August 2003 blackout in Northeastern America that disconnected 61,800 MW of power to an area spanning 8 states and 2 provinces and containing 50 million people [56]. The vital importance of the electrical infrastructure to society motivates the understanding and analysis of large blackouts.

Here are some substantial challenges:

- North American power transmission system data appears to show power tails in the probability distribution of blackout sizes, making the risk of large blackouts consequential. What is the origin and the implications of this distribution of blackout sizes? Can this probability distribution be changed within economic and engineering constraints to minimize the risk of blackouts of all sizes?
- Large blackouts are typically caused by long, intricate cascading sequences of rare events. Dependencies between the first few events can be assessed for a subset of the most likely or anticipated events and this type of analysis is certainly useful in addressing part of the problem (e.g. [48]). However, this combinatorial analysis gets overwhelmed and becomes infeasible for long sequences of events or for the huge number of all possible rare events and interactions, many of which are unanticipated, that cascade to cause large blackouts. How does one do risk analysis of rare, cascading, catastrophic events? Can one monitor or mitigate the risk of these cascading failures at a more global level without working out all the details?
- Much of the effort in avoiding cascading failure has focussed on reducing the chances of the start of a cascading failure. How do we determine whether power system design and operation is such that cascades will tend to propagate after they have started? That is, where is the "edge" for propagation of cascading failure?

The aim of global complex systems analysis of power system blackouts is to provide new insights and approaches that could address these challenges. We focus on global bulk properties of series of blackouts rather than on the details of a particular blackout. Concepts from complex systems, statistical physics, probability and risk analysis are combined with power system modeling to study blackouts from a top-down perspective.

1.1 Literature review

We briefly review some other approaches to complex systems and cascading failure in power system blackouts.

Chen and Thorp [17] and Chen, Thorp, and Dobson [18] model power system blackouts using the DC load flow approximation and standard linear programming optimization of the generation dispatch and represent in detail hidden failures of the protection system. The expected blackout size is obtained using importance sampling and it shows some indications of a critical point as loading is increased. The distribution of power system blackout size is obtained by rare event sampling and blackout risk assessment and mitigation methods are studied. Rios, Kirschen, Jawayeera, Nedic, and Allan [51] evaluate expected blackout cost using Monte Carlo simulation of a power system model that represents the effects of cascading line overloads, hidden failures of the protection system, power system dynamic instabilities, and the operator responses to these phenomena. Kirschen, Jawayeera, Nedic, and Allan [40] then apply correlated sampling and their Monte Carlo simulation to develop a calibrated reference scale of system stress that relates system loading to blackout size and test it on a 1000 bus power system. Hardiman, Kumbale, and Makarov [35] simulate and analyze cascading failure using the TRELSS software. In its "simulation approach" mode, TRELSS represents cascading outages of lines, transformers and generators due to overloads and voltage violations in large AC networks (up to 13000 buses). Protection control groups and islanding are modeled in detail. The cascading outages are ranked in severity and the results have been applied in industry to evaluate transmission expansion plans. Other modes of operation are available in TRELSS that can rank the worst contingencies and take into account remedial actions and compute reliability indices.

Ni, McCalley, Vittal, and Tayyib [48] evaluate expected contingency severities based on real time predictions of the power system state to quantify the risk of operational conditions. The computations account for current and voltage limits, cascading line overloads, and voltage instability. Zima and Andersson [59] study the transition into subsequent failures after an initial failure and suggest mitigating this transition with a wide-area measurement system.

Roy, Asavathiratham, Lesieutre, and Verghese [52] construct randomly generated tree networks that abstractly represent influences between idealized components. Components can be failed or operational according to a Markov model that represent both internal component failure and repair processes and influences between components that cause failure propagation. The effects of the network degree and the inter-component influences on the failure size and duration are studied. Pepyne, Panayiotou, Cassandras, and Ho [50] also use a Markov model for discrete state power system nodal components, but propagate failures along the transmission lines of a power systems network with a fixed probability. They study the effect of the propagation probability and maintenance policies that reduce the probability of hidden failures.

The challenging problem of determining cascading failure due to dynamic transients in hybrid nonlinear differential equation models is addressed by DeMarco [24] using Lyapunov methods applied to a smoothed model and by Parrilo, Lall, Paganini, Verghese, Lesieutre, and Marsden [49] using Karhunen-Loeve and Galerkin model reduction. Watts [58] describes a general model of cascading failure in which failures propagate through the edges of a random network. Network nodes have a random threshold and fail when this threshold is exceeded by a sufficient fraction of failed nodes one edge away. Phase transitions causing large cascades can occur when the network becomes critically connected by having sufficient average degree or when a highly connected network has sufficiently low average degree so that the effect of a single failure is not swamped by a high connectivity to unfailed nodes. Lindley and Singpurwalla [42] describe some foundations for causal and cascading failure in infrastructures and model cascading failure as an increase in a component failure rate within a time interval after another component fails.

Chen and McCalley [19] fit the empirical probability distribution of 20 years of North American multiple line failures with a cluster distribution derived from a negative binomial probability model. Carlson and Doyle have introduced a theory of highly optimized tolerance (HOT) that describes power law behavior in a number of engineered or otherwise optimized applications [6]. Stubna and Fowler [55] published an alternative view based on HOT of the origin of the power law in the NERC data. To apply HOT to the power system, it is assumed that blackouts propagate one dimensionally [55] and that this propagation is limited by finite resources that are engineered to be optimally distributed to act as barriers to the propagation [6]. The one dimensional assumption implies that the blackout size in a local region is inversely proportional to the local resources. Minimizing a blackout cost proportional to blackout size subject to a fixed sum of resources leads to a probability distribution of blackout sizes with an asymptotic power tail and two free parameters. The asymptotic power tail exponent is exactly –1 and this value follows from the one dimensional assumption. The free parameters can be varied to fit the NERC data for both MW lost and customers disconnected. However [55] shows that a better fit to both these data sets can be achieved by modifying HOT to allow some misallocation of resources.

The historically high reliability of power transmission systems in developed countries is largely due to estimating the transmission system capability and designing and operating the system with margins with respect to a chosen subset of likely and serious contingencies. The analysis is usually either deterministic analysis of estimated worst cases or Monte Carlo simulation of moderately detailed probabilistic models that capture steady state interactions [4]. Combinations of likely contingencies and some dependencies between events such as common mode or common cause are sometimes considered. The analyses address the first few likely and anticipated failures rather than the propagation of many rare or unanticipated failures in a cascade.

