A criticality approach to monitoring cascading failure risk and failure propagation in transmission systems

Ian Dobson, Benjamin A. Carreras, David E. Newman

***Abstract*— We consider the risk of cascading failure of electric power transmission systems as overall loading is increased. There is evidence from both abstract and power systems models of cascading failure that there is a critical loading at which the risk of cascading failure sharply increases. Moreover, as expected in a phase transition, at the critical loading there is a power tail in the probability distribution of blackout size. (This power tail is consistent with the empirical distribution of North American blackout sizes.) The importance of the critical loading is that it gives a reference point for determining the risk of cascading failure. Indeed the risk of cascading failure can be quantified and monitored by finding the closeness to the critical loading. This paper suggests and outlines ways of detecting the closeness to criticality from data produced from a generic blackout model. The increasing expected blackout size at criticality can be detected by computing expected blackout size at various loadings. Another approach uses branching process models of cascading failure to interpret the closeness to the critical loading in terms of a failure propagation parameter λ. We suggest a statistic for λ that could be applied before saturation occurs. The paper concludes with suggestions for a wider research agenda for measuring the closeness to criticality of a fixed power transmission network and for studying the complex dynamics governing the slow evolution of a transmission network.**

***Index Terms*— blackouts, power system security, stochastic processes, branching process, cascading failure, reliability, risk analysis, complex system, phase transition.**

I. INTRODUCTION

Cascading failure is the usual mechanism for large blackouts of electric power transmission systems. For example, long, intricate cascades of events caused the August 1996 blackout in Northwestern America that disconnected 30,390 MW to 7.5 million customers [29], [28], [39]) and the August 2003 blackout in Northeastern America that disconnected 61,800 MW to an area containing 50 million people [38]. The vital importance of the electrical infrastructure to society motivates the understanding and analysis of large blackouts.

Electric power transmission systems are complex networks of large numbers of components that interact in diverse ways. When component operating limits are exceeded, protection acts and the component "fails" in the sense of not being available to transmit power. Components can also fail in the sense of misoperation or damage due to aging, fire, weather, poor maintenance or incorrect settings. In any case, the failure causes a transient and causes the power flow in the component to be redistributed to other components according to circuit laws, and subsequently redistributed according to automatic and manual control actions. The transients and readjustments of the system can be local in effect or can involve components far away, so that a component disconnection or failure can effectively increase the loading of many other components throughout the network. In particular, the propagation of failures is not limited to adjacent network components. The interactions involved are diverse and include deviations in power flows, frequency, and voltage as well as operation or misoperation of protection devices, controls, operator procedures and monitoring and alarm systems. However, all the interactions between component failures tend to be stronger when components are highly loaded. For example, if a more highly loaded transmission line fails, it produces a larger transient, there is a larger amount of power to redistribute to other components, and failures in nearby protection devices are more likely. Moreover, if the overall system is more highly loaded, components have smaller margins so they can tolerate smaller increases in load before failure, the system nonlinearities and dynamical couplings increase, and the system operators have fewer options and more stress.

A typical large blackout has an initial disturbance or trigger events followed by a sequence of cascading events. Each event further weakens and stresses the system and makes subsequent events more likely. Examples of an initial disturbance are short circuits of transmission lines through untrimmed trees, protection device misoperation, and bad weather. The blackout events and interactions are often rare, unusual, or unanticipated because the likely and anticipated failures are already routinely accounted for in power system design and operation.

Blackouts are traditionally analyzed after the blackout by a thorough investigation of the details of the particular sequence of failures. This is extremely useful for finding areas of weakness in the power system and is good engineering practice for strengthening the transmission system [29], [38], [28], [39].

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We take a different and complementary approach and seek to determine the risk of series of blackouts from a global, top-down perspective. That is, we are not concerned with the deterministic details of a particular blackout, but rather the overall probability and risk of blackouts from a bulk systems perspective. Our overall approach draws from probability and statistics, power systems engineering, statistical physics, risk analysis, and modeling and simulation.

There are two measures of blackout size that immediately present themselves as useful for blackouts. