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Geometric description of bridge rectifier operational modes using regular polygons

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Indexing terms: Mathematical techniques, Power electronics, Convertors

Abstract: AC line currents in idealised 3-phase bridge rectifier systems trace out regular polygons. This fact is used to derive a geometric visualisation of rectifier operation and to explain operational modes in 6- and 12-pulse rectifiers from a geometric point of view.

1 Introduction

Insight into the operation of 6- and 12-pulse bridge rectifiers may be gained by considering the geometric figures described by currents and voltages in the AC lines supplying the rectifier. Many rectifier phenomena are due to interactions in the AC lines and may be simply represented in the plane of the geometric figures. In particular, 12-pulse rectifier modes, which are complicated to describe algebraically, are easier to understand from a geometric point of view. This geometric analysis is complementary to the usual algebraic analysis [1, 2] and provides a visualisation of rectifier operation. Geometric visualisation of rectifier operation contributes to the qualitative understanding of rectifiers required for checking and interpreting experimental or computer simulation results.

2 6-pulse rectifier modes

Consider an idealised and balanced AC line feeding a DC load via a 6-pulse diode bridge as shown in Fig. 1. First

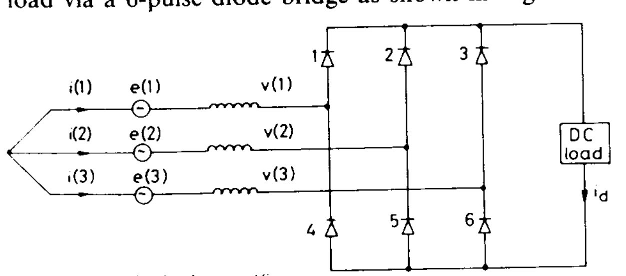

Fig. 1 *6-pulse bridge rectifier*

assume mode 1 operation so that commutation of current between the AC lines alternates with no commutation. The AC line currents may be regarded as the components of a 3 dimensional vector $\boldsymbol{i} = (i(1), i(2), i(3))$. The star connection of the AC lines constrains the current vector $\boldsymbol{i}$ to lie in the plane P determined by $i(1) + i(2) + i(3) = 0$. It is convenient to identify vectors in the P plane by their co-ordinates in 3-space; these coordinates may be evaluated by projecting the vector onto the line through one of the axes $\boldsymbol{a}_1, \boldsymbol{a}_2, \boldsymbol{a}_3$ shown in Fig. 2

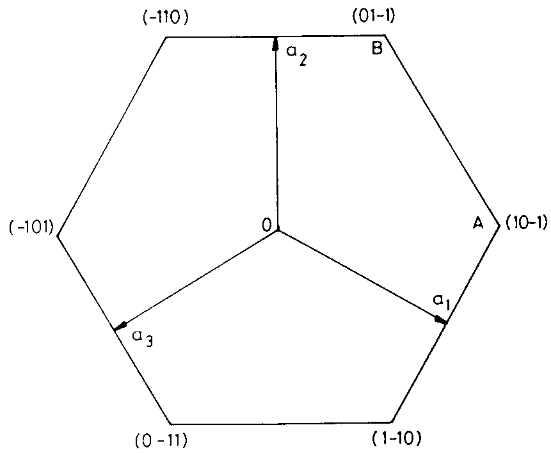

Fig. 2 *Representation of AC line currents in the P plane*

and determining the length of the projection as a multiple of the axis length. ($\boldsymbol{a}_1, \boldsymbol{a}_2, \boldsymbol{a}_3$ are covariant axes for the P plane; see Appendix 8). When the bridge is not commutating and there is a constant load current of unity, there are six possible AC line currents; these correspond to the six possible diode switching patterns that have exactly two diodes conducting. These AC line currents form the vertices of a regular hexagon in the P plane as shown in Fig. 2. For example, when the diodes are switched in the pattern $\begin{smallmatrix}1&0&0\\0&0&1\end{smallmatrix}$, the AC line currents are $(10-1)$ and the current vector is at vertex A of the hexagon. During the following commutation, when the diodes are switched in the pattern $\begin{smallmatrix}1&1&0\\0&0&1\end{smallmatrix}$, $i(3)$ remains constant and $i(1)$ and $i(2)$ are linearly related because their sum is the unity load current and hence the current vector traverses the hexagon edge AB. The commutation ends when $i(1)$ becomes zero; that is, when the current vector crosses the line through OB and the projection on $\boldsymbol{a}_1$ becomes zero. In this case the commutation ends when vertex B is reached. Thus in mode 1 operation with constant load current, the AC line currents describe a regular hexagon, pausing at the vertices when not commutating and traversing the edges during commutation.

The generator voltage vector $\boldsymbol{e} = (e(1), e(2), e(3))$ and bridge input voltage vector $\boldsymbol{v} = (v(1), v(2), v(3))$ also lie in the P plane. The vector $\boldsymbol{e}$ describes a circle in the P plane at the supply frequency and the sinusoidal components $e(1), e(2), e(3)$ may be obtained by the projection of $\boldsymbol{e}$ onto the axes a_1, a_2, a_3. (In general, the projection of circular motion onto an axis passing through the centre of the

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circle is sinusoidal even if the axis does not lie in the plane of the circle.) The length of $\boldsymbol{e}$ is proportional to the maximum generator voltage. This representation is similar to that used when representing 3-phase quantities using symmetrical components: vectors confined to the P plane are exactly those with no zero sequence component and the length of a vector rotating anticlockwise in the P plane is proportional to the positive sequence component of the quantity. The usual visualisation of positive sequence components has three rotating vectors projected onto a single axis; the P plane visualisation has a single rotating vector projected onto three fixed axes.

When the bridge is not commutating, the line currents are constant and $\boldsymbol{v}$ is equal to $\boldsymbol{e}$. The mode 1 switching conditions are shown in Fig. 3; a diode turns off when $\boldsymbol{i}$

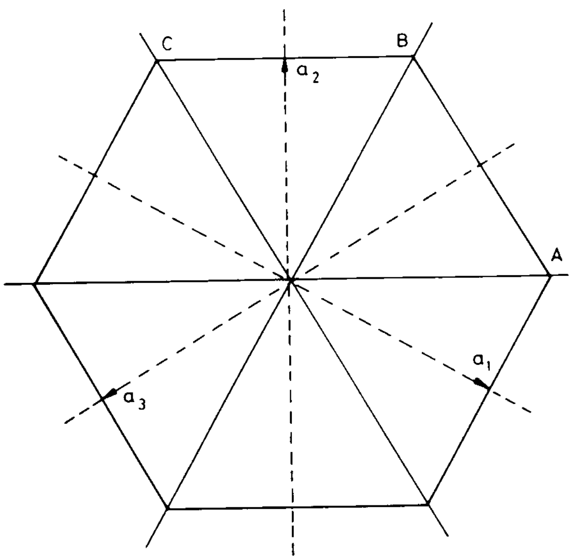

Fig. 3 *Mode 1 switching conditions*

crosses a solid line and a diode turns on when $\boldsymbol{v}$ crosses a dashed line. In particular, when the diode switching pattern is $\substack{100\\001}$, and the current vector is at hexagon vertex A, the condition for diode 2 to switch on and commutation to start is $v(1) = v(2)$ or that $\boldsymbol{v}$ crosses the dashed line containing the $\boldsymbol{a}_3$ axis. During commutation along AB, the diode switching pattern is $\substack{110\\001}$ and phases 1 and 2 are short-circuited so that $v(1) = v(2)$. As the AC line reactances are assumed to be balanced, $v(1)$ and $v(2)$ are equal to the average of $e(1)$ and $e(2)$, which is a sinusoid half the amplitude of $e(1)$ and lagging $e(1)$ by 60°. Thus $v(1)$ and $v(2)$ may be represented by the projection on $\boldsymbol{a}_1$ of a vector $\boldsymbol{w}$ half the length of $\boldsymbol{e}$ and lagging $\boldsymbol{e}$ by 60° as shown in Fig. 4. (For convenience the length of $\boldsymbol{e}$ is taken to be $\sqrt{2}$ in Fig. 4.) At the end of the commutation $\boldsymbol{e}$ reverts to $\boldsymbol{v}$ in all three components. Thus the line voltages are described by the components of $\boldsymbol{e}$ when there is no commutation and a selection of the components of $\boldsymbol{e}$ and $\boldsymbol{w}$ during commutation.

In mode 1 operation, $\boldsymbol{v}$ reverts to $\boldsymbol{e}$ before $\boldsymbol{e}$ has rotated past the $\boldsymbol{a}_2$ axis; that is, before the start condition for the next commutation is reached. Thus $\boldsymbol{e}$ rotates at most 60° during a mode 1 commutation. It is well known that the commutation angle u depends on the load current I_d according to

$$\cos u = 1 - \frac{2X}{\sqrt{3}\, e_{max}} I_d$$

where e_{max} is the maximum generator voltage and X is

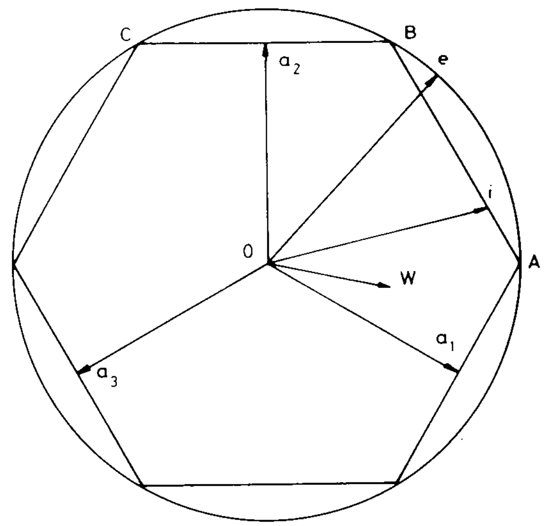

Fig. 4 *Commutation along hexagon edge AB*

the AC line reactance. For sufficiently high load currents, the commutation angle u exceeds 60° and mode 2 operation occurs; the start of commutation along AB is delayed until the end of the previous commutation, when $\boldsymbol{v}$ reverts to $\boldsymbol{e}$ and $v(2)$ suddenly becomes greater than $v(1)$ because $\boldsymbol{e}$ has already passed over the dashed line containing the $\boldsymbol{a}_3$ axis. The bridge is always commutating and the current vector traverses the hexagon edges without pausing at the vertices.

Mode 3 operation occurs when the commutation angle is so large that $\boldsymbol{w}$ rotates to lie along OB and $\boldsymbol{e}$ rotates to lie along OC during the commutation. Then $v(1)$, the projection of $\boldsymbol{w}$ on $\boldsymbol{a}_1$, and $v(3)$, the projection of $\boldsymbol{e}$ on $\boldsymbol{a}_3$, vanish simultaneously, the equality of $v(1)$ and $v(3)$ causes diode 4 to turn on, and the diode switching pattern becomes $\substack{110\\101}$. The commutation from diode 1 to diode 2 and the commutation from diode 6 to diode 4 occur simultaneously so that the current vector departs from hexagon edge AB by acquiring some velocity in the direction of BC. The commutation from diode 1 to diode 2 continues until the current vector reaches BC, when diode 2 has unity current and hence diode 1 has zero current and turns off. During the simultaneous commutations the current vector 'rounds off' the vertex of the hexagon and the AC lines do not supply all of the load current; the balance of the load current is supplied by the loop current passing through diodes 1 and 4 and the load.

3 12-pulse rectifier modes

The geometric method used to describe the 6-pulse rectifier modes extends readily to 12-pulse rectifier modes 1, 2 and 3.

Consider the 12-pulse rectifier system shown in Fig. 5, in which two 6-pulse bridges are connected in series. In

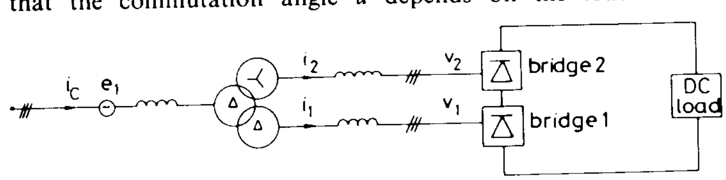

Fig. 5 *12-pulse bridge rectifier*

mode 1 operation, the commutations in the two bridges alternate and the currents i_1, i_2 in the AC lines *individual* to each bridge describe hexagons in the P plane; the 30° phase shift of the transformer causes i_2 to describe a hexagon which is displaced 30° from the hexagon described by i_1. The *common* AC line current vector i_c is the sum of i_1 and i_2 and describes a regular dodecagon as shown in Fig. 6. i_c pauses at the dodecagon vertices when