1.2 Blackout mechanisms

We review cascading failure mechanisms of large blackouts to provide context for the cascading failure modeling. Bulk electrical power transmission systems are complex networks of large numbers of components that interact in diverse ways. When component operating limits are exceeded protection acts and the component "fails" in the sense of not being available to transmit power. Components can also fail in the sense of misoperation or damage due to aging, fire, weather, poor maintenance or incorrect design or operating settings. In any case, the failure causes a transient and causes the power flow in the component to be redistributed to other components according to circuit laws, and subsequently redistributed according to automatic and manual control actions. The transients and readjustments of the system can be local in effect or can involve components far away, so that a component disconnection or failure can effectively increase the loading of many other components throughout the network. In particular, the propagation of failures is not limited to adjacent network components. The interactions involved are diverse and include deviations in power flows, frequency, and voltage as well as operation or misoperation of protection devices, controls, operator procedures and monitoring and alarm systems. However, all the interactions between component failures tend to be stronger when components are highly loaded. For example, if a more highly loaded transmission line fails, it produces a larger transient, there is a larger amount of power to redistribute to other components, and failures in nearby protection devices are more likely. Moreover, if the overall system is more highly loaded, components have smaller margins so they can tolerate smaller increases in load before failure, the system nonlinearities and dynamical couplings increase, and the system operators have fewer options and more stress.

A typical large blackout has an initial disturbance or trigger events followed by a sequence of cascading events. Each event further weakens and stresses the system and makes subsequent events more likely. Examples of an initial disturbance are short circuits of transmission lines through untrimmed trees, protection device misoperation, and bad weather. The blackout events and interactions are often rare, unusual, or unanticipated because the likely and anticipated failures are already routinely accounted for in power system design and operation. The complexity is such that it can take months after a large blackout to sift through the records, establish the events occurring and reproduce with computer simulations and hindsight a causal sequence of events.

2 Blackout data and risk

2.1 Power tails in North American blackout data

We consider the statistics of series of blackouts. The North American Electrical Reliability Council (NERC) has a documented list summarizing major blackouts of the North American power transmission system from 1984 to 1998 [45]. It is apparent that large blackouts are rarer than small blackouts, but how much rarer are they? One might expect a probability distribution of blackout sizes to fall off at most exponentially as the blackout size increases. However, analyses of the NERC data show that the probability distribution of the blackout sizes does not decrease exponentially with the size of the blackout, but rather has a power law tail [15, 7, 8, 16].

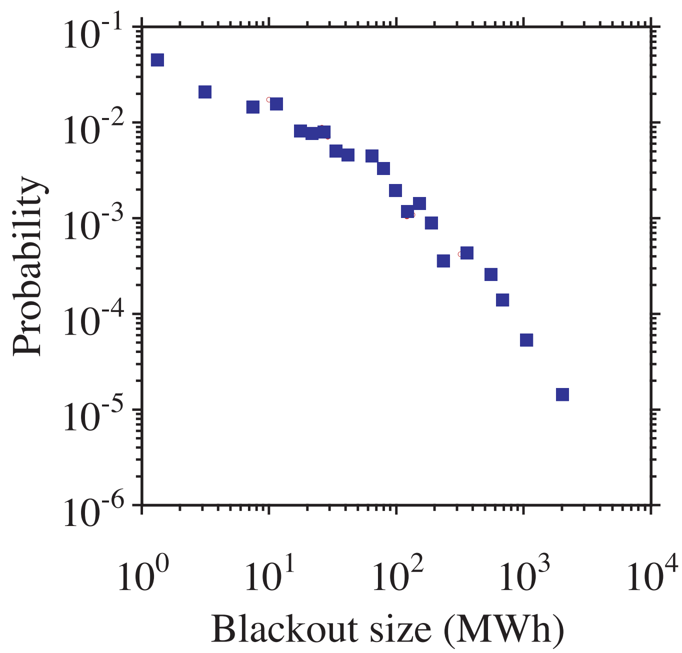

Fig. 1: Log-log plot of scaled PDF of energy unserved during North American blackouts 1984 to 1998.

For example, Fig. 1 plots on a log-log scale the empirical probability distribution of energy unserved in the North American blackouts. The fall-off with blackout size is close to a power dependence with an exponent between -1 and -2. (A power dependence with exponent -1 implies that doubling the blackout size only halves the probability and appears on a log-log plot as a straight line of slope -1). Thus the NERC data suggests that large blackouts are much more likely than might be expected. The power tails are of course limited in extent in a practical power system by a finite cutoff near system size corresponding to the largest possible blackout.

2.2 Blackout risk with respect to blackout size

Blackout risk is the product of blackout probability and blackout cost. Here we assume that blackout cost is roughly proportional to blackout size, although larger blackouts may well have costs (especially indirect costs) that increase faster than linearly. In the case of the exponential tail, large blackouts become rarer much faster than blackout costs increase so that the risk of large blackouts is negligible. However, in the case of a power law tail, the larger blackouts can become rarer at a similar rate as costs increase, and then the risk of large blackouts is comparable to, or even exceeding, the risk of small blackouts [12]. Thus power laws in blackout size distributions significantly affect the risk of large blackouts. Standard probabilistic techniques that assume independence between events imply exponential tails and are not applicable to systems that exhibit power tails.

Consideration of the probability distribution of blackout sizes leads naturally to a more sophisticated framing of the problem of avoiding blackouts. Instead of seeking only to limit blackouts in general, one can seek to manipulate the probability distribution of blackouts to jointly limit the frequency of small, medium and large blackouts. This elaboration is important because measures taken to limit the frequency of small blackouts may inadvertently increase the frequency of large blackouts when the complex dynamics governing transmission expansion are considered as discussed in section 8.

The strength of our conclusions is naturally somewhat limited by the short time period (15 years) of the available blackout data and the consequent limited resolution of the statistics. To further understand the mechanisms governing the complex dynamics of power system blackouts, modeling of the power system is indicated. We consider both abstract models of cascading failure and a power system blackout model in the following section.

3 Three models of cascading failure

This section summarizes three models of cascading failure that are used to explore aspects of blackouts. The first two models aim to represent some of the salient features of cascading failure in blackouts with an analytically tractable probabilistic model and the third model represents a power transmission system.

1. The CASCADE model is an abstract probabilistic model of cascading failure that captures the weakening of the system as the cascade proceeds [27, 32].
2. The branching process model is a useful approximation to the CASCADE model [28].
3. The OPA model for a fixed network is a power systems model that represents cascading line overloads and outages at the level of DC load flow and LP dispatch of generation [11].

While our main motivation is large blackouts, the abstract CASCADE and branching process models are sufficiently simple and general that they could be applied to cascading failure of other large, interconnected infrastructures [47].

3.1 CASCADE model

The features that the CASCADE model abstracts from the formidable complexities of large blackouts are the large but finite number of components, components that fail when their load exceeds a threshold, an initial disturbance loading the system, and the additional loading of components by the failure of other components. The initial overall system stress is represented by upper and lower bounds on a range of initial component loadings. The model neglects the length of times between events and the diversity of power system components and interactions. Of course, an analytically tractable model is necessarily much too simple to represent with realism all the aspects of cascading failure in blackouts; the objective is rather to help understand some global systems effects that arise in blackouts and in more detailed models of blackouts.

3.1.1 Description of CASCADE model

The CASCADE model [27, 32] has n identical components with random initial loads. For each component the minimum initial load is $L^{\min}$ and the maximum initial load is $L^{\max}$. For j=1,2,...,n, component j has initial load L_j that is a random variable uniformly distributed in $[L^{\min}, L^{\max}]$. $L_1, L_2, \cdots, L_n$ are independent.

Components fail when their load exceeds L^{fail}. When a component fails, a fixed amount of load P is transferred to each of the components.

To start the cascade, we assume an initial disturbance that loads each component by an additional amount D. Other components may then fail depending on their initial loads L_j and the failure of any of these components will distribute an additional load $P \geq 0$ that can cause further failures in a cascade.

Now we define the normalized CASCADE model. The normalized initial load ℓ_j is

$$\ell_j = \frac{L_j - L^{\min}}{L^{\max} - L^{\min}} \tag{1}$$

Then ℓ_j is a random variable uniformly distributed on $[0, 1]$.

Let

$$p = \frac{P}{L^{\max} - L^{\min}}, \qquad d = \frac{D + L^{\max} - L^{\text{fail}}}{L^{\max} - L^{\min}} \qquad (2)$$

Then the normalized load increment p is the amount of load increase on any component when one other component fails expressed as a fraction of the load range $L^{\max} - L^{\min}$. The normalized initial disturbance d is a shifted initial disturbance expressed as a fraction of the load range. Moreover, the failure load is $\ell_j = 1$.

3.1.2 *Distribution of the number of failures*

The distribution of the total number of component failures S is

$$P[S = r] = \begin{cases} \binom{n}{r} \phi(d)(d + rp)^{r-1}(\phi(1 - d - rp))^{n-r}, \\ \qquad\qquad\qquad r = 0, 1, ..., n - 1, \\ 1 - \sum_{s=0}^{n-1} P(S = s), \qquad r = n, \end{cases} \qquad (3)$$

where $p \geq 0$ and the saturation function is

$$\phi(x) = \begin{cases} 0 \ ; x < 0, \\ x \ ; 0 \leq x \leq 1, \\ 1 \ ; x > 1. \end{cases} \qquad (4)$$

When using (3) it is assumed that $0^0 \equiv 1$ and $0/0 \equiv 1$.

If $d \geq 0$ and $d + np \leq 1$, then there is no saturation ($\phi(x) = x$) and (3) reduces to the quasibinomial distribution

$$P[S = r] = \binom{n}{r} d(d + rp)^{r-1}(1 - d - rp)^{n-r}. \qquad (5)$$

The quasibinomial distribution was introduced by Consul [21] to model an urn problem in which a player makes strategic decisions and further studied by Burtin [5], Islam, O'Shaughnessy, and Smith [37], and Jaworski [38]. The saturation in (3) extends the parameter range of the quasibinomial distribution and the saturated distribution can represent highly stressed systems with a high probability of all components failing.

3.2 *Branching process*

The branching process approximation to the CASCADE model gives a way to quantify the propagation of cascading failures with a parameter λ and further simplifies the mathematical modeling [28].

In a Galton-Watson branching process [36, 1] the failures are regarded as produced in stages. The failures in each stage independently produce further failures in the next stage according to a probability distribution with mean λ. An exception is that the first stage assumes a probability distribution with mean θ to represent the initial disturbance. We assume in this section that each failure produces 0,1,2,3,... further failures according to a Poisson distribution. Thus, after the initial disturbance, each failure in each stage independently produces further failures in the next stage according to a Poisson distribution of mean λ.

The branching process is a transient discrete time Markov process and its behavior is governed by the parameter λ. The mean number of failures in stage k is $\theta\lambda^{k-1}$. In the subcritical case of $\lambda < 1$, the failures will die out (i.e., reach and remain at zero failures at some stage) and the mean number of failures in each stage decreases geometrically. In the supercritical case of $\lambda > 1$, although it possible for the process to die out, often the failures increase without bound. Of course, there are a large but finite number of components that can fail in a blackout and in the CASCADE model, so it is also necessary to account for the branching process saturating with all components failed.

The stages of the CASCADE model can be approximated by the stages of a saturating branching process by letting the number of components n become large while p and d become small in such a way that $\lambda = np$ and $\theta = nd$ remain constant. The number S of components failed in the saturating branching process is a saturating form of the generalized Poisson distribution:

For $\theta \geq 0$,

$$P[S = r] = \theta(r\lambda + \theta)^{r-1} \frac{e^{-r\lambda - \theta}}{r!} \quad ; \quad 0 \leq r \leq (n - \theta)/\lambda, \ r < n \qquad (6)$$

$$P[S = r] = 0 \ ; \quad (n - \theta)/\lambda < r < n, \ r \geq 0 \qquad (7)$$

$$P[S = r] = 1 - \sum_{s=0}^{n-1} g(s, \theta, \lambda, n) \qquad (8)$$

In the subcritical or critical case $\lambda \leq 1$, there is no saturation and (6)-(8) reduce to

$$P[S = r] = \theta(r\lambda + \theta)^{r-1} \frac{e^{-r\lambda - \theta}}{r!} \qquad (9)$$

which is the generalized (or Lagrangian) Poisson distribution introduced by Consul and Jain [23, 20, 22].

Further approximation of (6)-(8) yields [30]

$$P[S = r] \approx \frac{\theta \, e^{(1-\lambda)\frac{\theta}{\lambda}}}{\lambda\sqrt{2\pi}} r^{-1.5} e^{-r/r_0} \qquad (10)$$
$$; 1 \ll r < r_1 = \min\{n/\lambda, n\}, \quad \theta/\lambda \sim 1$$
$$\text{where} \qquad r_0 = (\lambda - 1 - \ln \lambda)^{-1}$$

In the approximation (10), the term $r^{-1.5}$ dominates for $r \lesssim r_0$ and the exponential term e^{-r/r_0} dominates for $r \gtrsim r_0$. Thus (10) reveals that the distribution of the number of failures has an approximate power law region of exponent -1.5 for $1 \ll r \lesssim r_0$ and an exponential tail for $r_0 \lesssim r < r_1$. Note that near criticality, $\lambda \approx 1$ and r_0 becomes large.

For a very general class of branching processes (not necessarily assuming that each failure produces further failures with a Poisson distribution), at criticality, the probability distribution of the total number of failures has a power law form with exponent -1.5. That is, as one doubles the number of failures the probability of that number of failures is divided by $2^{1.5}$. The universality of the -1.5 power law at criticality in the probability distribution of the total number of failures in a branching process suggests that this is a signature for this type of cascading failure. In particular, the generalized Poisson distributions (6)-(8) and (9) have a -1.5 power law at $\lambda = 1$.

The approximation of CASCADE by a branching process implies that the CASCADE model has approximately a -1.5 power law at $np = 1$. Moreover, the -1.5 power law is approximately consistent with the North American blackout data described in section 2.1.

Criticality or supercriticality in the branching process implies a high risk of catastrophic and widespread cascading failures. Maintaining sufficient subcriticality in the branching process according to a simple criterion such as requiring $\lambda < \lambda_{max} < 1$ would limit the propagation of failures and reduce this risk. The approximation of CASCADE as a branching process allows the criterion to be expressed in terms of system loading. However, implementing the criterion to reduce the risk of catastrophic cascading failure would require limiting the system throughput and this is costly. Managing the tradeoff between the certain cost of limiting throughput and the rare but very costly widespread catastrophic cascading failure may be difficult. Indeed we maintain in section 6 that for large blackouts, economic, engineering and societal forces may self-organize the system to criticality and that efforts to mitigate the risk should take account of these broader dynamics [12].

Our emphasis on limiting the propagation of system failures after they are initiated is complementary to more standard methods of mitigating the risk of cascading failure by reducing the risk of the first few likely failures caused by an initial disturbance as for example in [48].

The branching process approximation does capture some salient features of loading dependent cascading failure and suggests an approach to reducing the risk of large cascading failures by limiting the average propagation of failures. However, much work remains to establish the correspondence between these simplified global models and the complexities of cascading failure in real systems.

3.3 OPA blackout model for a fixed network

This section summarizes the OPA blackout model when the network is assumed to be fixed [11]. This model represents blackouts caused by probabilistic cascading line overloads and outages and is used to produce blackout statistics. Lines fail probabilistically and the consequent redistribution of power flows is calculated using the DC load flow approximation and a standard LP dispatch of generation. Cascading line outages leading to blackout are modeled. There is also a version of OPA that additionally represents the complex dynamics as the network evolves and this is discussed in section 6.2.

Cascading failure can happen at any time but tends to be more likely and more widespread at peak load when the network is most stressed. For simplicity, the daily peak load is chosen as representative of the loading during each day and the cascade is computed based on that peak load. Each day has the possibility of one cascade. The lines involved in the cascade are represented but the timing of events is neglected.

The OPA model represents transmission lines, loads and generators with the usual DC load flow assumptions. Starting from a solved base case, blackouts are initiated by random line outages. Whenever a line is outaged, the generation and load is redispatched using standard linear programming methods. The cost function is weighted to ensure that load shedding is avoided where possible. If any lines were overloaded during the optimization, then these lines are outaged with probability p_1. The process of redispatch and testing for outages is iterated until there are no more outages.

The OPA model does not attempt to capture the intricate details of particular blackouts, which may have a large variety of complicated interacting processes also involving, for example, protection systems, dynamics and human factors. However, the OPA model does represent in a simplified way a dynamical process of cascading overloads and outages that is consistent with some basic network and operational constraints.

4 Critical loading

As load increases, it is clear that cascading failure becomes more likely, but exactly how does it become more likely? Our results show that the cascading failure does not gradually and uniformly become more likely; instead there is a point of criticality or phase transition at which the cascading failure becomes more likely.

In complex systems and statistical physics, criticality is associated with power tails in probability distributions. Other indicators of criticality are changes in gradient (for a type 2 phase transition) or a discontinuity (for a type 1 phase transition) in some measured quantity as system passes through the critical point.

The importance of the critical loading is that it defines a reference point for increasing risk of cascading failure. The terminology of "criticality" comes from statistical physics and it is of course extremely useful to use the standard scientific terminology. However, while the power tails at critical loading indicate a substantial risk of large blackouts, it is premature at this stage of risk analysis to automatically presume that operation at criticality is bad because it entails some substantial risks. There is also economic gain from an increased loading of the power transmission system. Indeed, one of the objectives in pursuing the risk analysis of cascading blackouts is to determine and quantify the tradeoffs involved so that sensible decisions about optimal design and operation and blackout mitigation can be made.

4.1 *Qualitative effect of load increase on distribution of blackout size*

Consider cascading failure in a power transmission system in the impractically extreme cases of very low and very high loading. At very low loading, any failures that occur have minimal impact on other components and these other components have large operating margins. Multiple failures are possible, but they are approximately independent so that the probability of multiple failures is approximately the product of the probabilities of each of the failures. Since the blackout size is roughly proportional to the number of failures, the probability distribution of blackout size will have a tail bounded by an exponential. The probability distribution of blackout size is different if the power system were to be operated recklessly at a very high loading in which every component was close to its loading limit. Then any initial disturbance would necessarily cause a cascade of failures leading to total or near total blackout. It is clear that the probability distribution of blackout size must somehow change continuously from the exponential tail form to the certain total blackout form as loading increases from a very low to a very high loading. We are interested in the nature of the transition between these two extremes. Our results presented below suggest that the transition occurs via a critical loading at which there is a power tail in the probability distribution of blackout size. This concept is shown in Figure 2.

4.2 *Critical transitions as load increases in CASCADE*

This subsection describes one way to represent a load increase in the CASCADE model and how this leads to a parameterization of the normalized model. Then the effect of the load increase on the distribution of the number of components failed is described.

