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Combining Phasor Measurements to Monitor Cutset Angles

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Abstract

Power systems under stress can show large voltage angle differences between areas that can be monitored by wide area phasor measurements. One way to make this idea more specific is to choose a cutset of transmission lines that separates two power system areas and then define an angle difference across the cutset that is a suitable combination of the angle differences across lines of the cutset. We suggest that monitoring this cutset angle yields useful and specific information about power system stress.

1 Introduction

The voltage phasor angle difference between two ends of a transmission line becomes large when the line power flow is large or the line impedance is large. Similar relationships are expected to apply to the angle difference between two buses in different areas of a power grid. That is, a large angle difference indicates, in some general sense, a stressed power system with large power flows or increased impedance between the areas. Simulations of the grid before the August 2003 Northeastern blackout show increasing angle differences between Cleveland and West Michigan, suggesting that large angle differences could be a blackout risk precursor [2]. Wide area nomograms involving linear combinations of phasor angles have been suggested for monitoring of security boundaries [5]. A recent simulation study [7] of potential phasor measurements on the 39 bus New England test system shows that, of several phasor measurements, angle differences were the best in discriminating alert and emergency states. The increasing deployment of wide area measurement of phasor angles [3] spurs interest in finding ways to implement the general idea of using phasor angles to determine system stress.

Picking one bus in each of two areas and monitoring their angle difference has a problem that, although the angle difference is generally expected to increase with system stress, many factors influence the angle difference, including which two buses are chosen and the local power flows within each area. It is then harder to give a specific meaning to the angle difference and specify threshold values that indicate when the angle difference becomes dangerously large. This paper shows a way to combine multiple angle difference measurements to obtain a cutset angle that has a more specific meaning.

In section 2 we assume a DC load flow model of the power system and define the angle across a given cutset. Then the monitoring of this cutset angle from measurements is explained and illustrated. Section 3 expresses the cutset angle in terms of standard network matrices. Since the previous sections have assumed phasor measurements available at all buses, section 4 shows how the monitoring may be done in a network with fewer phasor measurements by using a standard network reduction. Section 5 tests the method on simulated measurements obtained from an AC model of the New England 39 bus test system. Section 6 shows the connections to classical circuit theory and section 7 concludes the paper.

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2 Angle across a cutset

This section defines the angle across the cutset and the cutset susceptance, explains the monitoring of the cutset angle, and considers how the monitored cutset angle changes when changes occur in the grid.

2.1 Definition of cutset angle

Consider a power grid with the DC load flow approximation. The structure and impedances of the base case grid are assumed to be known. First we assume that the voltage phasor angle at every bus is measured; the more practical case of fewer measurements is considered in section 4.

Write θ_i for the voltage angle at bus number i and $\hat{\theta}_j$ for the angle difference across line number j. The susceptance of line number j is b_j.

Choose a cutset of lines c that divides the network into area 1 and area 2. The cutset c need not be a minimal cutset. Any cutset c can be chosen but the cutset is fixed throughout the following discussion. The power flowing from area 1 to area 2 along line j of the cutset is $b_j\hat{\theta}_j$. Here, it is convenient initially to assume that the angle difference $\hat{\theta}_j$ on line j is defined so that $\hat{\theta}_j$ is positive for positive power flowing on line j from area 1 to area 2. The power P_c flowing through the cutset c is the sum of the powers flowing in each line of the cutset:

$$P_c = \sum_{j \in c} b_j \hat{\theta}_j \tag{1}$$

The cutset susceptance is

$$b_c = \sum_{j \in c} b_j \tag{2}$$

We define the angle across the cutset as

$$\hat{\theta}_c = \sum_{j \in c} \frac{b_j}{b_c} \hat{\theta}_j \tag{3}$$

which is a linear combination of the cutset line angle differences, weighted according to the line susceptances. Then (1), (2) and (3) imply that

$$P_c = b_c \hat{\theta}_c, \tag{4}$$

which expresses the power flowing through the cutset as the product of the cutset susceptance and the angle across the cutset.

2.2 Monitoring cutset angle

Now we discuss how we propose to monitor the angle across the cutset. The following quantities are assumed to be available:

- The susceptances of the lines of the cutset for a fixed base case DC load flow model of the grid. These line susceptances are denoted $\{b_j^0 \mid j \in c\}$. Then the base case cutset susceptance is

$$b_c^0 = \sum_{j \in c} b_j^0 \tag{5}$$

  Note that the base case line susceptances $\{b_j^0 \mid j \in c\}$ may be different than the susceptances of the lines of the cutset for the currently observed grid.

- The voltage angles of the buses incident on lines in the cutset. (Recall that this section assumes that all voltage angles are available from phasor measurements of the currently observed grid.) Then the voltage angles across the each of the lines in the cutset $\{\hat{\theta}_j \mid j \in c\}$ can be computed from these measurements.

Now, following (3),

$$\hat{\theta}_c^m = \sum_{j \in c} \frac{b_j^0}{b_c^0} \hat{\theta}_j \tag{6}$$

is used to compute and monitor the angle across the cutset. Thus $\hat{\theta}_c^m$ is computed from the base case DC load flow line susceptances and the voltage angles across the each of the lines in the cutset obtained from the measurements. In the case that the cutset line susceptances remain fixed at their base case values, and so $b_c = b_c^0$, then the monitored cutset angle $\hat{\theta}_c^m$ satisfies

$$P_c = b_c \hat{\theta}_c^m. \tag{7}$$

In fact, (7) holds under the weaker assumption that the ratios of cutset line susceptances remain fixed.

Table 1. Cutset susceptances of base case

cutset line	susceptance
14—15	46.08
16—17	112.36
26—28	21.10
26—29	16.00
$b_c^0 = 195.5$	

For example, consider the 39 bus New England test system shown in Figure 1. The 4 lines in the chosen cutset are shown by the thicker dashed lines and their susceptances in the base case power grid model are shown in Table 1. Adding the 4 lines susceptances gives the base case cutset susceptance $b_c^0 = 195.5$. The base case power flow from area 1 to area 2 across the cutset is $P_c^0 = -4.83$ per unit. The voltage angles at buses 14, 15, 16, 17, 26, 28, 29 are measured and used to compute the angle differences across the 4 cutset lines. Then (6) and the susceptances in Table 1 are used to compute the base case cutset angle $\hat{\theta}_c^0 = -1.4$ degree.

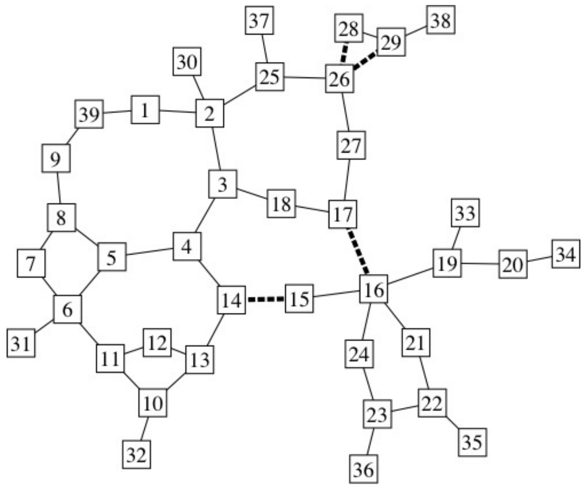

Figure 1. New England 39 bus test system. Cutset lines are shown by the thicker dashed lines. The cutset separates area 1 (on the left) from area 2 (upper right and lower right).

2.3 Effect of changing power injections

Now we consider how $\hat{\theta}_c^m$ computed with (6) behaves when power injections are changed.

Consider a power injection in area 1 and an equal load increase in area 2. Then P_c increases by the amount of the power injection, the cutset line susceptances and b_c are unchanged, and $\hat{\theta}_c^m$ increases proportionally to P_c. For example, in the 39 bus test system, increasing generation at bus 39 by 2.42 and load at bus 21 by 2.42. changes the base case power flowing across the cutset from $P_c^0 = -4.83$ to $P_c = -2.42$ so that $\hat{\theta}_c^m$ is correspondingly halved from $\hat{\theta}_c^0 = -1.4$ degree to become $\hat{\theta}_c^m = -0.7$ degree.

Now consider a power injection in area 1 and an equal load increase in area 1. The cutset power flow P_c is unchanged, the cutset line susceptances and b_c are unchanged, and $\hat{\theta}_c^m$ is unchanged. There is similarly no change for power dispatched entirely within area 2. Power redispatches entirely within a single area can change the angles across particular lines in the cutset but do not change $\hat{\theta}_c^m$.

2.4 Effect of changing line susceptances

Suppose that power injections are constant but that the susceptance of a line not in the cutset changes. The change in line susceptance could result from change in the circuit linearization as loading changes or from the line tripping and the line susceptance changing to zero. One special case occurs when the line is tripped and islands generation or load. In this special case, $\hat{\theta}_c^m$ changes because the effective power injection in the islanded area changes. This special case can be treated as in subsection 2.3. However, if there is no such islanding, the area power injections and the power P_c flowing through cutset are unchanged. The cutset line susceptances and b_c are also unchanged, and it follows from (7) that $\hat{\theta}_c^m$ is unchanged.

Now we consider the case of the susceptance of a line in the cutset changing. (We exclude the special case of islanding.) It is convenient in order to simplify notation to suppose that the line in the cutset that changes susceptance is line 1 of the grid. The base case line 1 susceptance is b_1^0 and the susceptance of line 1 changes to b_1. The susceptance of all the other lines is unchanged so that $b_j = b_j^0$ for $j \neq 1$.

The cutset angle after the susceptance of line 1 changes is

$$\hat{\theta}_c = \sum_{j \in c} \frac{b_j}{b_c} \hat{\theta}_j = \frac{b_1}{b_c} \hat{\theta}_1 + \sum_{\substack{j \in c \\ j \neq 1}} \frac{b_j}{b_c} \hat{\theta}_j \tag{8}$$

The measured cutset angle after the susceptance of line 1 changes is

$$\hat{\theta}_c^m = \sum_{j \in c} \frac{b_j^0}{b_c^0} \hat{\theta}_j = \frac{b_1^0}{b_c^0} \hat{\theta}_1 + \sum_{\substack{j \in c \\ j \neq 1}} \frac{b_j}{b_c^0} \hat{\theta}_j \tag{9}$$

Since the power flow through the cutset is unchanged,

$$b_c^0 \hat{\theta}_c^0 = P_c^0 = P_c = b_c \hat{\theta}_c \tag{10}$$

Combining (8), (9), and (10) yields an expression for the change in the measured cutset angle

$$\hat{\theta}_c^m - \hat{\theta}_c^0 = \frac{b_1^0 - b_1}{b_c^0} \hat{\theta}_1 \tag{11}$$

The change in the measured cutset angle is proportional to the change in admittance $b_1^0 - b_1$ and to the final angle across line 1.

For example, suppose that the line in the cutset joining buses 14 and 15 is tripped so that its susceptance changes from 46.08 to zero. Then the monitored cutset angle changes from the base case value of $\hat{\theta}_c^0 = -1.4$ degree to $\hat{\theta}_c^m = -1.0$ degree. The dependence of the cutset angle $\hat{\theta}_c$ on the cutset susceptance when the power flows do not change shows that the cutset angle includes information about grid impedances not detectable from power flow information.

In summary, monitoring the cutset angle $\hat{\theta}_c^m$ detects changes in power flow through the cutset and changes in the cutset susceptances, but $\hat{\theta}_c^m$ is unchanged by dispatch or susceptance changes within one of the areas. Thus monitoring $\hat{\theta}_c^m$ yields specific information about changes to the grid with respect to the chosen cutset.

3 Formulation of cutset angle using network matrices

It is useful to express the cutset angle, susceptance and power flow in terms of standard network matrices. For example, this allows the cutset angle to be computed from the B matrix of the DC load flow. The formulation also allows arbitrary orientation of the cutset lines.

Let θ be the vector of bus angles and P be the vector of bus power injections. The DC load flow equations of the base case grid are

$$P = B\theta \tag{12}$$

where

$$B = A\Lambda A^T \tag{13}$$

and Λ is the diagonal matrix of line susceptances

$$\Lambda = \text{diag}\{b_1, b_2, \cdots, b_{\text{nline}}\}$$

and A is the incidence matrix

$$A_{ij} = \begin{cases} 1 & \text{bus } i \text{ is sending end of line } j \\ -1 & \text{bus } i \text{ is receiving end of line } j \\ 0 & \text{otherwise.} \end{cases} \tag{14}$$

In the DC load flow equations (12), it is convenient *not* to delete the rows of the power and angle vectors corresponding to a slack bus and *not* to delete the corresponding row and column of B [4].

