THE INSTITUTION OF ELECTRICAL ENGINEERS

IEE

IEE PROCEEDINGS-A covering Physical Science, Measurement and Instrumentation, Management and Education, and Reviews

Vol. 134, 1987

Published by
The IEE, Savoy Place, London WC2R 0BL, United Kingdom
Publishing Department: PO Box 8, Southgate House, Stevenage, Herts. SG1 1HQ, United Kingdom

IEE Proceedings Part A

Physical Science · Measurement and Instrumentation · Management and Education · Reviews

The Institution of Electrical Engineers, Savoy Place, London WC2R 0BL, United Kingdom.
Publishing Department: PO Box 8, Southgate House, Stevenage, Herts. SG1 1HQ, United Kingdom

Volume 134 1987 UK ISSN 0143-702X

Pages in issues of *IEE Proceedings Part A*, 1987

Pages	Issue No.	Date
1–96	1	January
97–236	2	February
237–308	3	March
309–368	4	April
369–464	5	May
465–576	6	June
577–632	7	July
633–704	8	September
705–760	9	November
761–840	10	December

Secretary of the IEE: H.H.W. Losty, DEng., FEng., FIEE
Managing Editor: Bernard Dunkley; *Assistant Editors:* Rachael Jones and Matthew Bacon
Graphics Manager: Maurice Partridge

Typeset by: Santype International Limited, Salisbury, Wiltshire, United Kingdom
Printed by: Unwin Brothers Ltd., Old Woking, Surrey, United Kingdom

Representation and simulation of AC/DC convertor systems using fixed and varying electrical axes

I. Dobson, MA

Indexing terms: Mathematical techniques, Simulation, Convertors, Power electronics

Abstract: An electrical axes formulation of Kron's tensor analysis is presented and applied to the representation and digital simulation of AC/DC bridge convertor systems. The electrical, transformer, and switching connections of a general convertor system are represented as fixed and varying electrical axes so that the system equations may be automatically formed and solved. The resulting general switching circuit simulation is capable of analysing transients in 12-pulse convertor systems with nonideal loads. The circuit theory used in deriving the simulation is suggestive of useful methods of describing and visualising convertor system phenomena.

List of principal symbols and conventions

Vectors and tensors

$\boldsymbol{i}$ = currents
$\boldsymbol{e}$ = voltages (total)
$\boldsymbol{e}^S$ = voltages across voltage sources
$\boldsymbol{e}^O$ = voltages across nonconducting switching devices
$\boldsymbol{R}$ = resistance tensor
$\boldsymbol{L}$ = inductance tensor
$\boldsymbol{C}^u_w$ = connection matrix (the example shown relates the U and W axes)

Axes systems

C = primitive axes system (distinguished from a connection matrix by the lack of indices)
W = basis for circuit currents
Z = varying basis for circuit currents
X = basis for switched circuit currents
V = axes used to calculate voltages across nonconducting switching devices
$U1$, $U2$ = axes systems intermediate between C and W
N = switching constraint axes; a subset of the C axes

Dual axes systems are indicated by dashes; M' is the axes system dual to M
A vector expressed relative to the M axes is indicated by superscript m
A vector expressed relative to the M' axes is indicated by subscript m

For example:
$\boldsymbol{i}^m$ = column vector containing the M-co-ordinates of $\boldsymbol{i}$
$\boldsymbol{e}_m$ = row vector containing the M'-co-ordinates of $\boldsymbol{e}$
Individual components of vectors are indicated with round brackets:
$\boldsymbol{i}^m(2)$ = second component of $\boldsymbol{i}^m$
A summation convention applies if the same index appears twice in a product, first as a subscript and then further to the right as a superscript
For example:

$$\boldsymbol{e}_m \boldsymbol{i}^m = \sum_{\alpha} \boldsymbol{e}_m(\alpha)\boldsymbol{i}^m(\alpha)$$

Other symbols

t = time
p = differentiation with respect to time
y = switched circuit state vector
$\boldsymbol{I}$ = unit matrix
q = number of circuit coils

Paper 4986A (P6), received 22nd August 1985

The author was formerly with EASAMS Ltd., Frimley Road, Camberley, Surrey, working on contract for the United Kingdom Atomic Energy Authority and Euratom/UKAEA Fusion Association, Abingdon, Oxon OX14, United Kingdom. He is now studying at the School of Electrical Engineering, Cornell University, Ithaca, NY 14853, USA

1 Introduction

Detailed calculations of the performance of AC/DC bridge convertor systems by standard techniques is made difficult by the frequent switching of the bridge diodes or thyristors. For example, consider calculating the transient response of a 12-pulse convertor system with a varying load current; it is supposed that the AC phases and individual switching devices are explicitly represented but that nonlinear transformer effects and detailed semiconductor effects are neglected. Hand calculations for 12-pulse systems at this level of detail are limited to idealised steady-state conditions and are impractical for the precise determination of transients. Computer simulation is possible, but care must be taken to represent the circuit switching so that the system differential equations may be obtained and solved effectively. The number of active switching elements, the great variety of possible conduction patterns and output processing problems generally make the use of conventional circuit analysis packages difficult; simulation taking special account of the switching processes is indicated.

Previous authors have successfully simulated 6-pulse representations of convertor systems at this level of detail [1, 2] using Kron's tensor analysis [3, 4]. However, these simulations are specific to particular 6-pulse circuit configurations and do not represent the phase shift transformer connections necessary for 12-pulse systems. This paper approaches the simulation of 12-pulse bridge convertor systems by considering the more general problem of simulation of a switching circuit with any arrangement of impedances, voltage sources, transformers, and switching devices such as thyristors and diodes. Tensor analysis techniques due to Kron are formulated in terms of electrical axes and generally applied to represent bridge convertor systems and derive the simulation equations. The general approach requires a careful treatment of the fundamentals of applying tensors to elementary circuit theory so that, for example, engineering concepts such as system constraints or degrees of freedom may be discussed with reference to a general switching circuit. The main advantage of the general approach is that the resulting simulation is potentially useful for the analysis of a wide range of switching circuits, including the various 6-pulse and 12-pulse configurations of AC/DC convertor systems. Moreover, the circuit theory used in deriving the simulation is a coherent framework for analysis of AC/DC convertor systems and is suggestive of useful methods of describing and visualising convertor system phenomena.

After reviewing the required tensor theory with an emphasis on the underlying axes systems, the representation of the electrical and transformer connections of a convertor system is described with reference to a basic convertor circuit which includes a 6-pulse diode bridge and a phase shift transformer. The circuit connections, or, equivalently, the constraints or degrees of freedom of circuit currents, may be directly represented and manipulated in terms of electrical axes. The varying axes concept is first introduced by analysing steady state currents in 6-pulse and 12-pulse examples of convertor systems. These examples show how varying electrical axes are a natural choice for describing convertor system currents and how varying electrical axes may be visualised using the geometric methods of Reference 5.

Simulation of a general switching circuit is then considered. The fixed circuit connections are represented by a fixed choice of electrical axes and the subsequent choice of varying axes represents the varying circuit connections due to circuit switchings. Simulation equations for a general switching circuit may be written conveniently with respect to the varying axes. The general techniques developed are illustrated by applying them to a 12-pulse bridge convertor system with a resistive and inductive load in parallel with a freewheel diode and a filtering capacitor. The simulation specification, limitations and implementation are summarised and the switching power supply systems which have been analysed with the simulation are indicated.

2 Electrical axes systems

The elementary circuit theory of Kron's tensor analysis [3, 4] is presented in terms of the axes system, vector, and duality concepts of standard linear algebra. Current and voltage vectors are explained with due attention to the underlying axes systems and the representation of circuit constraints and degrees of freedom with suitably chosen axes is demonstrated. Duality of currents and voltages is used in the mathematical treatment of circuit theory in Hirsch and Smale [6]. Nering [7] emphasises the advantages of a careful treatment of duality in linear algebra applications.

2.1 Circuit coils

The circuit to be analysed is considered by Kron to be composed of interconnected coils, where a coil is a general circuit element which may contain a resistance, inductance, voltage source or switching device connected in series. Each coil has an orientation so that a positive direction of current through the coil may be distinguished. The number of coils in the circuit is denoted by q.

2.2 Circuit paths as axes for currents

The basic elements of axes systems for currents are directed paths through the circuit coils. A directed path may be specified by a column vector $\boldsymbol{m}$ in which $\boldsymbol{m}(\alpha)$ is the number of times (positive, negative or zero) the path passes through coil α. A set of paths $M = \{\boldsymbol{m}^1, \boldsymbol{m}^2, \ldots, \boldsymbol{m}^q\}$ may be specified by placing the column vectors corresponding to the M-paths in a $q \times q$ connection matrix C^c_m so that

$C^c_m(\alpha, \beta)$ = number of times path β passes through coil α

Paths may be added to each other or multiplied by a constant by performing the corresponding operations on their column vectors. It is useful to choose the set of M-paths to be independent; that is, so that no M-path is expressible as a linear combination of the other M-paths.

Consider the special case of paths $C = \{\boldsymbol{c}^1, \boldsymbol{c}^2, \ldots, \boldsymbol{c}^q\}$ in which $\boldsymbol{c}^\alpha$ passes exactly once through coil α and through no other coils. The C-paths are used as a primitive set of paths from which other sets of paths may be defined. For example, the columns of C^c_m express the M-paths as linear combinations of C-paths:

$$\boldsymbol{m}^\beta = \sum_{\alpha=1}^{q} C^c_m(\alpha, \beta)\boldsymbol{c}^\alpha \quad \beta = 1, 2, \ldots, q \tag{1}$$

A current of x amperes in path $\boldsymbol{m}$ is written as the product $x\boldsymbol{m}$ and is equivalent to currents of $x\boldsymbol{m}(\alpha)$ amperes in coil α for $\alpha = 1, 2, \ldots, q$. The coefficient x specifies the magnitude of the current and the path $\boldsymbol{m}$ specifies the distribution of the current in the circuit coils. (Paths may be used in this way because they behave like dimensionless unit currents.) For the special case of the C-paths, $x\boldsymbol{c}^\alpha$, or x amperes in C-path α, specifies a current of x amperes in coil α and zero current in the other coils. Any circuit currents may be specified by a suitable linear combination of the C-paths. If the coefficient of $\boldsymbol{c}^\alpha$ is written as $\boldsymbol{i}^c(\alpha)$, the linear combination is

$$\boldsymbol{i} = \sum_{\alpha=1}^{q} \boldsymbol{i}^c(\alpha)\boldsymbol{c}^\alpha \tag{1a}$$

where $\boldsymbol{i}$ denotes the circuit currents. More generally, a linear combination of M-paths may be used to specify circuit currents. The coefficient of $\boldsymbol{m}^\beta$ is written $\boldsymbol{i}^m(\beta)$ and

$$\boldsymbol{i} = \sum_{\beta=1}^{q} \boldsymbol{i}^m(\beta)\boldsymbol{m}^\beta \tag{1b}$$

The coefficients $\boldsymbol{i}^m(1), \boldsymbol{i}^m(2), \ldots, \boldsymbol{i}^m(q)$ may be listed in the column vector

$$\boldsymbol{i}^m = \begin{bmatrix} \boldsymbol{i}^m(1) \\ \boldsymbol{i}^m(2) \\ \vdots \\ \boldsymbol{i}^m(q) \end{bmatrix}$$

For a given choice of M-paths, a range of circuit currents may be specified by varying the coefficients $\boldsymbol{i}^m$. That is, the M-paths are an axes system and circuit currents may be described by the coordinates $\boldsymbol{i}^m$. The superscript m indicates that the vector is expressed relative to the M axes. In this notation, a distinction is made between the circuit currents, which are denoted by $\boldsymbol{i}$, and their representation as a list of co-ordinates relative to an axes system such as M, which is denoted by i^m.

The C-co-ordinates of the current vector form a list of the currents in the circuit coils and are related to the M-co-ordinates of the current vector by

$$i^c(\alpha) = \sum_{\beta=1}^{q} C_m^c(\alpha, \beta) i^m(\beta) \quad \alpha = 1, 2, \ldots, q$$

which may also be written

$$i^c = C_m^c i^m \tag{2}$$

using the repeated index to imply the summation. Eqn. 2 may be derived by equating the right-hand sides of eqns. 1a and 1b, substituting for m^β using eqn. 1, and equating the C axes coefficients. Eqn. 2 shows how the co-ordinates of currents may be transformed between two axes systems by the connection matrix relating the axes systems.

2.3 Dual paths and constraints on currents

A dual path may be specified by a row vector m' in which $m'(\alpha)$ is the number of times the dual path acts on coil α. A dual path acts on paths to give numerical results; it is a linear function of paths. The action of dual path m' on any path m is given by the matrix product

$$[m'(1) m'(2), \ldots, m'(q)] \begin{bmatrix} m(1) \\ m(2) \\ \vdots \\ m(q) \end{bmatrix}$$

A set of dual paths $M' = \{m'_1, m'_2, \ldots, m'_q\}$ may be specified by placing the row vectors corresponding to the M' dual paths in a $q \times q$ matrix C_c^m. If a set of axes $M = \{m^1, m^2, \ldots, m^q\}$ has already been chosen, the M' axes are chosen to satisfy

$$\sum_{\gamma=1}^{q} m'_\alpha(\gamma) m^\beta(\gamma) = \begin{cases} 1 & \text{if } \alpha = \beta \\ 0 & \text{if } \alpha \neq \beta \end{cases} \tag{2a}$$

Thus m'_α is the function of paths which evaluates to zero at each of the m^β, $\beta = 1, 2, \ldots, q$ except for m^α, where it evaluates to unity. Condition 2a may also be written

$$C_c^m C_m^c = I \tag{3}$$

and the M and M' axes related in this reciprocal way are said to be dual to each other. The axes system dual to the C axes is the set of dual paths $C' = \{c'_1, c'_2, \ldots, c'_q\}$ in which c'_α acts exactly once on coil α and does not act on other coils. The rows of C_c^m express the M' dual paths as linear combinations of the C' dual paths:

$$m'_\beta = \sum_{\alpha=1}^{q} c'_\alpha C_c^m(\beta, \alpha)$$

Dual paths may be used as linear constraints on currents. For example, if coils 1, 2 and 4 are joined in a star connection, and the positive direction of each coil points towards the node of the star, Kirchhoff's current law constraint at the node is $k = c'_1 + c'_2 + c'_4$, or, if expressed as a vector relative to the C' axes, $k_c = (1\ 1\ 0\ 1\ 0 \ldots 0)$. The application of the constraint to circuit currents i may be written

$$k_c i^c = 0 \tag{3a}$$

Note that a multiple or the negative of k_c also describes the same constraint. The constraint may also be expressed relative to the M' axes as k_m, which is related to k_c by

$$k_c = k_m C_c^m$$

This equation may be multiplied by C_m^c to give

$$k_c C_m^c = k_m \tag{4}$$

as a consequence of the duality of M and M' (eqn. 3). Eqn. 4 is a plausible formula for computing k_m because

$$k_c i^c = k_c(C_m^c i^m) = (k_c C_m^c) i^m = k_m i^m$$

for all possible currents i.

2.4 Dual axes systems for voltages

The expression of voltages relative to dual path axes is similar to the expression of currents relative to path axes. A voltage of x volts in dual path m' is written as the product xm' and is equivalent to voltages of $xm'(\alpha)$ volts across coil α for $\alpha = 1, 2, \ldots, q$. Circuit voltages e may be specified by a linear combination of the M' dual paths as

$$e = \sum_{\beta=1}^{q} e_m(\beta) m'_\beta \tag{5}$$

The coefficients of the M' dual paths may be listed in a row vector as

$$e_m = (e_m(1), e_m(2), \ldots, e_m(q))$$

The M' dual paths are an axes system and circuit voltages may be described by the coordinates e_m. The subscript m indicates that the vector is expressed relative to the M' axes. Circuit voltages e expressed relative to the C' axes are written as e_c; e_c is a list of the voltages across the circuit coils.

Voltage co-ordinates transform between the M' and C' axes according to

$$e_c = e_m C_c^m \tag{5a}$$

This voltage co-ordinate transformation may be derived similarly to the current co-ordinate transformation of eqn. 2. Multiplication by C_m^c and application of the duality condition 3 gives

$$e_c C_m^c = e_m \tag{5a'}$$

Thus the connection matrix C_m^c transforms voltage co-ordinates from the C' to the M' axes and current co-ordinates from the M to the C axes (eqn. 2). Derivations of transformation 5a' in which the proper distinction between axes and dual axes is not made are confusing because current and voltage co-ordinates seem, incorrectly, to be referred to a single axes system. The reason for current and voltage co-ordinates being transformed in 'opposite directions' by C_m^c is then necessarily unclear. The approach adopted here allows eqn. 5a' to be derived from the standard formula for transforming vector co-ordinates (eqn. 5a) and the dual relationship between the two axes systems.

Eqn. 5 expresses the circuit voltages e as a linear combination of the dual paths $m'(1), m'(2), \ldots, m'(q)$. As dual paths may be evaluated on paths, e may also be regarded as a function to be evaluated on paths. Evaluating e at a path is equivalent to finding the voltage across or around the path. In particular, the result of evaluating e at path m^α is

$$em^\alpha = \left(\sum_{\beta=1}^{q} e_m(\beta) m'_\beta\right) m^\alpha = \sum_{\beta=1}^{q} e_m(\beta)(m'_\beta m^\alpha) = e_m(\alpha) \tag{6}$$

Eqn. 6 shows that $e_m(\alpha)$ may be interpreted as the voltage across or around the path m^α. Given path axes M, circuit voltages may be specified by the co-ordinates e_m, which are the voltages around the M-paths as well as the coefficients of the M'-paths implicitly chosen dual to the M axes.

2.5 Axes, constraints and degrees of freedom

The choosing of axes systems representing circuit constraints and degrees of freedom is now considered. An axes system W with fewer than q axes may be chosen in which the expressible currents are exactly those satisfying linear constraints on currents such as Kirchhoff's current law (KCL).

Suppose, for example, that a circuit has $s + 1$ nodes so that s independent KCL constraints may be chosen. Then any KCL constraint may be expressed as a linear combination of the s independent constraints. The last s M' axes are chosen to be the s independent constraints and then M axes are chosen to satisfy the duality condition (eqn. 3) with respect to the last s M' axes. That is, the last s rows of C_c^m are set to the C'-co-ordinates of the s constraints and then C_m^c is required to satisfy

$$(\text{last } s \text{ rows of } C_c^m)\, C_m^c = \text{last } s \text{ rows of } I \qquad (6a)$$

It is convenient to define the W axes to be the first $r = q - s$ M axes. The following reasoning shows that the duality condition forces the W axes to be closed paths, or meshes.

The KCL constraint k at a particular circuit node may be expressed as a linear combination of the last s M' axes:

$$k = \sum_{\alpha=r+1}^{q} k_m(\alpha) m'_\alpha$$

The duality condition 6a implies that

$$k w^\beta = k m^\beta = 0 \quad \beta = 1, 2, \ldots, r$$

This is the condition that the path w^β enters the node the same number of times as it leaves it. If this argument is applied to each circuit node, it may be concluded that each of the W-paths is a mesh. The duality between the W axes and the last s M' axes, or the constraints on currents, requires the W axes to satisfy the constraints, or, equivalently, to be meshes.

The KCL constraints expressed relative to the M' axes are the last s rows of the identity matrix:

$$\begin{bmatrix} 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & 1 & \cdots & 0 \\ & & & \vdots & & & \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

Therefore the currents satisfying KCL have the last s M-co-ordinates zero. Moreover, any choice for the W-co-ordinates (i.e. the first r M-co-ordinates) specifies currents satisfying the constraints. Each W-co-ordinate specifies an amount of current and the corresponding W axis specifies how that current is distributed in the circuit coils. As the W axes are meshes, the current is distributed in the circuit coils so that KCL is satisfied. That is, circuit currents expressible as i^w satisfy KCL because they may be written as a linear combination of the W axes, which themselves satisfy KCL. Each W axis describes a degree of freedom of circuit currents satisfying KCL.

The W axes not only satisfy KCL; they are independent (as the M axes are independent) and equal in number to the number of degrees of freedom of circuit currents. Therefore not only do circuit currents expressible as i^w satisfy KCL; all currents satisfying KCL may be expressed as i^w. Thus i^w is a state vector capable of completely describing the possible circuit currents with the minimum number of co-ordinates. An axes system such as W, with independent axes satisfying the circuit constraints and exactly as many axes as there are degrees of freedom of circuit currents, is called a basis for currents in the circuit. A basis is a complete and minimal description of the degrees of freedom of circuit currents satisfying the circuit constraints; each axis in a basis describes one of the degrees of freedom. KCL constraints may be applied by restricting currents to those expressible relative to a basis for circuit currents. The method is not limited to KCL constraints; it may be applied to any linear constraints on circuit currents.