We assume for convenience that the system has $n = 1000$ components. Suppose that the system is operated so that the initial component loadings vary from $L^{\min}$ to $L^{\max} = L^{\text{fail}} = 1$. Then the average initial component loading $L = (L^{\min} + 1)/2$ may be increased by increasing $L^{\min}$. The initial disturbance $D = 0.0004$ is assumed to be the same

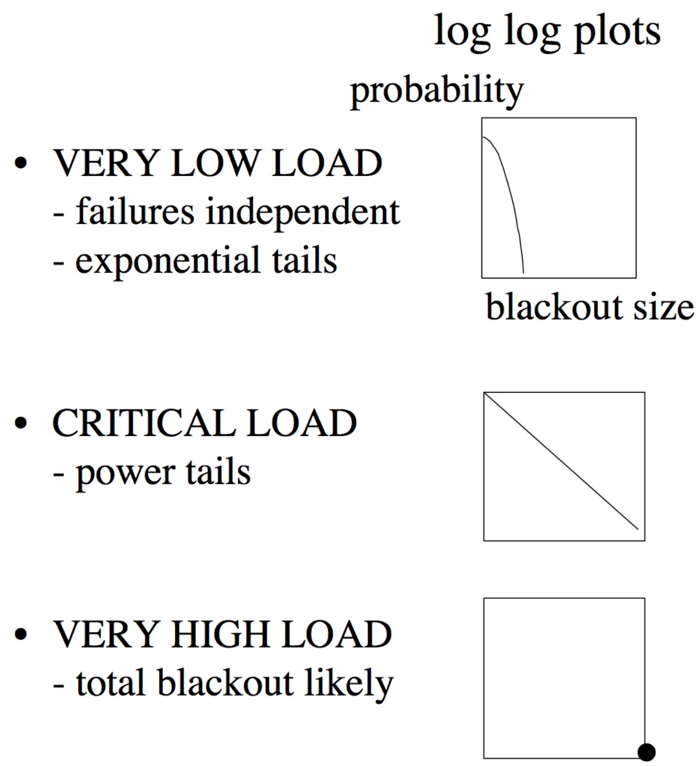

Fig. 2: Log-log plots sketching idealized blackout size probability distributions for very low, critical, and very high power system loadings.

as the load transfer amount $P = 0.0004$. These modeling choices for component load lead via the normalization (2) to the parameterization

$$p = d = \frac{0.0004}{2 - 2L}, \qquad 0.5 \leq L < 1. \tag{11}$$

The increase in the normalized power transfer p with increased L may be thought of as strengthening the component interactions that cause cascading failure.

The distribution for the subcritical and nonsaturating case $L = 0.6$ has an approximately exponential tail as shown in Figure 3. The tail becomes heavier as L increases and the distribution for the critical case $L = 0.8$, $np = 1$ has an approximate power law region over a range of S. The power law region has an exponent of approximately -1.4 and this compares to the exponent of -1.5 obtained by the analytic approximation discussed in subsection 3.2. The distribution for the supercritical and saturated case $L = 0.9$ has an approximately exponential tail for small r, zero probability of intermediate r, and a probability of 0.80 of all 1000 components failing. If an intermediate number of components fail in a saturated case, then the cascade always proceeds to all 1000 components failing.

The increase in the mean number of failures as the average initial component loading L is increased is shown in Figure 4. The sharp change in gradient at the critical loading $L = 0.8$ corresponds to the saturation of (3) and the consequent increasing probability of all components failing. Indeed, at $L = 0.8$, the change in gradient in Figure 4 together with the power law region in the distribution of S in Figure 3 suggest a type two phase transition in the system. If we interpret the number of components failed as corresponding to blackout size, the power law region is consistent with the North American blackout data discussed in section 2. In particular, North American blackout data suggest an empirical distribution of blackout size with a power tail with exponent between –1 and –2. This power tail indicates a significant risk of large blackouts that is not present when the distribution of blackout sizes has an exponential tail.

The model results show how system loading can influence the risk of cascading failure. At low loading there is an approximately exponential tail in the distribution of number of components failed and a low risk of large cascading failure. There is a critical loading at which there is a power law region in the distribution of number of components failed and a sharp increase in the gradient of the mean number of components failed. As loading is increased past the critical loading, the distribution of number of components failed saturates, there is an increasingly significant probability of all components failing, and there is a significant risk of large cascading failure.

4.3 *Critical transitions as load increases in OPA*

Criticality can be observed in the fast dynamics OPA model as load power demand is slowly increased as shown in Fig. 5. (Random fluctuations in the pattern of load are superimposed on the load increase in order to provide statistical data.) At a critical loading, the gradient of the expected blackout size sharply increases. Moreover, the PDF of blackout size shows power tails at the critical loading as shown in Fig. 6. OPA can also display complicated critical point behavior corresponding to both generation and transmission line limits [11].

As noted in section 1.1, the cascading hidden failure model of Chen and Thorp also shows some indications of criticality as load is increased [17, 18].

5 Quantifying proximity to criticality

At criticality there is a power tail, a sharp increase in mean blackout size, and an increased risk of cascading failure. Thus criticality gives a reference point or a power system operational limit with respect to cascading failure. That is, we are suggesting adding an increased risk of cascading failure limit to the established power system operating limits such as thermal, voltage, and transient stability. How does one practically monitor or measure margin to criticality?

One approach is to increase loading in a blackout simulation incorporating cascading failure mechanisms until criticality is detected by a sharp increase in mean blackout size. The mean blackout size is calculated at each loading level by running the simulation repeatedly with some random variation in the system initial conditions so that a variety of cascading outages are simulated. This approach is straightforward and likely to be useful, but it is not fast and it seems that it would be difficult or impossible to apply to real system data. Also it could be challenging to describe and model a good sample of the diverse interactions involved in cascading failure in a fast enough simulation. This approach, together with checks on the power law behavior of the distribution of blackout size, was used to find criticality in several power system and abstract models of cascading failure [11, 17, 18, 32, 28]. Confirming criticality in this way in a range of power system models incorporating more detailed or different cascading failure mechanisms

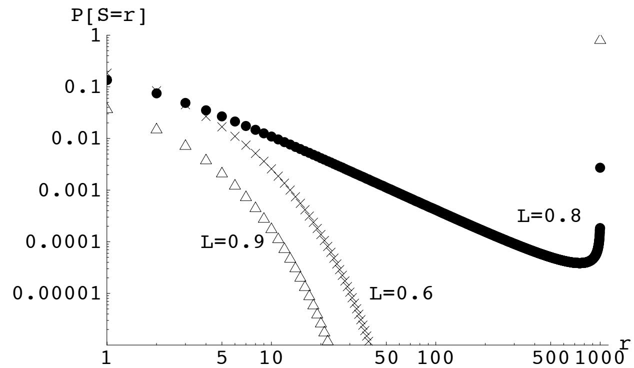

Fig. 3: Log-log plot of distribution of number of components failed S for three values of average initial load L. Note the power law region for the critical loading $L = 0.8$. $L = 0.9$ has an isolated point at $(1000, 0.80)$ indicating probability 0.80 of all 1000 components failed. Probability of no failures is 0.61 for $L = 0.6$, 0.37 for $L = 0.8$, and 0.14 for $L = 0.9$.

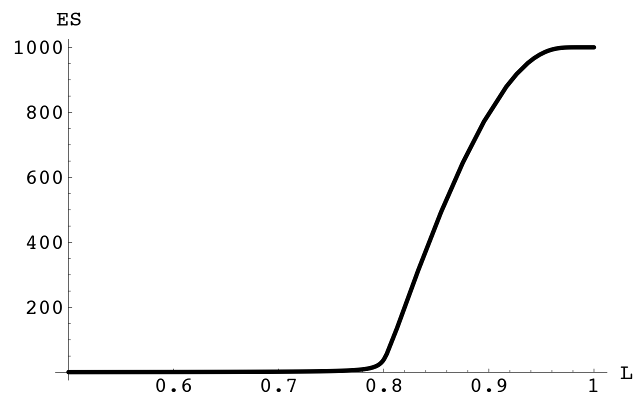

Fig. 4: Mean number of components failed ES as a function of average initial component loading L. Note the change in gradient at the critical loading $L = 0.8$. There are $n = 1000$ components and ES becomes 1000 at the highest loadings.

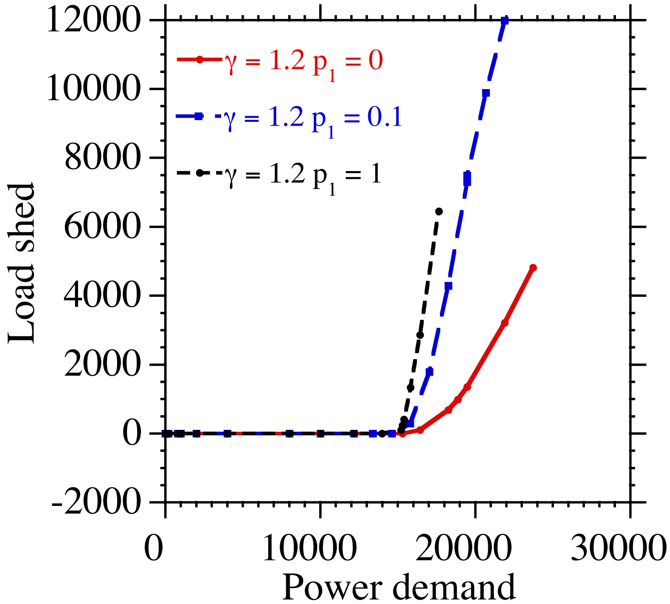

Fig. 5: Average blackout size in OPA as loading increases. Critical loading occurs at kink in curves where average blackout size sharply increases.

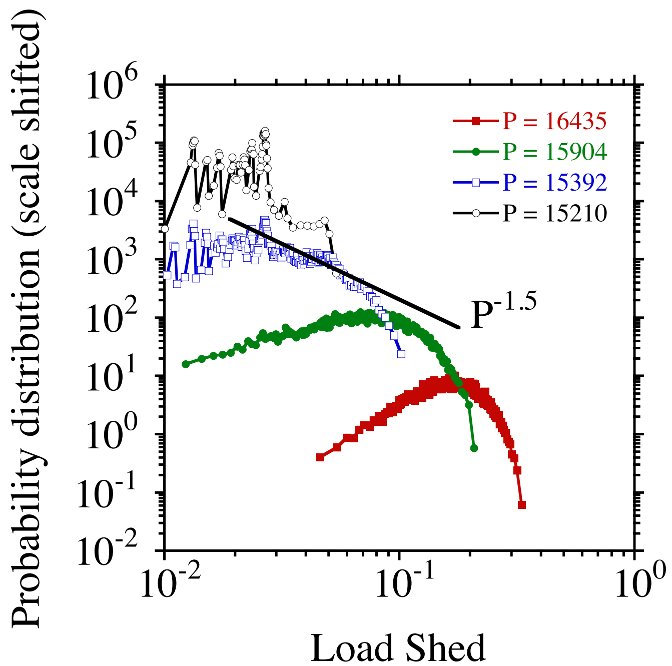

Fig. 6: Blackout size PDF at critical loading P=15392 and other loadings.

would help to establish further the key role that criticality plays in cascading failure.

Another approach that is currently being developed [13, 30, 31] is to monitor or measure from real or simulated data how much failures propagate after they are initiated. Branching process models such as the Galton-Watson process described in section 3.2 have a parameter λ that measures both the average failure propagation and proximity to criticality. In branching process models, the average number of failures is multiplied by λ at each stage of the branching process. Although there is statistical variation about the mean behavior, it is known [1] that for subcritical systems with $\lambda < 1$, the failures will die out and that for supercritical systems with $\lambda > 1$, the number of failures can exponentially increase. (The exponential increase will in practice be limited by the system size and any blackout inhibition mechanisms; current research seeks to understand and model the blackout inhibition mechanisms.)

The idea is to statistically estimate λ from simulated or real failure data. Essentially this approach seeks to approximate and fit the data with a branching process model. The ability to estimate λ and any other parameters of the branching process model would allow the computation of the corresponding distribution of blackout size probability and hence estimates of the blackout risk.

Note that the cascading failure limit measures overall system stress in terms of how failures propagate once started; it is complementary to measures to limit cascading failure by inhibiting the start of cascade such as the n-1 criterion.

6 Self-organization and slow dynamics of network evolution

6.1 Qualitative description of self-organization

We qualitatively describe how the forces shaping the evolution of the power network could give rise to self-organizing dynamics. The power system contains many components such as generators, transmission lines, transformers and substations. Each component experiences a certain loading each day and when all the components are considered together they experience some pattern or vector of loadings. The pattern of component loadings is determined by the power system operating policy and is driven by the aggregated customer loads at substations. The power system operating policy includes short term actions such as generator dispatch as well as longer term actions such as improvements in procedures and planned outages for maintenance. The operating policy seeks to satisfy the customer loads at least cost. The aggregated customer load has daily and seasonal cycles and a slow secular increase of about 2% per year.

The probability of component failure generally increases with component loading. Each failure is a limiting or zeroing of load in a component and causes a redistribution of power flow in the network and hence a discrete increase in the loading of other system components. Thus failures can cascade. If a cascade of events includes limiting or zeroing the load at substations, it is a blackout. A stressed power system experiencing an event must either redistribute load satisfactorily or shed some load at substations in a blackout. A cascade of events leading to blackout usually occurs on a time scale of minutes to hours and is completed in less than one day.

It is customary for utility engineers to make prodigious efforts to avoid blackouts and especially to avoid repeated blackouts with similar causes. These engineering responses to a blackout occur on a range of time scales longer than one day. Responses include repair of damaged equipment, more frequent maintenance, changes in operating policy away from the specific conditions causing the blackout, installing new equipment to increase system capacity, and adjusting or adding system alarms or controls. The responses reduce the probability of events in components related to the blackout, either by lowering their probabilities directly or by reducing component loading by increasing component capacity or by transferring some of the loading to other components. The responses are directed towards the components involved in causing the blackout. Thus the probability of a similar blackout occurring is reduced, at least until load growth degrades the improvements made. There are similar, but less intense responses to unrealized threats to system security such as near misses and simulated blackouts.

The pattern or vector of component loadings may be thought of as a system state. Maximum component loadings are driven up by the slow increase in customer loads via the operating policy. High loadings increase the chances of cascading events and blackouts. The loadings of components involved in the blackout are reduced or relaxed by the engineering responses to security threats and blackouts. However, the loadings of some components not involved in the blackout may increase. These opposing forces driving the component loadings up and relaxing the component loadings are a reflection of the standard tradeoff between satisfying customer loads economically and security. The opposing forces apply over a range of time scales. We suggest that the opposing forces, together with the underlying growth in customer load and diversity give rise to a dynamic equilibrium.

These ideas of complex dynamics by which the network evolves are inspired by corresponding concepts of self-organized criticality (SOC) in statistical physics. As a brief introduction to the concept, a self-organized critical system is one in which the nonlinear dynamics in the presence of perturbations organize the overall average system state near to, but not at, the state that is marginal to major disruptions. Self-organized critical systems are characterized by a spectrum of spatial and temporal scales of the disruptions that exist in remarkably similar forms in a wide variety of physical systems [2, 3, 39]. In these systems, the probability of occurrence of large disruptive events decreases as a power function of the event size. This is in contrast to many conventional systems in which this probability decays exponentially with event size.

6.2 OPA blackout model for a slowly evolving network

The OPA blackout model [14, 25, 9, 10] represents the essentials of slow load growth, cascading line outages, and the increases in system capacity caused by the engineering responses to blackouts. Cascading line outages leading to blackout are regarded as fast dynamics and are modeled as described in section 3.3 and the lines involved in a blackout are predicted. The slow dynamics model the growth of the load demand and the engineering response to the blackout by upgrades to the grid transmission capability. The slow dynamics represents the complex dynamics outlined in section 6.1. The slow dynamics is carried out by the following small changes applied at each day: All loads are multiplied by a fixed parameter that represents the daily rate of increase in electricity demand. If a blackout occurs, then the lines involved in the blackout have their line flow limits increased slightly. The generation is increased at randomly selected generators subject to coordination with the limits of nearby lines when the generator capacity margin falls below a threshold. The OPA model is "top-down" and represents the processes in greatly simplified forms, although the interactions between these processes still yield complex (and complicated!) behaviors. The simple representation of the processes is desirable both to study only the main interactions governing the complex dynamics and for pragmatic reasons of model tractability and simulation run time.

6.3 Self-Organization

We propose one way to understand the origin of the dynamics and distribution of power system blackouts. Indeed, we suggest that the slow, opposing forces of load increase and network upgrade in response to blackouts shape the system operating margins so that cascading blackouts occur with a frequency governed approximately by a power law relationship between blackout probability and blackout size. That is, these forces drive the system to a dynamic equilibrium just below and near criticality.

The load increase is a force weakening the power system (reducing operating margin) and the system upgrades are a force strengthening the system (increasing operating margin). If the power system is weak, then there will be more blackouts and hence more upgrades of the lines involved in the blackout and this will eventually strengthen the power system. If the power system is strong, then there will be fewer blackouts and fewer line upgrades, and the load increase will weaken the system. Thus the opposing forces drive the system to a dynamic equilibrium that keeps the system near a certain pattern of operating margins relative to the load. This process is observed in OPA results. Note that engineering improvements and load growth are driven by strong, underlying economic and societal forces that are not easily modified.

Moreover, when the generator upgrade process is suitably coordinated with the line upgrades and load increase, OPA results show power tails in the PDF of blackout sizes. For example, OPA results for the IEEE 118 bus network and an artificial 382 bus tree-like network are shown in Figure 7. Both the power law region of the PDF and the consistency with the NERC blackout data are evident. We emphasize that this criticality was achieved by the internal dynamics modeled in the system and is in this sense self-organizing to criticality.

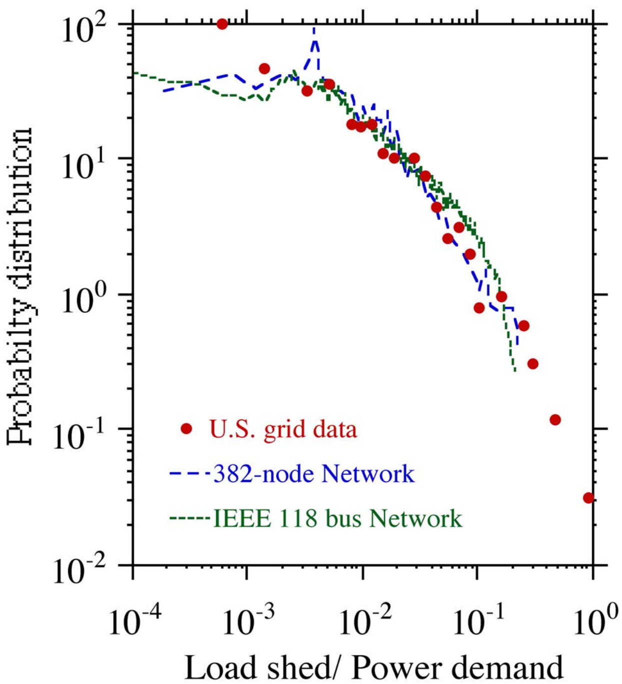

Fig. 7: Blackout size PDF resulting from self-organization showing OPA results on 2 networks. The NERC blackout data is also shown for comparison.

6.4 Blackout mitigation

While much remains to be learned about these complex dynamics, it is clear that these global dynamics have important implications for power system control and operation and for efforts to reduce the risk of blackouts.