The incidence matrix A relates the bus angles θ to the line angle differences $\hat{\theta}$:

$$\hat{\theta} = A^T\theta \tag{15}$$

The row vector σ defines the buses in area 1 by

$$\sigma_i = \begin{cases} 1 & \text{bus } i \text{ in area 1} \\ 0 & \text{bus } i \text{ in area 2.} \end{cases} \tag{16}$$

Then the lines in the cutset c can be indicated by the row vector

$$c = \sigma A \tag{17}$$

since then it follows from (14) that

$$c_j = \begin{cases} 1 & \text{line } j \text{ in cutset has sending end} \\ & \text{ in area 1} \\ -1 & \text{line } j \text{ in cutset has receiving end} \\ & \text{ in area 1} \\ 0 & \text{line } j \text{ not in cutset.} \end{cases} \tag{18}$$

Moreover,

$$P_c = \sigma P \tag{19}$$

is both the sum of the powers injected in area 1 and the power that flows from area 1 to area 2 through the cutset c.

Applying σ to the DC load flow equations (12) gives

$$P_c = \sigma P = \sigma B\theta \tag{20}$$

which can be rewritten as

$$P_c = \sigma A\Lambda A^T\theta = c\Lambda\hat{\theta} \tag{21}$$

which can be recognized as the matrix form of (1). Note how the row vector $c\Lambda$ contains the susceptances in the cutset:

$$(c\Lambda)_j = \begin{cases} b_j & \text{line } j \text{ in cutset has sending} \\ & \text{ end in area 1} \\ -b_j & \text{line } j \text{ in cutset has receiving} \\ & \text{ end in area 1} \\ 0 & \text{line } j \text{ not in cutset.} \end{cases} \tag{22}$$

Each cutset line susceptance appears once in $c\Lambda$, with sign depending on the orientation of the cutset line. Now (18) and (22) imply that

$$c\Lambda c^T = \sum_{j\in c} b_j = b_c \tag{23}$$

Therefore the cutset impedance b_c defined in (2) can be re-expressed as

$$b_c = c\Lambda c^T = \sigma B\sigma^T \tag{24}$$

Moreover, the cutset angle difference $\hat{\theta}_c$ defined in (3) can be re-expressed as

$$\hat{\theta}_c = \frac{c\Lambda}{b_c}\hat{\theta} \tag{25}$$

or

$$\hat{\theta}_c = \frac{\sigma B\,\theta}{\sigma B\sigma^T} \tag{26}$$

The derivation starting from the DC load flow equations (12) can now be summarized: Multiply (12) on the left by σ to obtain

$$P_c = \sigma P = \sigma B\theta = \sigma B\sigma^T\frac{\sigma B\,\theta}{\sigma B\sigma^T} = b_c\hat{\theta}_c \tag{27}$$

The formula for the monitored cutset angle becomes

$$\hat{\theta}_c^m = \frac{\sigma B^0\,\theta}{\sigma B^0\sigma^T}, \tag{28}$$

where B^0 is the matrix for the base case DC load flow and θ contains the measured angles.

4 Fewer phasor measurements

Application of the method above to monitor the angle across a given cutset requires phasor angle measurements on every bus incident on a cutset line. Since this is restrictive, we show how a standard reduction of the network can allow the method to be applied to the practical case of a grid with phasor angle measurements on a given subset of buses.

4.1 Method with fewer measurements

Write θ_m and P_m for the angle and power injected at buses with phasor measurements and $\theta_{\overline{m}}$ and $P_{\overline{m}}$ for the angle and power injected at buses with no phasor measurements. Order the buses so that the measured buses come first and

$$\theta = \begin{pmatrix} \theta_m \\ \theta_{\overline{m}} \end{pmatrix}, \quad P = \begin{pmatrix} P_m \\ P_{\overline{m}} \end{pmatrix},$$

and the DC load flow equations (12) become

$$\begin{pmatrix} P_m \\ P_{\overline{m}} \end{pmatrix} = \begin{pmatrix} B_{mm} & B_{m\overline{m}} \\ B_{\overline{m}m} & B_{\overline{mm}} \end{pmatrix} \begin{pmatrix} \theta_m \\ \theta_{\overline{m}} \end{pmatrix}. \tag{29}$$

Eliminating $\theta_{\overline{m}}$ in the usual way gives

$$P_m - B_{m\overline{m}} B_{\overline{mm}}^{-1} P_{\overline{m}} = (B_{mm} - B_{m\overline{m}} B_{\overline{mm}}^{-1} B_{\overline{m}m})\theta_m$$

and, letting

$$P_{eq} = P_m - B_{m\overline{m}} B_{\overline{mm}}^{-1} P_{\overline{m}} \tag{30}$$

$$B_{eq} = B_{mm} - B_{m\overline{m}} B_{\overline{mm}}^{-1} B_{\overline{m}m} \tag{31}$$

we obtain an equivalent grid connecting the buses with measurements with the DC load flow equations

$$P_{eq} = B_{eq}\theta_m \tag{32}$$

to which the preceding computations can be applied.