The W axes are specified relative to the C axes by the $q \times r$ matrix C_w^c, which is the first r columns of C_m^c. The last s M-co-ordinates of currents satisfying KCL, which are always zero, may be suppressed from the right hand side of eqn. 2 to give the transformation formula

$$i^c = C_w^c i^w$$

One consequence of the W axes being meshes is that Kirchhoff's voltage law (KVL), which requires the voltage around any mesh to be zero, may be expressed by requiring the circuit voltages e to evaluate to zero on each W axis:

$$e w^\alpha = 0 \quad \alpha = 1, 2, \ldots, r \qquad (7)$$

Applying eqn. 6 and noting that the first r M' axes are the W' axes allows KVL to be written as

$$e_{w'} = 0$$

Eqn. 7 may also be considered from a point of view in which the W-meshes are constraints on voltages and act on the voltages e satisfying KVL to give zero (see also eqn. 3a in the case of constraints on currents).

The W axes may also be used to represent the voltages across specific types of circuit components around the W-meshes; these partial voltages are generally nonzero. For example, the voltage source voltages e^S around a mesh may be nonzero. Eqn. 5a' is also valid for partial voltages such as e^S and the first r columns of eqn. 5a' give the transformation law

$$e_c^S C_w^c = e_w^S$$

A significant symmetry is apparent when axes and dual axes are chosen to represent circuit constraints and degrees of freedom. Suppose there are r degrees of freedom for circuit currents. Then r of the M axes may be chosen to be independent meshes, which are degrees of freedom for currents as well as constraints on voltages, and $q - r$ of the M' axes may be chosen to be constraints on currents, which are also degrees of freedom for voltages. The duality condition then determines the choice of the remaining M and M' axes.

To summarise, circuit currents and voltages may be expressed as vectors relative to electrical axes systems. The axes and dual axes systems may be chosen to describe precisely the circuit connections, or, equivalently, the constraints and degrees of freedom of the currents and voltages satisfying Kirchhoff's laws in the circuit. The approach is particularly suitable for the analysis of circuits with varying connections due to switchings. A nonconducting switching device provides a constraint on the switching device current and a degree of freedom for the switching device voltage whereas a conducting switching device provides a degree of freedom for circuit currents and a constraint on the switching device voltage. This suggests that the varying circuit connections of a switching circuit may be described by a correspondingly varying axes system.

2.6 Impedance tensors

Circuit resistance $\boldsymbol{R}$ and inductance L are 2-dimensional tensors expressible relative to 2 circuit axes. Circuit resistance $\boldsymbol{R}$ may be specified relative to the C' and C axes by the diagonal matrix $\boldsymbol{R}_{cc}$ in which the diagonal is a list of the coil resistances. More generally, $\boldsymbol{R}_{uw}$ is the matrix which determines the resistive voltage drops relative to the U' axes due to currents expressed relative to the W axes to be $\boldsymbol{R}_{uw} \boldsymbol{i}^w$. Both dimensions of the resistance tensor transform like voltages. For example

$$\boldsymbol{R}_{uw} = \boldsymbol{R}_{cc} \, C^c_u \, C^c_w$$

Similar remarks apply to inductance L, except that the $\boldsymbol{L}_{cc}$ matrix may contain off-diagonal elements which are mutual inductances between coils.

2.7 Example

The simple circuit of Fig. 1 has two independent KCL

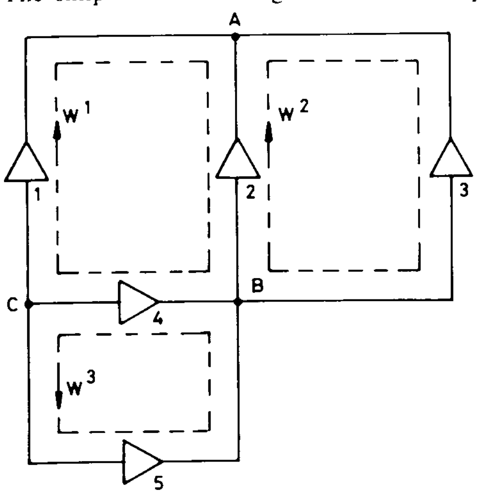

Fig. 1 *Simple example circuit*

constraints; these may be chosen as the constraints corresponding to the circuit nodes A and C:

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

These two constraints may be chosen to be the last two M' axes. As there are five coil currents subject to two independent constraints, circuit currents have $5 - 2 = 3$ degrees of freedom and three independent W axes may be chosen dual to and satisfying the constraints. That is, C^c_w has three independent columns satisfying

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix} C^c_w = 0$$

By inspection, a suitable choice for the W axes is

$$C^c_w = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 0 \\ -1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

Meshes are intuitively represented by superimposing them on the circuit diagram; the W axes are the meshes shown in Fig. 1.

The choice of the W axes and the last two M' axes is sufficient to determine the remaining M and M' axes by duality. (For example, the independence of the M axes and duality with the last two M' axes determines the last two M axes.) Hence

$$C^c_m = \begin{bmatrix} 1 & 0 & 0 & \frac{2}{8} & \frac{2}{8} \\ -1 & 1 & 0 & \frac{3}{8} & -\frac{1}{8} \\ 0 & -1 & 0 & \frac{3}{8} & -\frac{1}{8} \\ -1 & 0 & -1 & -\frac{1}{8} & \frac{3}{8} \\ 0 & 0 & 1 & -\frac{1}{8} & \frac{3}{8} \end{bmatrix}$$

$$C^m_c = \begin{bmatrix} \frac{4}{8} & -\frac{2}{8} & -\frac{2}{8} & -\frac{2}{8} & -\frac{2}{8} \\ \frac{2}{8} & \frac{3}{8} & -\frac{5}{8} & -\frac{1}{8} & -\frac{1}{8} \\ -\frac{2}{8} & \frac{1}{8} & \frac{1}{8} & -\frac{3}{8} & \frac{5}{8} \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Possible circuit currents and voltages are given by

$$\boldsymbol{e}_c = (2 \quad 1 \quad 1 \quad 1 \quad 1) \qquad \boldsymbol{e}_m = (0 \quad 0 \quad 0 \quad 1 \quad 1)$$

$$\boldsymbol{e}_w = (0 \quad 0 \quad 0)$$

$$\boldsymbol{i}^c = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \\ -1 \end{bmatrix} \quad \boldsymbol{i}^m = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \quad \boldsymbol{i}^w = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

The three W axes completely define the circuit connections in terms of the degrees of freedom of currents (or constraints on voltages) and the last two M' axes completely define the circuit connections in terms of the degrees of freedom of voltages (or constraints on currents). Most circuit calculations do not require explicit calculation of all the M and M' axes.

3 Representation of a basic convertor circuit

The representation of the connections and components of a convertor system is considered in the case of the basic circuit shown in Fig. 2. The approach presented for this

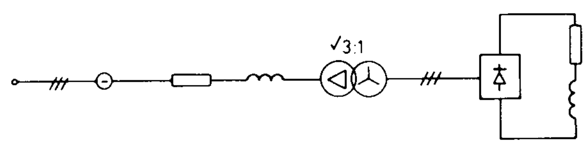

Fig. 2 *Basic convertor circuit*

circuit generalises readily to 12-pulse convertor circuits. The circuit consists of a 3-phase AC supply and phase shift transformer feeding a 6-pulse diode bridge and a simple load. The transformer has a 4-limb iron core and a turns ratio approximating $\sqrt{3}:1$. Coils are chosen for the circuit as shown in Fig. 3.

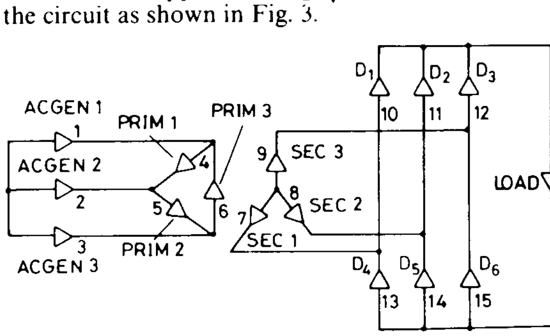

Fig. 3 *Choice of coils for basic convertor circuit*

3.1 Electrical and magnetic connections

The electrical and magnetic connections are modelled as integer linear constraints on the coil currents. The electrical connections are given by KCL and may be completely described by an independent set of eight constraints:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1	0	0	−1	0	1										
2	0	1	0	1	−1	0										
3	0	0	1	0	1	−1										
4							1	0	0	−1	0	0	1	0	0	0
5							0	1	0	0	−1	0	0	1	0	0
6							0	0	1	0	0	−1	0	0	1	0
7							0	0	0	0	0	0	−1	−1	−1	1
8							1	1	1	0	0	0	0	0	0	0

The coil numbers are shown above the corresponding matrix columns and the constraint numbers are shown next to the corresponding rows.

The transformer turns ratio is taken to be 26 : 15, which approximates $\sqrt{(3)}:1$ to within 0.1%. For a 4-limb iron core transformer, there is for each phase a closed flux path in the iron passing through the primary and secondary windings of that phase only. By applying the Maxwell-Ampere law around the closed iron path,

$$26\ (\text{primary current}) + 15\ (\text{secondary current}) = \int \boldsymbol{H}\,dl = \frac{1}{\mu}\int \boldsymbol{B}\,dl \tag{10}$$

The idealisation of perfect iron may be obtained by letting the magnetic permeability μ tend to infinity. The magnetic field $\boldsymbol{B}$ remains finite and therefore the right-hand side of eqn. 10 becomes zero. Applying this argument separately to the three transformer phases yields the transformer magnetic constraints:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
9	0	0	0	26	0	0	15	0	0	0	0	0	0	0	0	0
10	0	0	0	0	26	0	0	15	0	0	0	0	0	0	0	0
11	0	0	0	0	0	26	0	0	15	0	0	0	0	0	0	0

The magnetic connections of a general iron cored transformer may be similarly represented by a set of integer linear constraints in which each constraint corresponds to an independent magnetic circuit mesh.

3.2 *Calculation of W axes*

The 11 electrical and magnetic constraints completely describe the circuit connections. However, the simulation requires the connection information in the form of degrees of freedom of currents rather than in the form of constraints on currents. In terms of axes systems, it is required to calculate C^c_w defining W axes which are a basis for circuit currents. As there are 16 coil currents subject to 11 independent constraints, the number of degrees of freedom of the circuit currents is $16 - 11 = 5$. The W axes may be chosen to be any independent set of 5 meshes satisfying the 11 constraints.

In a circuit with electrical constraints only, the W axes may usually be chosen by inspection. However, the axes choice for a convertor system with complex transformer connections is simplified if a systematic approach is adopted in which the constraints are successively applied in stages. At each stage, an intermediate axes system satisfying the constraints applied up to that stage is calculated.

The circuit constraints are grouped as follows:

(*a*) electrical constraints (constraints 1–7)
(*b*) magnetic constraints (constraints 9–11)
(*c*) star connection of transformer windings (constraint 8)

Independent paths satisfying the first seven electrical constraints may be chosen by inspection to form the $U1$ axes:

$C^c_{u1} =$

	1	2	3	4	5	6	7	8	9
1 ACGEN1	1	0	−1						
2 ACGEN2	−1	1	0						
3 ACGEN3	0	−1	1						
4 PRIM1	1	0	0						
5 PRIM2	0	1	0						
6 PRIM3	0	0	1						
7 SEC1				1	0	0	0	0	0
8 SEC2				0	1	0	0	0	0
9 SEC3				0	0	1	0	0	0
10 D1				1	0	0	1	0	0
11 D2				0	1	0	−1	1	0
12 D3				0	0	1	0	−1	1
13 D4				0	0	0	1	0	0
14 D5				0	0	0	−1	1	0
15 D6				0	0	0	0	−1	1
16 LOAD				0	0	0	0	0	1

The remaining constraints are projected onto the U_1 axes by transformation with C^c_{u1}:

$$
\begin{array}{c} \\ 8 \\ 9 \\ 10 \\ 11 \end{array}
\begin{array}{c}
\begin{array}{cccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & \cdots & 16 \end{array} \\
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & \cdots & 0 \\
0 & 0 & 0 & 26 & 0 & 0 & 15 & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & 0 & 26 & 0 & 0 & 15 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & 0 & 0 & 26 & 0 & 0 & 15 & 0 & \cdots & 0
\end{bmatrix}
\end{array} C^c_{u1}
$$

$$
= \begin{array}{c}
\begin{array}{ccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{array} \\
\begin{bmatrix}
0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\
26 & 0 & 0 & 15 & 0 & 0 & 0 & 0 & 0 \\
0 & 26 & 0 & 0 & 15 & 0 & 0 & 0 & 0 \\
0 & 0 & 26 & 0 & 0 & 15 & 0 & 0 & 0
\end{bmatrix}
\end{array}
$$

The axis numbers are shown above the corresponding matrix columns. (Note that applying the first set of constraints and then applying the projections of the remaining constraints always has the same effect as applying all the constraints at once.) Choosing the $U1$ axes so that exactly one axis passes through each transformer winding and postponing the application of the star constraint ensure that the projected constraints have a simple form.

The magnetic constraints are applied by choosing $U2$ axes. The calculation is performed relative to the $U1$ axes to ensure that the previously applied constraints remain satisfied. An appropriate choice of $U2$ axes satisfying the magnetic constraints is given by

$$
C^{u1}_{u2} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{array}
\begin{array}{c}
\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\
\begin{bmatrix}
-15 & 0 & 0 & & & \\
0 & -15 & 0 & & & \\
0 & 0 & -15 & & & \\
26 & 0 & 0 & & & \\
0 & 26 & 0 & & & \\
0 & 0 & 26 & & & \\
 & & & 1 & 0 & 0 \\
 & & & 0 & 1 & 0 \\
 & & & 0 & 0 & 1
\end{bmatrix}
\end{array}
$$

The remaining star constraint is projected onto the $U2$ axes:

$$(0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0)\, C^{u1}_{u2} = (26 \quad 26 \quad 26 \quad 0 \quad 0 \quad 0)$$

W axes which are a basis for circuit currents may be chosen relative to the $U2$ axes by choosing meshes satisfying the star constraint:

$$
C^{u2}_{w} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}
\begin{array}{c}
\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{bmatrix}
1 & 0 & & & \\
0 & 1 & & & \\
-1 & -1 & & & \\
 & & 1 & 0 & 0 \\
 & & 0 & 1 & 0 \\
 & & 0 & 0 & 1
\end{bmatrix}
\end{array}
$$

The W axes may be expressed relative to the C axes by calculating

$C^c_w = C^c_{u1} C^{u1}_{u2} C^{u2}_w =$

	1	2	3	4	5
1 ACGEN1	−30	−15	0	0	0
2 ACGEN2	15	−15	0	0	0
3 ACGEN3	15	30	0	0	0
4 PRIM1	−15	0	0	0	0
5 PRIM2	0	−15	0	0	0
6 PRIM3	15	15	0	0	0
7 SEC1	26	0	0	0	0
8 SEC2	0	26	0	0	0
9 SEC3	−26	−26	0	0	0
10 D1	26	0	1	0	0
11 D2	0	26	−1	1	0
12 D3	−26	−26	0	−1	1
13 D4	0	0	1	0	0
14 D5	0	0	−1	1	0
15 D6	0	0	0	−1	0
16 LOAD	0	0	0	0	1

Note that axes w^1 and w^2 are meshes which are composed of multiple disjoint loops. Mentally superimposing w^1 and w^2 on the circuit diagram is one way to understand the operation of the phase shift transformer connection.

3.3 Circuit components

The coil resistances and inductances may be specified in the R_{cc} and L_{cc} matrices and transformed to the W axes:

$$R_{ww} = R_{cc}\, C^c_w C^c_w$$

$$L_{ww} = L_{cc}\, C^c_w C^c_w$$

Coils 1, 2 and 3 contain ideal sinusoidal voltage sources; their voltages are known functions of time.

Each bridge diode is represented by a coil containing an impedance in series with an ideal diode. (If required, the forward voltage drop of the diode may be represented by including a constant voltage source in the coil.) The ideal diode is an on/off switch controlled by its current and voltage; it switches off (stops conducting) when its current becomes negative and switches on (conducts) when its voltage becomes positive. The ideal diode part of the coil has zero impedance when the diode is on. Following Williams and Smith [1], the off diode is assumed to have a very large impedance so that the circuit branches containing off diodes are effectively deleted from the circuit. The circuit resulting from the deletions is conveniently referred to as the switched circuit.

The idealisation of the diode switching allows the circuit state to be divided into a discrete and a continuous part; the discrete part, or switching state, describes the switched circuit by specifying which diodes are off and the continuous part describes the switched circuit currents. One of the main tasks of the simulation is to form and solve the differential equations for each of the switched circuits arising during the run.

4 Switching axes examples and visualisation

4.1 Varying axes for an ideal 6-pulse convertor

Consider the 6-pulse convertor shown in Fig. 4 operating in mode 1. The circuit is first supposed to be commutating with switching state defined by $^{110}_{001}$, where 0 indicates that the diode in the corresponding position is off and 1 indicates that it is on. X axes describing the degrees of freedom of currents in the switched circuit may be chosen by

$$C^c_x = \begin{matrix} 1 \text{ ACGEN1} \\ 2 \text{ ACGEN2} \\ 3 \text{ ACGEN3} \\ 4 \text{ D1} \\ 5 \text{ D2} \\ 6 \text{ D3} \\ 7 \text{ D4} \\ 8 \text{ D5} \\ 9 \text{ D6} \\ 10 \text{ LOAD} \end{matrix} \begin{bmatrix} \frac{1}{2} & -1 \\ \frac{1}{2} & 1 \\ -1 & 0 \\ \frac{1}{2} & -1 \\ \frac{1}{2} & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 1 & 0 \end{bmatrix}$$

The x^1 axis describes the circuit degree of freedom by which the load current is supplied; it is scaled by a factor of $\frac{1}{2}$ so that $i^x(1)$ is equal to the load current. The x^2 axis describes the degree of freedom by which the circuit commutates. Moreover, if the AC line impedances are assumed to be balanced, the X axes describe decoupled degrees of freedom for currents and remain a convenient choice of axes even under transient conditions. This may be demonstrated by observing that x^1 is orthogonal to x^2

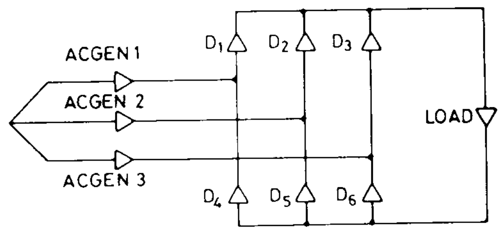

Fig. 4 *Coils of a 6-pulse AC DC convertor*

(their vector dot product is zero) and that balanced AC line impedances means that the 3 × 3 submatrices of R_{cc} and L_{cc} corresponding to the AC line coils are proportional to the identity matrix. As x^1 and x^2 intersect only in the AC line coils, it follows that $L_{xx} = L_{cc}\, C^c_x C^c_x$ and $R_{xx} = R_{cc}\, C^c_x C^c_x$ are both diagonal. It follows that $i^x(1)$ and $i^x(2)$ are decoupled when the circuit differential equations are written with respect to the X axes:

$$L_{xx}\, p i^x + R_{xx}\, i^x + e^S_x = 0$$

Thus the progress of the commutating current is decoupled from fluctuations in the DC current. (Note, however, that the start and finish time of the commutation may depend on all the circuit currents.)

The X axes defined above are convenient for describing system currents during the commutation in switching state $^{110}_{001}$. When the commutation finishes, the switching state becomes $^{010}_{001}$, the system reduces to one degree of freedom and it is convenient to change the X axes to one axis describing that degree of freedom:

$$C^c_x = \begin{matrix} 1 \text{ ACGEN1} \\ 2 \text{ ACGEN2} \\ 3 \text{ ACGEN3} \\ 4 \text{ D1} \\ 5 \text{ D2} \\ 6 \text{ D3} \\ 7 \text{ D4} \\ 8 \text{ D5} \\ 9 \text{ D6} \\ 10 \text{ LOAD} \end{matrix} \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

When the next commutation starts, the switching state becomes $^{010}_{101}$ and the degrees of freedom of currents may be described by two axes:

$$C^c_x = \begin{matrix} 1 \text{ ACGEN1} \\ 2 \text{ ACGEN2} \\ 3 \text{ ACGEN3} \\ 4 \text{ D1} \\ 5 \text{ D2} \\ 6 \text{ D3} \\ 7 \text{ D4} \\ 8 \text{ D5} \\ 9 \text{ D6} \\ 10 \text{ LOAD} \end{matrix} \begin{bmatrix} -\frac{1}{2} & -1 \\ 1 & 0 \\ -\frac{1}{2} & 1 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ \frac{1}{2} & 1 \\ 0 & 0 \\ \frac{1}{2} & -1 \\ 1 & 0 \end{bmatrix}$$

Similar choices of X axes describing the degrees of freedom of circuit currents may be made at each switching in the full cycle of operation. Note how the problem of specifying circuit currents is conveniently split into first specifying the X axes, which take account of and describe the varying circuit connections, and then specifying the amounts of current flowing in the axes, or i^x. The X axes chosen for this 6-pulse example also describe decoupled degrees of freedom for currents and are therefore helpful in imagining the operation of the circuit. Intuition of rectifier phenomena by means of current loops is a natural procedure; electrical axes methods of circuit analysis correspond exactly to this intuition and may be used to strengthen the intuition and notate it precisely.

The varying X axes and the circuit currents may be visualised by projecting them onto the AC line coils or, equivalently, projecting them onto the first three C axes. As interactions in the AC lines account for much of the circuit behaviour, this projection preserves much of the information about the circuit dynamics. The star connection of the AC lines has co-ordinates (1 1 1 0 0 0 0 0 0) and becomes (1 1 1) when projected onto the AC coils. If the projected vectors are regarded as points in 3-space, the star connection constrains the X axes and the AC line currents to lie in the plane normal to (1 1 1). If a constant load current is assumed, the trajectory of the AC line currents in this plane is a regular hexagon centred on the origin as shown in Fig. 5. (A detailed

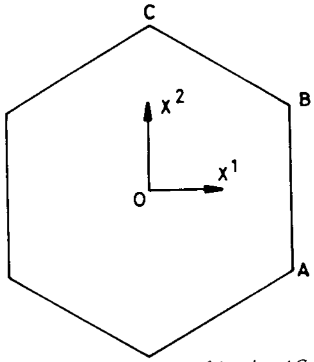

Fig. 5 *Trajectory of 6-pulse AC line currents*

explanation of this representation is given in Reference 5.) When the bridge is not commutating, the AC line currents are constant and the trajectory is stationary at one of the hexagon vertices. During commutation, the constant load current constrains the current vector to traverse a hexagon edge. In the second mode of operation, in which the start of commutation is delayed until the previous commutation is finished, the trajectory is similar except that it does not pause at the hexagon vertices. Fig. 5 also shows the projected X axes when the switching state is $\substack{110\\001}$. During commutation the AC line currents traverse hexagon edge AB; it is evident from Fig. 5 that this motion is conveniently described relative to the X axes by $i^x(1)$ being held fixed at the load current and $i^x(2)$ varying from $-\frac{1}{2}$ to $\frac{1}{2}$ as the commutation proceeds. When the switching state becomes $\substack{010\\001}$, the AC line currents pauses at hexagon vertex B and the new X axis is chosen; its projection lies along the line OB. When the switching state becomes $\substack{010\\101}$, the AC line currents traverse hexagon edge BC and the two new X axes have projections rotated 60° anticlockwise from the axes shown in Fig. 5. Plotting the projected quantities in the plane of Fig. 5 shows how the varying choice of X axes simplifies the dynamics of the circuit currents. Moreover, as the two X axes used during commutation only intersect in the AC line coils, orthogonality of the projections of x^1 and x^2 in the plane of Fig. 5 implies the orthogonality of x^1 and x^2 and the consequent decoupling of the system equations.

4.2 Varying axes for an ideal 12-pulse convertor

The above analysis extends usefully to 12-pulse convertors [5]. This subsection describes the projected axes and AC line currents for the 12-pulse convertor shown in Fig. 6. The decoupling of current degrees of freedom is readily observable from a diagram of the projected axes despite the additional complexity of the 12-pulse case.

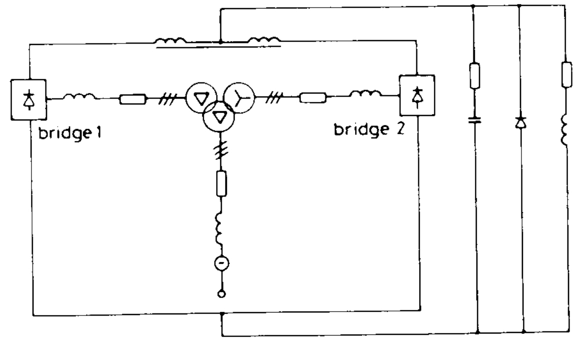

Fig. 6 *12-pulse convertor system*

If constant bridge output currents and 12-pulse mode 1 or 2 operation is assumed, the currents in the AC lines individual to each bridge traverse hexagons as shown in Fig. 7. Hexagon $A_2B_2C_2$ ... is rotatated 30° relative to hexagon $A_1B_1C_1$ ... by the phase shift of the transformer. The common AC line currents are the sum of the individual AC line currents and traverse the regular dodecagon PQR ... shown in Fig. 7. In mode 1 operation the bridges

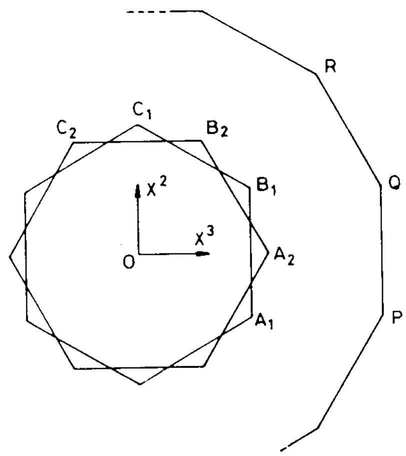

Fig. 7 *Trajectories of 12-pulse AC line currents*

commutate alternately and the trajectory pauses at the dodecagon vertices between commutations. In mode 2 operation, the bridges interact via the common AC line impedances so that commutations in one bridge are delayed until commutation in the other bridge has finished. The dodecagon is then traversed without pauses at the vertices.

Convenient varying axes describing degrees of freedom of currents may be chosen for the 12-pulse case similarly to the 6-pulse case. For example, when bridge 1 is commutating so that the common AC line currents traverse dodecagon edge PQ, there are three degrees of freedom and a convenient choice of X axes has x^1 and x^2 describing the supply of load current and commutation in bridge 1 and x^3 describing the supply of load current through bridge 2. (Co-ordinates of x^2 and x^3 with respect to the C axes are given by the second and third columns of the C_z^c matrix at the end of Section 8.2. For convenience, Fig. 7 shows the projection of x^3 at a reduced scale.) The projections of x^1 and x^3 lie along OA_2 and the projection of x^2 lies along OC_1. If balanced AC line impedances are assumed, the orthogonality of the projections of x^2 and x^3 in Fig. 7 and their nonintersection in coils outside the AC line show that x^2 and x^3 represent decoupled degrees of freedom for circuit currents. In other words, the commutation in bridge 1 is decoupled from the supply of current to the load in the bridge 2. Similar decoupling occurs during the commutation in mode 2, but the timing of the start of commutation is determined by the end of the commutation in the other bridge. This decoupling is useful when deriving simple equivalent circuits which approximate the transient behaviour of convertor systems [8].

5 Internal structure of a switching circuit simulation

For simplicity, the circuit is supposed to consist of resistances, inductances, diodes and time dependent voltage sources. (The circuit representation is extended to include thyristors, capacitors and more general voltage sources in Section 7.)

The purpose of the simulation is to solve the set of piecewise differential equations

$$py = f(y, t) \tag{12}$$

starting from a specified initial state. y is a state vector for currents in the switched circuit, f is the set of switched circuit differential equations for y, and p is differentiation with respect to time t. All currents and voltages in the switched circuit are calculable from y, py, t (inductor voltages are calculable from py and voltage source voltages are calculable from t) and y, py, t define the continuous part of the circuit state. y is a minimal state vector for the switched circuit; each component of y corresponds to a degree of freedom of the switched circuit currents. Both y and f vary with the switching state. Moreover, the nature and timing of the switchings, or transitions between the switching states, depend on the evolution of y and py.

The simulation has an integration module to update y, py, t and a switching module to update the switching state. The integration and switching modules are interdependent; the integration module advances the solution of eqn. 12 assuming a particular switching state and the switching module detects and accurately locates any switching in a given time interval assuming the variation of y, py is known over that interval. The simulation proceeds from an initial state at time t_0 by using the integration module to calculate values of y, py in the time interval $[t_0, t_1]$, assuming that the switching state does not change in $[t_0, t_1]$. The variation of y, py in $[t_0, t_1]$ is then used by the switching module to determine whether there was any switching in $[t_0, t_1]$. If there were none, the switching state did in fact remain at its initial value throughout the interval and the process may be repeated starting from t_1 with the updated circuit state. If there were switchings in the interval, and the first switching occurred at time t_{sw}, the state vector y is interpolated at t_{sw} and the simulation proceeds from t_{sw} with the updated circuit state. If the values of y, py, t are output as they are calculated, the output will contain 'overshoots' corresponding to the solution past a switching point with a previous, incorrect switching state. These overshoots are artifacts of the solution method and may be removed easily by an output processing program.

The switching module contains switching rules to determine the switchings occurring during the switching test interval $[t_0, t_1]$. Off diodes with voltages increasing through zero and on diodes with currents decreasing through zero are detected and the switching time is located precisely using standard root finding procedures. The switching rules may be written in a general way so that no account is taken of the particular configuration of switches in the circuit or the operational mode of the circuit. The switching rules are described in Reference 9; for the purposes of this paper it is sufficient to note that the switching rules require the currents through on diodes and the voltages across off diodes to be calculable at any given time t in the switching test interval.

The integration module calculates values of y at specific time points in the switching test interval. If the switching module requires diode currents and voltages at the general time t, it must interpolate to calculate $y(t)$ and use eqn. 12 to calculate py from y and t. Calculation of the required diode currents and voltages from y, py, t and the switching state is considered in the following Section.

6 Switching axes

6.1 Requirements and brief description

It is assumed that the switching module has just located a switching and determined the new switching state. The switched circuit differential equations f must be formed and the initial condition of the new state vector y must be calculated. When the switching module tests for the next switching, it will require the on diode currents and the off diode voltages to be readily calculable from y, py and t.

These requirements may be met by constructing switching axes Z which are closely related to the switching state. Calculations are referred to the Z axes while the switching state persists. At each circuit switching, the Z axes are recalculated relative to the W axes from the switched circuit constraints on currents. The Z axes consist of two systems of subaxes, X and V; the X axes are a basis for currents in the switched circuit and the V axes are used to calculate the voltages across off diodes.

6.2 Calculation of Z axes from the switching constraints

For convenience of notation, let the N axes be the C axes paths which correspond to off diodes. The number of off diodes is denoted by s. Then the switching constraints on currents may be written relative to the C' axes by selecting the corresponding rows from the identity matrix C^c_c to form the matrix C^n_c. The switching constraints may be projected onto the W' axes:

$$C^n_w = C^n_c C^c_w$$

The discussion now assumes that the projected switching constraints are independent; the exceptional case of dependent constraints is treated in Appendix 13. The last s Z' axes are chosen to be the switching constraints, or, equivalently, the N' axes. Thus C^n_z is the last s rows of the identity matrix

$$C^n_z = \begin{bmatrix} 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & 1 & 0 & \cdots & 0 \\ & & & & \vdots & & & \\ 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \tag{12a}$$

The form of C^n_z requires the Z axes to pass through exactly one off diode or through none; the former are the V axes and the latter are the X axes. Thus the Z axes divide naturally into two systems of subaxes, X and V, and eqn. 12a may be rewritten as

$$C^n_x = 0 \tag{12b}$$

$$C^n_v = I \tag{12c}$$

Independent Z axes are calculated relative to the W axes by finding an invertible transformation C_z^w satisfying

$$C_w^n C_z^w = C_z^n \tag{13}$$

Eqn. 13 requires the Z axes to be dual to the last s Z' axes (N' axes): rewriting the left-hand side of eqn. 13 as

$$C_w^n C_z^w = C_w^n C_c^w C_z^c = C_c^n C_z^c$$

exhibits eqn. 13 in the form of duality condition 3. C_z^w is a square invertible matrix because the Z axes are independent and equal in number to the W axes. Hence the Z and W axes are equivalent choices of axes describing the fixed circuit connections. Eqn. 12a shows that the Z axes are chosen so that the Z-co-ordinates of the switching constraints assume a particular and convenient form.

6.3 X axes and the switched circuit differential equations

The X axes satisfy the fixed circuit constraints because they are calculated relative to the W axes and satisfy the switching constraints because they satisfy eqn. 12b. In particular, the X axes are meshes because they satisfy the fixed electrical constraints. Moreover, as the X axes are independent and equal in number to the number of degrees of freedom of the switched circuit currents, they are a basis for switched circuit currents. Therefore the state vector y may be set to i^x. As the X axes are meshes, the switched circuit differential equations may be derived relative to the X axes by applying KVL around the X axes:

$$0 = e_x = L_{xx} p i^x + R_{xx} i^x + e_x^S + e_x^O \tag{13a}$$

where e^O is the vector of voltages across off diodes. e_x^O vanishes because the X axes do not pass through any off diodes. L and R may be calculated relative to the X axes by transformation from the W axes:

$$L_{xx} = L_{ww} C_x^w C_x^w$$

$$R_{xx} = R_{ww} C_x^w C_x^w$$

and e_x^S may be calculated from the known function of time e_c^S:

$$e_x^S = e_w^S C_x^w = e_c^S C_w^c C_x^w$$

If L_{xx} has a nonzero determinant, eqn. 13a may be rearranged as

$$p i^x = -(L_{xx})^{-1}(R_{xx} i^x + e_x^S) \tag{13b}$$

Eqn. 13b states the set of switched circuit differential equations in a form suitable for numerical integration. The number of differential equations is the smallest possible because they are expressed relative to x axes which are a basis for switched circuit currents. The condition of a nonzero L_{xx} determinant requires each circuit mesh to have some inductance; numerical calculation accuracy further requires that the circuit inductance must be sufficient to give an L_{xx} matrix with a determinant that is not very small.

6.4 Calculation of the initial value of y

Suppose the X axes of the previous switching state (the one in which the switching occurred) are denoted by $XOLD$. The value of y is interpolated at the switching time to give i^{xold}; it is required to calculate i^x, the initial value of y at the switching time. The circuit currents may be expressed relative to the W axes:

$$i^w = C_{xold}^w i^{xold}$$

If the calculation of the Z axes is additionally constrained so that C_x^w contains a square diagonal matrix of side equal to the number of X axes, the corresponding rows of the transformation

$$i^w = C_x^w i^x$$

allow i^x to be calculated from i^w. Note that it is not necessary to distinguish the switching state to which i belongs because current is continuous across switchings.

In the case of a diode switching off, the calculation effectively projects i^{xold} onto i^x. i^{xold} and i^x specify the same circuit currents since the current which is zeroed in the projection passes through the diode and has already been determined to be zero at the switching time.

6.5 Calculation of diode currents and voltages

The calculation of the currents in on diodes and the voltages across off diodes from the state vector information y, py, t will now be demonstrated.

The current through an on diode or, indeed, any circuit coil, is given by the appropriate component of

$$i^c = C_x^c i^x = C_w^c C_x^w i^x$$

The voltage across off diodes e^O is readily calculated relative to the W axes by applying KVL around the W axes:

$$0 = e_w = L_{wx} p i^x + R_{wx} i^x + e_w^S + e_w^O$$

$$e_w^O = -(L_{wx} p i^x + R_{wx} i^x + e_w^S) \tag{14}$$

Transform this equation to the V axes to obtain

$$e_v^O = e_w^O C_v^w$$

$$= -(L_{vx} p i^x + R_{vx} i^x + e_v^S) \tag{14a}$$

Transformation by C_n^v, which in this version of the algorithm is the identity matrix, gives the required off diode voltages e_n^O in the terms of readily calculable quantities:

$$e_n^O = e_v^O C_n^v$$

$$= -(L_{vx} p i^x + R_{vx} i^x + e_v^S) C_n^v$$

where

$$L_{vx} = L_{ww} C_v^w C_x^w$$

$$R_{vx} = R_{ww} C_v^w C_x^w$$

$$e_v^S = e_c^S C_v^c$$

The calculation of e_n^O may also be explained as follows: e_w^O may be calculated from readily available quantities using eqn. 14. However, e_w^O is the contribution of off diodes around the W axes to the total mesh voltage and each W axis may pass through several off diodes. Thus calculating e_w^O gives simultaneous equations for e_n^O. Transforming eqn. 14 to the V axes effectively solves these simultaneous equations. The choice of the V axes to pass through exactly one off diode ensures the simple relationship between e_n^O and e_v^O.

7 Representation of additional circuit components

The circuit representation considered above may be extended to include several additional circuit components.

Capacitors may be introduced by augmenting the state vector y with capacitor voltages e^{CAP}. For convenience of notation, let the A axes be the C axes paths corresponding to coils with nonzero capacitance. Then the circuit state vector becomes

$$y = \begin{bmatrix} i^x \\ e_a^{CAP} \end{bmatrix}$$

and the differential equations $py = f(y, t)$ become

$$pi^x = -(L_{xx})^{-1}(R_{xx} i^x + (e_c^S + e_c^{CAP})C_x^c)$$
$$pe_a^{CAP} = Q_{aa} i^a = Q_{aa} C_x^a i^x \qquad (17)$$

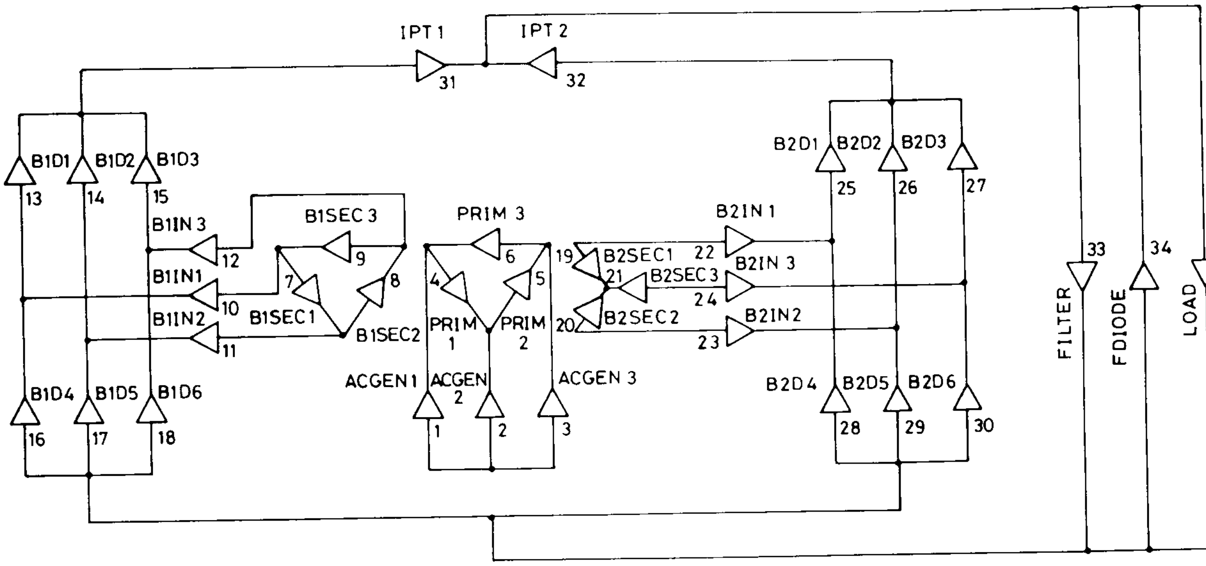

Fig. 8 *Choice of coils for 12-pulse convertor system*

where Q is the elastance tensor and C_x^a is formed by selecting the appropriate rows from C_x^c. The ability to simulate capacitors is useful for studying capacitative switching power supplies and rectifying circuits with filters.

Voltage sources e^S may be generalised to any function of circuit currents or time as the circuit currents are available from the state vector y. (The right-hand side of eqn. 17 remains a function of y and t.) In particular, time-variable and nonlinear resistances may be represented.

Thyristors may be modelled by modifying the diode switching rules so that switch on is inhibited if a gate pulse is not on. The firing control circuitry is modelled by on/off gate pulse functions which may depend on time or circuit currents. The dependence of gate pulse functions and voltage generators on circuit currents may be used to represent simple feedback control of the thyristor firing angles.

8 Axes calculations for a 12-pulse convertor system

Circuit and switched circuit axes are calculated for the 12-pulse convertor system shown in Fig. 6. An AC line supplies two 6-pulse diode bridges through a phase shift transformer with a single delta primary and separate delta and star secondaries. The delta windings have equal numbers of turns and the delta/star turns ratio approximates $\sqrt{3}:1$. The bridges are connected in parallel through an interphase transformer to a resistive and inductive load with a capacitative filter and a freewheel diode. The impedances of any step-down transformers are assumed to be included with the AC line impedances. Coils may be chosen for the circuit as shown in Fig. 8. The interphase transformer is represented by specifying the self and mutual inductances of coils IPT1 and IPT2 in the L_{cc} matrix.