The success of mitigation efforts in self-organized critical systems is strongly influenced by the dynamics of the system. Unless the mitigation efforts alter the self-organization forces driving the system, the system will be pushed to criticality. To alter those forces with mitigation efforts may be quite difficult because the forces are an intrinsic part of our society. Then the mitigation efforts can move the system to a new dynamic equilibrium while remaining near criticality and preserving the power tails. Thus, while the absolute frequency of disruptions of all sizes may be reduced, the underlying forces can still cause the relative frequency of large disruptions to small disruptions to remain the same.

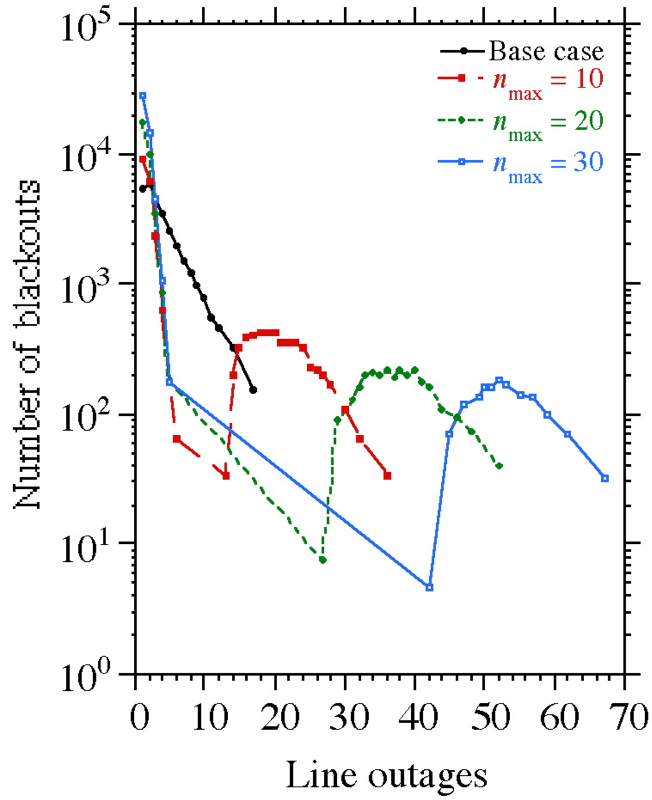

Fig. 8: Number of blackouts as number of line outages varies for differing inhibition of line outages ($n_{\max}$ is the maximum number of line overloads for which outages are inhibited). Results are obtained using OPA model on the IEEE 118 bus system.

Indeed apparently sensible efforts to reduce the risk of smaller blackouts can sometimes increase the risk of large blackouts. This occurs because the large and small blackouts are not independent but are strongly coupled by the dynamics. For example the longer term response to small blackouts can influence the frequency of large blackouts in such a way that measures to reduce the frequency of small blackouts can eventually reposition the system to have an increased risk of large blackouts. The possibility of an overall adverse effect on risk from apparently sensible mitigation efforts shows the importance of accounting for complex system dynamics when devising mitigation schemes [12]. For example [12], Figure 8 shows the results of inhibiting small numbers of line outages using the OPA model with self-organization on the IEEE 118 bus system. One of the causes of line outages in OPA is the outage of lines with a probability p_1 when the line is overloaded. The results show the effect of inhibiting these outages when the number of overloaded lines is less than $n_{\max}$. The inhibition corresponds to more effective system operation to resolve these overloads. Blackout size is measured by number of line overloads. The inhibition is, as expected, successful in reducing the smaller numbers of line outages, but eventually, after the system has repositioned to its dynamic equilibrium, the number of larger blackouts has increased. The results shown in Figure 8 are distributions of blackouts in the self-organized dynamic equilibrium and reflect the long-term effects of the inhibition of line outages. It is an interesting open question to what extent power transmission systems are near their dynamic equilibrium, but operation near dynamic equilibrium is the simplest assumption at the present stage of knowledge of these complex dynamics.

Similar effects are familiar and intuitive in other complex systems. For example, more effectively fighting small forest fires allows the forest system to readjust with increased brush levels and closer tree spacing so that when a forest fire does happen by some chance to progress to a larger fire, a huge forest fire is more likely [12].

7 Conclusions

We have summarized and explained an approach to series of cascading failure blackouts at a global systems level. This way of studying blackouts is complementary to existing detailed analyses of particular blackouts and offers some new insights into blackout risk, the nature of cascading failure, the occurrence of criticality, and the complex system dynamics of blackouts.

The power law region in the distribution of blackout sizes in North American blackout data [15, 16] has been reproduced by power system blackout models [11, 14, 18] and some abstract models of cascading failure [32, 28] and engineering design [55]. The power law profoundly affects the risk of large blackouts, making this risk comparable to, or even exceeding the risk of small blackouts. The power law also precludes many conventional statistical models with exponential-tailed distributions and new approaches need to be developed such as [32, 28, 31, 19].

We think that the power law in the distribution of blackout sizes arises from cascading failure when the power system is loaded near a critical loading. Several power system blackout models [11, 18] and abstract models of cascading failure [32, 28] show evidence of a critical loading at which the probability of cascading failure sharply increases. We suggest that determining the proximity to critical loading from power system simulations or data is an important problem. It seems that Monte-Carlo simulation methods will be able to usefully compute the proximity to critical loading [11, 18, 40]. Moreover, branching process models of cascading failure provide ways of quantifying with a parameter λ the extent to which failures propagate after they are started. We are pursuing practical methods of estimating λ from real or simulated failure data [28, 30, 31].

A novel and much larger view of the power system dynamics considers the opposing forces of growing load and the upgrade of the transmission network in response to real or simulated blackouts. Our simulation results show that these complex dynamics can self-organize the system to be near criticality [14]. These complex dynamics are driven by strong societal and economic forces and the difficulties or tradeoffs in achieving long-term displacement of the power system away from the complex systems equilibrium caused by these forces should not be underestimated. Indeed we have simulated a simple example of a blackout mitigation method that successfully limits the frequency of small blackouts, but in the long term increases the frequency of large blackouts as the transmission system readjusts to its complex systems equilibrium [12]. In the light of this example, we suggest that the blackout mitigation problem be reframed as jointly mitigating small and large blackouts.

There are good prospects for extracting engineering and scientific value from the further development of models, simulations and computations and we hope that this paper encourages further developments and practical applications in this emerging and exciting area of research. There is an opportunity for systems research to make a substantial contribution to understanding and managing the risk of cascading failure blackouts.

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