In particular, we first choose a cutset c of the equivalent grid by specifying the area 1 measured buses with σ_{eq} and then monitor the following equivalent cutset angle difference:

$$\hat{\theta}_c^m = \frac{\sigma_{eq} B_{eq}^0 \theta_m}{\sigma_{eq} B_{eq}^0 \sigma_{eq}^T} \tag{33}$$

For example, suppose for the sake of illustration that there are phasor measurements available only at odd numbered buses of the 39 bus New England system. Applying the network reduction yields the equivalent network shown in Figure 2. The reduced grid has more equivalent lines joining the measured nodes and correspondingly large cutsets. For example, the cutset of 4 lines in the 39 bus New England system shown in Figure 1 corresponds to a cutset of 9 lines in the reduced odd numbered bus New England system shown in Figure 2. The susceptances of the 9 lines in the cutset of the reduced system are shown in Table 2.

Although the cutsets of the full and reduced systems separate their respective power grids in a roughly corresponding way, it is important to note that that these cutsets differ in their susceptance, the power flowing through them, and the angle across them. This is caused by the cutset lines of the reduced system accounting not only for the lines of the unreduced cutset but also a portion of the grid adjacent to the lines of the unreduced cutset. Power injections at even numbered nodes near the cutset are accounted for differently in the full and reduced networks and this accounts for the difference in power flow through the cutsets.

Table 2. Cutset susceptances of reduced network

cutset line	susceptance
3—15	4.290
5—15	7.140
13—15	23.647
17—15	32.433
17—19	15.634
17—21	22.583
17—23	7.454
25—29	7.563
27—29	16.619
$b_{ceq}^0 = 137.36$	

4.2 Relation of reduced cutset to original grid

We know from sections 2.3 and 2.4 that the angle across the cutset in the reduced system is a function of the power flow across the reduced cutset and the susceptances of the cutset lines in the reduced grid. But how do the susceptances of the cutset lines in the reduced grid depend on the susceptances of lines in the original grid? This section shows which lines in the original grid impact the susceptances of the cutset lines in the reduced grid and hence the reduced cutset angle.

In any particular case, the lines present in a network can easily be determined from the B matrix since

$$\text{a line connects bus } i \text{ to bus } j \iff B_{ij} \neq 0.$$

Therefore it is straightforward to determine the lines of the reduced grid from B_{eq} after B_{eq} is computed using (31). However, we need to examine this more closely in order to understand how the cutset lines of the reduced grid and their susceptances depend on the the lines of the original grid and their susceptances.

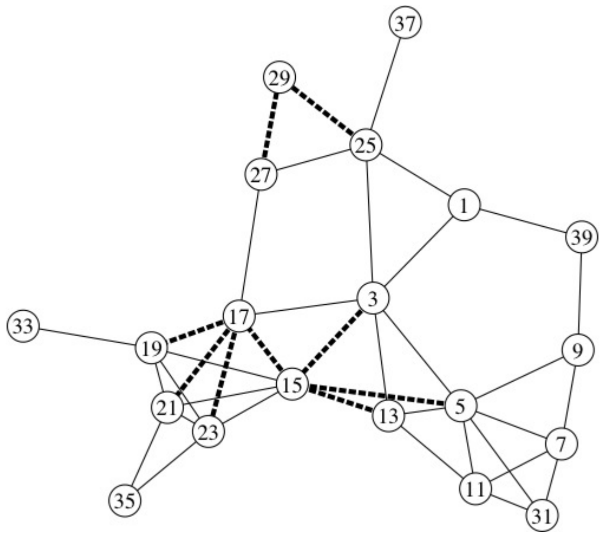

Figure 2. Reduction of the New England 39 bus test system to the odd numbered buses. Cutset lines are shown by the thicker dashed lines.

Consider any line of the reduced grid that connects odd numbered bus i to odd numbered bus j, where i and j are different. The susceptance $(B_{eq})_{ij}$ of this line in the reduced grid arises from either or both of the two terms in

$$B_{eq} = B_{mm} - B_{m\overline{m}} B_{\overline{mm}}^{-1} B_{\overline{m}m} \tag{34}$$

In the first term, $(B_{mm})_{ij} \neq 0$ precisely when there is a line joining bus i to bus j in the unreduced grid. Now we consider when the second term of (34) has a nonzero matrix entry.

The nonzero elements of the ith row of $B_{m\overline{m}}$ correspond to the even numbered buses which are connected by a line to bus i. If we call all the buses connected to bus i by a line the neighbors of i, then the nonzero elements of the ith row of $B_{m\overline{m}}$ correspond to the even numbered neighbors of bus i. Similarly, the nonzero elements of the jth column of $B_{\overline{m}m}$ correspond to the even numbered neighbors of bus j. $B_{\overline{mm}}$ is the B matrix of the even numbered buses only. The even numbered buses form a subnetwork of the grid that generally consists of several connected components. Each component consists of the even numbered buses that are connected by lines to other even numbered buses without passing through an odd numbered bus. It follows that $B_{\overline{mm}}^{-1}$ has a block matrix structure with the blocks corresponding to the connected components. Each block of $B_{\overline{mm}}^{-1}$ will generically be a full matrix with no zeros. Then we can see that the i, j element of the second term of (34) is generally nonzero when bus i has at least one even numbered neighbor in the same component as an even numbered neighbor of bus j. That is, the i, j element of the second term is nonzero when there is a path of multiple lines from bus i to bus j in which all the intermediate buses are even numbered.

The susceptance of the line in the reduced grid joining i and j is the negative of $(B_{eq})_{ij}$. This susceptance depends on the susceptance of the line joining i and j in the unreduced grid (if present) and the susceptances of the lines joining i and j to their even numbered neighbors and the susceptances of the lines in the component that joins these neighbors. One consequence is that the susceptance of the reduced grid cutset depends on not only on the lines of the unreduced cutset but some adjacent lines too.

All the lines of the unreduced grid whose susceptance contribute to the cutset susceptance are shown as thick or thin dashed lines in Figure 3. The dashed lines can be obtained visually starting from the cutset of the unreduced system by "moving" that cutset "over" any even number buses, but not allowing the cutset to move over any odd numbered buses. Then the dashed lines are formed as the union of all the lines included in one of the moving cutsets.

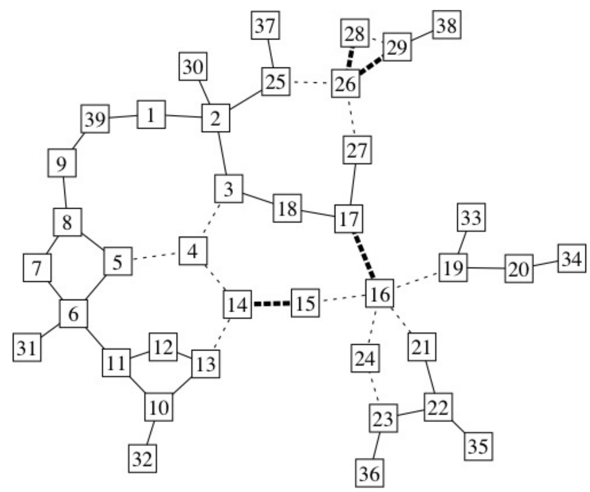

Figure 3. New England 39 bus test system with both thick and thin dashed lines showing the lines that affect the reduced grid cutset susceptance. The lines in the original cutset are shown by the thicker dashed lines and the additional lines affecting the reduced grid cutset susceptance are shown with thin dashed lines.

5 Testing on an AC system

This section tests the monitoring of cutset angle $\hat{\theta}_c^m$ on an AC model of the New England 39 bus system. The case considered is the same as in the case at the end of section 4. In particular, it is assumed for purpose of illustration that there are phasor measurements only at the odd numbered buses and the same cutset of 9 lines of the reduced system is chosen as shown in Figure 2 and Table 2. The goal of the testing is to determine whether the dependencies of the cutset angle on power injections and line tripping that are exact in the DC load flow model remain approximately true for the monitored cutset angle in the AC load flow model.

For each case considered, to evaluate the monitored cutset angle $\hat{\theta}_c^m$, the voltage angles at the odd numbered buses are computed from the AC model of the New England 39 bus system and used to obtain the angle differences across lines in the cutset. Then the angle differences are combined according to (33) using the cutset line susceptances of the reduced DC network shown in Table 2.

For the base case, the power flow through the cutset of the reduced system is $P_c^0 = -3.57$ per unit and the cutset angle is $\hat{\theta}_c^{m0} = -2.60$ degree. As discussed in the preceding section, the power flow and angle for the cutset of the reduced system are different than the power flow and angle for the cutset of the unreduced system that separates the grid in a roughly similar way.

5.1 Effect of changing power injections

We tested how $\hat{\theta}_c^m$ computed with (33) behaves when power injections are changed. We confirmed in several cases that if the power injections do not change the power flow across the cutset of the reduced system, then there are only very small changes in $\hat{\theta}_c^m$. This testing considered only power injections at odd and even numbered nodes that were clearly in area 1 or area 2 of the reduced system. We do not yet understand how power injections at the odd and even numbered buses near the cutset should affect $\hat{\theta}_c^m$.

5.2 Effect of changing line susceptances

We tested how $\hat{\theta}_c^m$ computed with (33) behaves when a sample of lines were tripped. When lines were tripped that islanded load or generation, this changes the power flow through the cutset and hence changes $\hat{\theta}_c^m$. When lines that were clearly in area 1 or 2 of the reduced system were tripped and there was no islanding, then we confirmed that $\hat{\theta}_c^m$ remained within 0.1 degree of $\hat{\theta}_c^{m0} = -2.60$ degree. However, when lines of the unreduced system involved in the cutset of the reduced system were tripped, $\hat{\theta}_c^m$ changed and generally increased. The increase in $\hat{\theta}_c^m$ when a cutset line is tripped can be attributed to the decreased cutset susceptance.

We conclude that, if islanding effects are excluded, the monitored cutset angle in the AC power flow remains nearly constant when lines not involved in the cutset of the reduced system are tripped. The monitored cutset angle in the AC power flow can and usually does change when lines involved in the cutset of the reduced system are tripped.

6 Cutset angle in classical network theory

The cutset angle $\hat{\theta}_c$ can be derived as an instance of classical circuit theory using a nonstandard choice of basis. Wai-Kai Chen in [1] explains a generalized cutset analysis[1] of a resistive network with voltage and current sources. Chen's analysis has different "through" and "across" circuit quantities than ours; that is, Chen's voltages correspond to our angles, Chen's currents correspond to our power flows, and Chen's admittances correspond to our susceptances. We now state Chen's equations rewritten in terms of our quantities. Let the rows of the matrix Q specify a basis for the cutset vector space of the network. Write $\hat{\theta}_q$ for the (generalized) cutset angles, P_q for the (generalized) cutset power flows, B_q for the (generalized) cutset admittance matrix, and P_b for the branch power flows. Then

$$\hat{\theta} = Q^T \hat{\theta}_q \tag{35}$$

$$P_q = Q P_b \tag{36}$$

$$B_q = Q \Lambda Q^T \tag{37}$$

Then, in the generalized cutset coordinates,

$$P_q = B_q \hat{\theta}_q, \tag{38}$$

which is essentially Chen's equation (2.81).

Now we make a special choice of the basis for the cutset vector space of the network by choosing the first row of the Q matrix to be the cutset c so that

$$Q = \left(\frac{c}{Q_\perp} \right) \tag{39}$$

and so that the remaining rows $Q_\perp$ of Q are basis vectors of the cutset vector space chosen orthogonal to c in the sense that $c \Lambda Q_\perp^T = 0$. (It may not be possible to choose all rows of $Q_\perp$ to consist of vectors with entries ± 1 and zero corresponding to the usual cutsets, but

[1] in [1] a nonminimal or minimal cutset is called a cut

this causes no fundamental difficulty.) Then in the basis (39), multiplying (35) on the left with $c\Lambda$ gives

$$c\Lambda\hat{\theta} = c\Lambda Q^T\hat{\theta}_q = c\Lambda c^T\hat{\theta}_{q1} = b_c\,\hat{\theta}_{q1}, \tag{40}$$

where $\hat{\theta}_{q1}$, the first component of $\hat{\theta}_q$, is the generalized cutset coordinate associated with c. Comparison of (40) with (25) shows that $\hat{\theta}_c = \hat{\theta}_{q1}$ is the generalized cutset coordinate associated with c in the basis (39). Moreover, in the basis (39),

$$B_q = Q\Lambda Q^T = \left(\begin{array}{c|c} c\Lambda c^T & 0 \\ \hline 0 & Q_\perp\Lambda Q_\perp^T \end{array}\right) = \left(\begin{array}{c|c} b_c & 0 \\ \hline 0 & Q_\perp\Lambda Q_\perp^T \end{array}\right)$$

and the first components of (36) and (38) become, respectively,

$$P_{q1} = cP_b = \sigma AP_b = \sigma P = P_c \qquad \text{and}$$
$$P_{q1} = b_c\hat{\theta}_{q1}.$$

Hence the first component of (38) may be written as $P_c = b_c\hat{\theta}_c$, which is (27). Thus we have found a nonstandard cutset basis (39) including c in which the cutset angle $\hat{\theta}_c$ is the generalized cutset coordinate associated with the basis element c.

7 Conclusion

We suggest monitoring combinations of phasor angle measurements that indicate the power system stress relative to a given cutset that separates two areas of the power system.

We formulate a concept of cutset angle in the context of a DC load flow model that can be calculated from the phasor measurements of angle differences across lines in the cutset. The cutset angle gives information about the power flows and impedances of the cutset. In particular, the cutset angle is proportional to the power flow between the areas and also depends on the susceptances of cutset lines and in particular on whether cutset lines have tripped. The cutset angle is insensitive to changes in power flow within one of the areas or line tripping within one of the areas.

In this paper we develop the cutset angle concept from scratch and also show how it results from a nonstandard choice of basis in general circuit theory. The cutset angle is simple to define and intuitive, so that it seems that it should have been defined or applied before. However, our searches have not yet found any reference to cutset angle (or analogs involving other “across” variables such as cutset voltage) in the circuit literature. This literature is vast, so we will continue to search to find out whether cutset angle is a new concept. We would welcome any advice.

The approach generalizes to the practical case of measurement of phasor angles at a subset of grid nodes by applying the method to a reduced power grid. Initial testing on AC load flows of the 39 bus New England IEEE test system suggests that the monitored cutset angle approximately preserves its properties of giving information about the power flows and impedances of the lines that play a role in the cutset of the reduced power grid.

The value of monitoring the cutset angle is that it gives specific information about power flows and impedances related to a given cutset separating two areas of the power grid. Moreover, the fact that the cutset angle is a meaningful quantity in circuit theory makes it likely to be a more useful quantity to monitor than an arbitrary combination of angles. The cutset angle augments the usual notions of power flow between two areas with information about the cutset angle and impedance.

Monitoring the cutset angle can be compared to monitoring an angle difference between two buses. Both of these approaches monitor scalars. The cutset angle computation is slightly more complicated and requires some weights obtained from a DC load flow model. The cutset angle gives specific information related to the cutset whereas the angle difference between two buses depends on many factors.

Now we conclude the paper by briefly suggesting some possible future directions.

We would like to test the cutset angle monitoring on a larger, less reduced power network and find out how well it can work on cutsets corresponding to known critical corridors. It could be beneficial to vary the choice of cutset or add phasor measurement at a few key locations. We do not yet have strategies to choose practical cutsets that give the most useful information.

Since the cutset angle gives specific information, it should be easier to interpret changes in cutset angle and determine thresholds of power system stress in terms of cutset angle. For example, one could choose a stressed, but operable case and determine the cutset angles corresponding to a contingency list of dangerous line trippings in the cutset. If the contingencies are comprehensive and all considered serious, then the minimum cutset angle for these contingencies gives a plausible threshold for an alarm.

It may also be feasible to more precisely confirm line trippings by checking changes in the cutset angle. A general approach to detecting line trippings from discrete changes in phasor measurements such as [6] generates lines that are likely to have tripped. A cutset containing some of these lines could be chosen and the changes in its cutset angle could be computed. Since there are fewer combinations of lines in and out in the cutset than in the entire system and hence fewer possible discrete changes in the cutset angle to consider, it should be possible to more easily confirm that the cutset angle change corresponds to a particular cutset line tripping.

Monitoring the cutset angle can be compared to monitoring a vector of angle differences between many pairs of buses. The cutset angle is a scalar giving specific information related to the cutset chosen. The cutset angle essentially condenses specific information from the vector of angle differences. There is, of course, more information in the vector of angle differences, but it may be harder to interpret. One interesting possibility, instead of monitoring a vector of angle differences, is to choose several key cutsets and monitor a vector of cutset angles to get specific information related to these cutsets.

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