8.1 Calculation of W axes

It is convenient to divide the circuit into four subcircuits as shown in Fig. 9. The calculation of the W axes follows

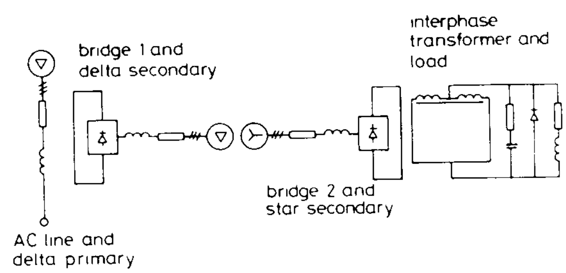

Fig. 9 *Subcircuits of 12-pulse convertor system*

the stages indicated in Section 3. Connections within the subcircuits are applied to obtain the $U1$ axes and then connections between the subcircuits are applied to obtain the $U2$ axes. Finally, the remaining star secondary windings constraint is applied to give W axes satisfying all the constraints. The use of subcircuits saves effort when other systems including similar subcircuits are specified.

Electrical connections within subcircuits may be specified by a choice of U_1 axes:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1 ACGEN1	1	0	−1																
2 ACGEN2	−1	1	0																
3 ACGEN3	0	−1	1																
4 PRIM1	1	0	0																
5 PRIM2	0	1	0																
6 PRIM3	0	0	1																

$C^c_{u1} =$

7 B1SEC1	−1	0	0	0	0	0										
8 B1SEC2	0	−1	0	0	0	0										
9 B1SEC3	0	0	−1	0	0	0										
10 B1IN1	1	0	−1	0	0	0										
11 B1IN2	−1	1	0	0	0	0										
12 B1IN3	0	−1	1	0	0	0										
13 B1D1	1	0	−1	1	0	0										
14 B1D2	−1	1	0	−1	1	0										
15 B1D3	0	−1	1	0	−1	1										
16 B1D4	0	0	0	1	0	0										
17 B1D5	0	0	0	−1	1	0										
18 B1D6	0	0	0	0	−1	1										
19 B2SEC1							−1	0	0	0	0	0				
20 B2SEC2							0	−1	0	0	0	0				
21 B2SEC3							0	0	−1	0	0	0				
22 B2IN1							1	0	0	0	0	0				
23 B2IN2							0	1	0	0	0	0				
24 B2IN3							0	0	1	0	0	0				
25 B2D1							1	0	0	1	0	0				
26 B2D2							0	1	0	−1	1	0				
27 B2D3							0	0	1	0	−1	1				
28 B2D4							0	0	0	1	0	0				
29 B2D5							0	0	0	−1	1	0				
30 B2D6							0	0	0	0	−1	1				
31 IPT1													1	0	0	0
32 IPT2													0	1	0	0
33 FILTER													1	1	−1	−1
34 FDIODE													0	0	−1	0
35 LOAD													0	0	0	1

The submatrices of C^c_{u1} correspond to subcircuits; all other entries are zero because connections between the subcircuits are not applied at this stage.

With this choice of $U1$ axes, the projections of the remaining constraints take the simple forms

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	26	0	0	26	0	0	0	0	15	0	0								
2	0	26	0	0	26	0	0	0	0	15	0								
3	0	0	26	0	0	26	0	0	0	0	15								
4									−1	0	0	0	0	0	0	1	0	0	0
5									0	0	0	0	0	0	−1	0	1	0	0
6									0	1	1	1	0	0	0	0	0	0	0

The first three constraints are the magnetic constraints connecting the transformer windings. (The transformer is assumed to have a fourth limb so that there are three independent flux paths through each triple of one primary and two secondary windings. The ideal magnetic connections may then be derived as in Section 3.) Constraints 4 and 5 connect the bridges to the interphase transformer and load. They express the equivalence of current in the DC output of each bridge to the current in the corresponding winding of the interphase transformer. Constraint 6 is the star connection of the secondary windings.

The first five projected constraints may be applied by choosing $U2$ axes:

$C^{u1}_{u2} =$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	−1	0	0				−15	0	0					
2	0	−1	0				0	−15	0					
3	0	0	−1				0	0	−15					
4	1	0	0											
5	0	1	0											
6	0	0	1											
7				1	0									
8				0	1									
9						1								
10						0	26	0	0					
11						0	0	26	0					
12						0	0	0	26					

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
13						0				1	0			
14						0				0	1			
15						0						1		
16						1						0		
17						0						1		
18						0							1	0
19						0							0	1

The remaining star constraint projected onto the $U2$ axes is

$$(0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 26\ \ 26\ \ 26\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0)$$

and is applied by choosing W axes 7 and 8 in terms of $U2$ axes 7, 8, 9 according to the C^{u2}_w submatrix

	7	8
7	1	0
8	0	1
9	−1	−1

and choosing W axes 1–6 and 9–13 respectively equal to $U2$ axes 1–6 and 10–14.

W axes representing the fixed circuit connections may be expressed in terms of the C axes by

$$C^c_w = C^c_{u1} C^{u1}_{u2} C^{u2}_w =$$

	1	2	3	4	5	6	7	8	9	10	11	12	13
1 ACGEN1	−1	0	1				−30	−15					
2 ACGEN2	1	−1	0				15	−15					
3 ACGEN3	0	1	−1				15	30					
4 PRIM1	−1	0	0				−15	0					
5 PRIM2	0	−1	0				0	−15					
6 PRIM3	0	0	−1				15	15					
7 B1SEC1	−1	0	0	0	0	0							
8 B1SEC2	0	−1	0	0	0	0							
9 B1SEC3	0	0	−1	0	0	0							
10 B1IN1	1	0	−1	0	0	0							
11 B1IN2	−1	1	0	0	0	0							
12 B1IN3	0	−1	1	0	0	0							
13 B1D1	1	0	−1	1	0	0							
14 B1D2	−1	1	0	−1	1	0							
15 B1D3	0	−1	1	0	−1	1							
16 B1D4	0	0	0	1	0	0							
17 B1D5	0	0	0	−1	1	0							
18 B1D6	0	0	0	0	−1	1							
19 B2SEC1							−26	0	0	0	0		
20 B2SEC2							0	−26	0	0	0		
21 B2SEC3							26	26	0	0	0		
22 B2IN1							26	0	0	0	0		
23 B2IN2							0	26	0	0	0		
24 B2IN3							−26	−26	0	0	0		
25 B2D1							26	0	1	0	0		
26 B2D2							0	26	−1	1	0		
27 B2D3							−26	−26	0	−1	1		
28 B2D4							0	0	1	0	0		
29 B2D5							0	0	−1	1	0		
30 B2D6							0	0	0	−1	1		
31 IPT1						1					0	0	0
32 IPT2						0					1	0	0
33 FILTER						1					1	−1	−1
34 FDIODE						0					0	−1	0
35 LOAD						0					0	0	1

8.2 Calculation of switching axes

Suppose the circuit switching state is known to be

0 1 0	0 1 0	
1 0 1	1 0 0	0
bridge1	bridge2	freewheel diode

Then the N axes are $\{c^{13}, c^{15}, c^{17}, c^{25}, c^{27}, c^{29}, c^{30}, c^{34}\}$ and the switching constraints projected onto the W' axes are the corresponding rows of the C^c_w matrix:

$C^n_w =$

	1	2	3	4	5	6	7	8	9	10	11	12	13
13 B1D1	1	0	−1	1	0	0							
15 B1D3	0	−1	1	0	−1	1							
17 B1D5	0	0	0	−1	1	0							
25 B2D1							26	0	1	0	0	0	0
27 B2D3							−26	−26	0	−1	1	0	0
29 B2D5							0	0	−1	1	0	0	0
30 B2D6							0	0	0	−1	1	0	0
34 FDIODE							0	0	0	0	0	−1	0

Eqn. 13 is then solved for C^w_z to determine the Z axes. As integer arithmetic is exact and fast on a computer, it is convenient to scale the V axes during the solution of eqn. 13 so that the calculation is performed in integers and yields an integer C^w_z matrix. The scaling is introduced into the calculation by placing scale factors along the diagonal of C^n_v and their reciprocals along the diagonal of C^v_n. In this example, V axes 9, 10, 11, 12 were scaled by a factor of 26 and the diagonal of C^n_v contains 1, 1, 1, 26, 26, 26, 26, 1.

$C^w_z =$

	← *X* axes				→	←			*V* axes				→
	1	2	3	4	5	6	7	8	9	10	11	12	13
1	0	−1		1		1	0	1					
2	1	−1		1		0	−1	0					
3	0	0		1		0	0	0					
4	0	1		0		0	0	−1					
5	0	1		0		0	0	0					
6	1	0		0		0	0	0					
7			1		0				1	0	1	1	0
8			−1		0				−1	−1	−1	0	0
9			−26		0				0	0	−26	−26	0
10			−26		0				0	0	0	−26	0
11			−26		0				0	0	0	0	0
12			0		0				0	0	0	0	−1
13			0		1				0	0	0	0	0

Hence the Z axes may be calculated relative to the C axes:

$C^c_z = C^c_w C^w_z =$

	← *X* axes				→	←			*V* axes				→
	1	2	3	4	5	6	7	8	9	10	11	12	13
1 ACGEN1	0	1	−15	0	0	−1	0	−1	−15	15	−15	−30	0
2 ACGEN2	−1	0	30	0	0	1	1	1	30	15	30	15	0
3 ACGEN3	1	−1	−15	0	0	0	−1	0	−15	−30	−15	15	0
4 PRIM1	0	1	−15	0	0	−1	0	−1	−15	0	−15	−15	0
5 PRIM2	−1	1	15	−1	0	0	1	0	15	15	15	0	0
6 PRIM3	0	0	0	−1	0	0	0	0	0	−15	0	15	0
7 B1SEC1	0	1	0	−1	0	−1	0	−1	0	0	0	0	0
8 B1SEC2	−1	1	0	−1	0	0	1	0	0	0	0	0	0
9 B1SEC3	0	0	0	−1	0	0	0	0	0	0	0	0	0
10 B1IN1	0	−1	0	0	0	1	0	1	0	0	0	0	0
11 B1IN2	1	0	0	0	0	−1	−1	−1	0	0	0	0	0
12 B1IN3	−1	1	0	0	0	0	1	0	0	0	0	0	0
13 B1D1	0	0	0	0	0	1	0	0	0	0	0	0	0
14 B1D2	1	0	0	0	0	−1	−1	0	0	0	0	0	0
15 B1D3	0	0	0	0	0	0	1	0	0	0	0	0	0
16 B1D4	0	1	0	0	0	0	0	−1	0	0	0	0	0
17 B1D5	0	0	0	0	0	0	0	1	0	0	0	0	0
18 B1D6	1	−1	0	0	0	0	0	0	0	0	0	0	0
19 B2SEC1	0	0	−26	0	0	0	0	0	−26	0	−26	−26	0
20 B2SEC2	0	0	26	0	0	0	0	0	26	26	26	0	0
21 B2SEC3	0	0	0	0	0	0	0	0	0	−26	0	26	0
22 B2IN1	0	0	26	0	0	0	0	0	26	0	26	26	0
23 B2IN2	0	0	−26	0	0	0	0	0	−26	−26	−26	0	0

	X axes					V axes							
	1	2	3	4	5	6	7	8	9	10	11	12	13
24 B2IN3	0	0	0	0	0	0	0	0	0	26	0	−26	0
25 B2D1	0	0	0	0	0	0	0	0	26	0	0	0	0
26 B2D2	0	0	−26	0	0	0	0	0	−26	−26	0	0	0
27 B2D3	0	0	0	0	0	0	0	0	0	26	0	0	0
28 B2D4	0	0	−26	0	0	0	0	0	0	0	−26	−26	0
29 B2D5	0	0	0	0	0	0	0	0	0	0	26	0	0
30 B2D6	0	0	0	0	0	0	0	0	0	0	0	26	0
31 IPT1	1	0	0	0	0	0	0	0	0	0	0	0	0
32 IPT2	0	0	−26	0	0	0	0	0	0	0	0	0	0
33 FILTER	1	0	−26	0	−1	0	0	0	0	0	0	0	1
34 FDIODE	0	0	0	0	0	0	0	0	0	0	0	0	1
35 LOAD	0	0	0	0	1	0	0	0	0	0	0	0	0

Examination of C_z^c shows that no X axis passes through an off diode and that each V axis passes through exactly one off diode.

9 General switching circuit simulation

9.1 Outline specification

The techniques described above may be used to design a simulation to solve the piecewise differential equations of a general switching circuit. The circuit may contain the following components:

(*a*) resistors, inductors, capacitors
(*b*) diodes, thyristors, switches
(*c*) voltage sources.

The circuit components are connected together by electrical and ideal magnetic connections. Mutual inductances may be specified. Diode or thyristor switchings are governed by general switching rules independent of the particular circuit arrangement or operational mode. The firing delay angles of thyristors may vary and switches may be opened or closed at preset times. The voltage sources may depend on time or circuit currents and may be used to model sinusoidal generators, batteries, nonlinear resistors, or arbitrary time or current dependent voltage sources.

The circuit components are input to the simulation as a list of coils and the electrical and ideal magnetic connections of the circuit are input as the integer connection matrix C_w^c. The switched circuit differential equations are automatically reformed at each switching using the switching axes theory of Section 6 and solved by numerical integration. The choice of integration methods includes Gear's method so that fast transients may be simulated [2] and switching points are located accurately by interpolation [2]. A complete description of the circuit state is output at each simulation time increment. When the program run is complete, an interactive output processing program is used to select, calculate and display the desired output quantities. The basic outputs are current and voltage waveforms from any part of the circuit and a record of the circuit switching states.

The main application of the simulation to AC/DC convertor systems is the detailed calculation of system behaviour when the switching processes are to be taken into account explicitly, particularly the determination of behaviour under transient conditions or with nonideal loads.

9.2 Assumptions and limitations

One of the main concerns when applying a simulation is that the system is represented by an equivalent circuit at an appropriate level of detail so that useful conclusions may be drawn from the results or understanding of the system may be gained. Most parts of the system are represented by simple lumped circuit linear components and the transformer connections and switchings are explicitly but simply represented. These assumptions are customary for conventional convertor calculations.

Transformer connections are assumed to be linear and are represented either ideally or by constant mutual inductances. The simple characterisation of diodes and thyristors as being either off with infinite impedance or on with the characteristics of the other components in the coil cannot take account of effects such as those caused by stored charge within the device. The crudity of the switching representation at the semiconductor level implies that the simulation may be properly applied only to systems in which semiconductor effects are of a higher order than the class of effects being studied. Detailed discussion of the applicability of the general switching rules is outside the scope of this paper, but it is clear that they may depend only on circuit effects derivable from a knowledge of y and py over the switching test interval.

The simulation design does not allow for current sources or variable inductances.

9.3 Implementation and application

The equations and simulation design considerations of this paper have been tested by writing the Connie switching circuit simulation [9, 10]. Connie has been used to analyse high-voltage supplies providing pulsed power for plasma physics fusion experiments at Culham Laboratory. These supplies are typically required to power nonstandard loads under rapidly changing conditions. It has been found that the level of modelling detail assumed in this paper is a useful one in the detailed analysis of these power supply systems. Applications to plasma heating power supplies have included calculation of the transient response of a 12-pulse convertor system module to load current interruptions and analysis of transient overvoltages in a Marx generator switched capacitative power supply. Connie has also proved useful in optimising the design of the switching circuits used to power the magnetic windings of fusion experiments. The method of analysis is clearly of use in fusion engineering and may well be applicable to switching circuits in other fields.

Connie is written in Fortran and uses National Algorithm Group library routines [11] for standard calculations such as numerical integration and interpolation. A typical solution rate for a 12-pulse convertor circuit is half a minute of central processor run time per supply cycle on a Prime 750 computer.

10 Conclusions

Tensor treatment of elementary network theory is clarified by the explicit use of both electrical axes and the duality of currents and voltages. Electrical axes systems may be chosen to represent circuit connections in the physically intuitive forms of constraints and degrees of freedom of circuit currents and voltages. In particular, electrical axes may be chosen to represent the electrical and transformer connections of a general AC/DC convertor system. The electrical and transformer connections are uniformly treated as linear constraints on circuit currents. Integer connection matrices specifying the electrical axes may be calculated systematically. Electrical axes analysis of convertor systems lends itself to description and visualisation of convertor system phenomena.

A general switching circuit simulation in which nonconducting diodes or thyristors are effectively deleted from the circuit has been designed and implemented using fixed and varying electrical axes to represent the fixed and varying circuit connections. The varying axes system is convenient for the expression and solution of the simulation equations and is recalculated automatically whenever the circuit switches. Tensor techniques have previously been used with advantage to simulate particular switching circuits; electrical axes allow these techniques to be uniformly applied to a wide range of switching circuits. The simulation is capable of analysing transients in 12-pulse AC/DC bridge convertor systems with nonideal loads.

11 Acknowledgments

This work was performed under contract M1A 54228 to Culham Laboratory (United Kingdom Atomic Energy Authority and Euratom/UKAEA Fusion Association). The author gratefully acknowledges the use of facilities at Culham Laboratory and the permission of the United Kingdom Atomic Energy Authority to publish this paper. Revision of this paper was supported in part by NSF under grant number ECS8352211. The author would like to thank Mr. J.W. Gray, Mr. J.H. Hay and Dr. J.B. Hicks of Culham Laboratory for their advice and support.

12 References

1 WILLIAMS, S., and SMITH, I.R.: 'Fast digital computation of 3-phase thyristor bridge circuits', *Proc. IEE*, 1973, **120**, (7), pp. 791–795

2 HICKS, J.B.: 'Prediction of the performance of synchronous-generator/bridge-convertor/ohmic heating circuit installations'. Proceedings of the Tenth Symposium on Fusion Technology, Padua, Italy, 1978, Vol. 2, pp. 371–376

3 KRON, G.: 'Tensor analysis of networks' (Wiley, 1939, reprinted by MacDonald, 1965)

4 KRON, G.: 'Tensors for circuits' (Dover, 1959)

5 DOBSON, I.: 'Geometric description of bridge rectifier operational modes using regular polygons', *IEE Proc. A*, 1987, **134**, (1), pp. 85–88

6 HIRSCH, M.W., and SMALE, S.: 'Differential equations, dynamical systems, and linear algebra' (Academic Press, 1974), Chap. 10, Section 5

7 NERING, E.D.: 'Linear algebra and matrix theory' (Wiley, 1970, 2nd edn.), Chap. 4, Section 3

8 DOBSON, I.: 'Calculating DC impedances equivalent to the supply impedances of bridge rectifier systems using regular polygons'. Reference EDD/ESG/83/N13, UKAEA Culham Laboratory, Abingdon, Oxon OX14 3DB, United Kingdom, November 1983

9 DOBSON, I.: 'CONNIE: a general AC-DC convertor system model; technical description'. Reference CLM-PDN 1/83, UKAEA Culham Laboratory, Abingdon, Oxon OX14 3DB, United Kingdom, November 1983

10 DOBSON, I.: 'CONNIE; a general AC-DC convertor system model; users guide'. Reference CLM-PDN 2/83, UKAEA Culham Laboratory, Abingdon, Oxon OX14 3DB, United Kingdom, November 1983

11 National Algorithm Group Library Manual, Mark 7

13 Appendix: Dependent switching constraints

The switching constraints becoming dependent when projected from the C' axes to the W' axes correspond to KVL not providing sufficient independent constraints to uniquely determine the voltages across off diodes. If, for example, a single branch of the circuit contains two off diodes in series, KVL specifies the total voltage across the diodes but not the sharing of this voltage between the diodes. Another example is when all the diodes of a 6-pulse bridge are off; there are six diode voltages to calculate and only five independent KVL constraints.

When dependent projected switching constraints arise, a subset of constraints with the maximum number of independent constraints is chosen. If the number of independent constraints is b, the last b Z' axes are chosen as the independent constraints. The Z axes may then be calculated according to eqn. 13; the properties of the resulting X axes are unaffected, but the V axes are related to the N axes by an $s \times b$ C_v^n matrix of the form

$$C_v^n = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ & & \vdots & & \\ 0 & 0 & 0 & \cdots & 1 \\ * & * & * & \cdots & * \\ & & \vdots & & \\ * & * & * & \cdots & * \end{bmatrix}$$

where the first b rows correspond to the independent subset of constraints and the last $s - b$ rows are not filled with zeros. e_v^O may be calculated from eqn. 14a, but the form of C_v^n shows that e_v^O does not uniquely specify the required off diode voltages e_n^O. The calculation requires further, arbitrary constraints on e_n^O to be assumed. The further constraints must be chosen arbitrarily at this level of modelling detail because the physical principles determining the further constraints are not represented in the model.