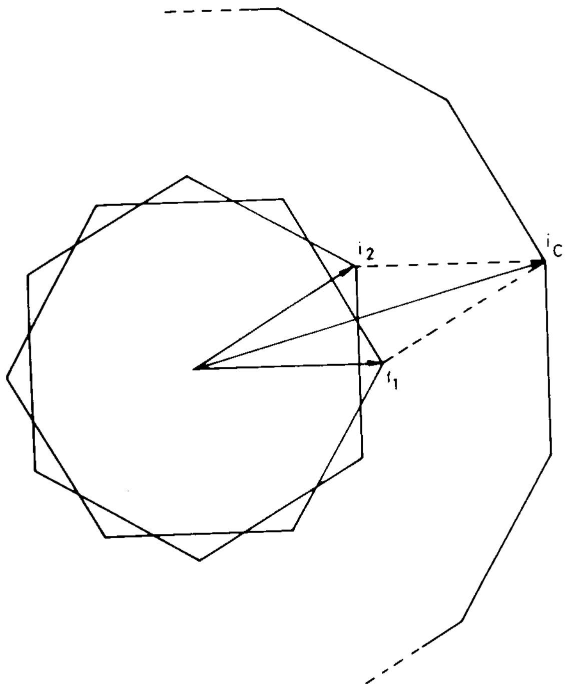

Fig. 6 *12-pulse AC line currents*

neither bridge is commutating and traverses a dodecagon edge when one of the bridges is commutating.

To describe the AC line voltages, it is convenient to define e_2 to be the displacement of the generator voltages e_1 through 30°; e_2 describes the generator voltages supplied to bridge 2 when the transformer phase shift is taken into account and is shown in Fig. 7. When neither

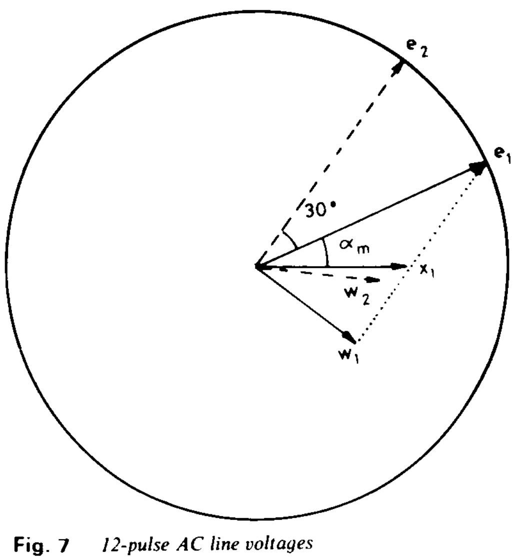

Fig. 7 *12-pulse AC line voltages*

bridge is commutating, the bridge input voltage vectors $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ are $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ respectively. When bridge 2 is commutating, the shortcircuited components of $\boldsymbol{v}_2$ revert to $\boldsymbol{w}_2$ just as in the 6-pulse case. The shortcircuited components of $\boldsymbol{v}_1$ revert to $\boldsymbol{w}_1$ in proportion to the ratio of the inductance of the common AC line to the inductance of the common and individual AC lines combined. That is, if the ratio of common to total AC line inductance is k, $\boldsymbol{v}_1$ reverts to $\boldsymbol{x}_1 = (1 - k)\boldsymbol{e}_1 + k\boldsymbol{w}_1$(see Fig. 7). k is a measure of the amount of interaction between the two bridges. The angle by which $\boldsymbol{x}_1$ lags $\boldsymbol{e}_1$ is denoted by α_m; elementary trigonometry reproduces the formula given in Reference 2 for α_m in terms of k:

$$\tan \alpha_m = \frac{k\sqrt{3}}{4 - 3k}$$

In mode 1 operation, the start condition for commutation of bridge 1 occurs 30° after the start of commutation in bridge 2; bridge 2 commutations occupy less than 30° so that bridge 2 is not commutating and $\boldsymbol{v}_1$ is equal to $\boldsymbol{e}_1$ when $\boldsymbol{e}_1$ rotates past a dashed line. In mode 2 operation, the commutation angle exceeds 30° and the relevant components of $\boldsymbol{v}_1$ continue to be determined by $\boldsymbol{x}_1$, causing the start of the bridge 1 commutation to be delayed until the bridge 2 commutation ends. The angle of delay is denoted by α. The maximum value of α is α_m, for if α exceeds α_m, $\boldsymbol{x}_1$ passes the dashed line and commutation in bridge 1 starts before the bridge 2 commutation ends. The simultaneous commutations distinguish mode 3 operation. The description of AC line voltages when bridge 1 is commutating is similar and may be obtained by interchanging the labels 1 and 2 in the above description.

The common AC line current vector $\boldsymbol{i}_c$ describes a dodecagon in modes 1 and 2; in mode 2 there is no pause at the vertices. In mode 3, the simultaneous commutation of the 2 bridges causes $\boldsymbol{i}_c$ to describe a dodecagon with rounded corners.

4 Discussion

The AC line currents of a less idealised rectifier may depart from accurate polygonal trajectories; however, visualising the circuit dynamics in the P plane may still provide insights. For example, the effect of varying the load current during a commutation may be imagined by superimposing on the polygonal trajectory a motion perpendicular to the polygon edge being traversed. Varying the load current when there is no commutation and the current vector is stationary at a polygon vertex may be imagined by correspondingly varying the length of the current vector.

Geometric models are potentially useful for 3-phase rectifier connections other than bridge connections. Any 3-phase currents or voltages which sum to zero may be represented in the P plane and circuit symmetry and an assumption of constant load current will generally cause the AC line currents to describe regular polygons. The P plane representation is also suitable for visualising decoupled natural modes of bridge rectifier currents [3].

When analysing data from a real or simulated rectifier it is desirable to be able to plot AC line quantities in the P plane. 3-phase line current data expressed as $(i(1), i(2), i(3))$ may be displayed in the P plane by plotting $(i(2) - i(3))/\sqrt{2}$ as a vertical co-ordinate against $(\sqrt{3}/\sqrt{2})i(1)$ as a horizontal co-ordinate.

5 Conclusions

A geometric model of idealised 3-phase bridge rectifier operation based on the regular figures described by AC line currents and voltages has been presented. The method has been illustrated by giving an account of 6- and 12-pulse rectifier modes from a geometric point of view. Extension of the method to visualise phenomena in less idealised or more general rectifier connections has been suggested. Geometric methods can improve the qualitative understanding of rectifier phenomena and are a pleasing way to exhibit the symmetry of rectifier connections.

6 Acknowledgments

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8 Appendix

Axes $\boldsymbol{a}_1$, $\boldsymbol{a}_2$, and $\boldsymbol{a}_3$ are covariant axes convenient for determining the 3-space co-ordinates of points in the P plane. A short calculation shows how the first co-ordinate of a point $\boldsymbol{x} = (x(1), x(2), x(3))$ in the P plane may be found by using the $\boldsymbol{a}_1$ axis:

$\boldsymbol{a}_1$ has co-ordinates $(1, -\frac{1}{2}, -\frac{1}{2})$ and its length is $\sqrt{3}/\sqrt{2}$. The length of the projection of $\boldsymbol{x}$ onto $\boldsymbol{a}_1$ is

$$\frac{\boldsymbol{a}_1 \cdot \boldsymbol{x}}{|\boldsymbol{a}_1|} = \frac{x(1) - \frac{1}{2}x(2) - \frac{1}{2}x(3)}{|\boldsymbol{a}_1|} = \frac{\frac{3}{2}x(1)}{|\boldsymbol{a}_1|} = \frac{\sqrt{3}}{\sqrt{2}}\, x(1)$$

as $x(1) + x(2) + x(3) = 0$. Therefore the length of this projection expressed as a multiple of $|\boldsymbol{a}_1|$ is $x(1)$. It follows, for example, that ideal 3-phase voltages with unity peak magnitude are represented in the P plane by a voltage vector describing a circle of radius $\sqrt{3}/\sqrt{2}$.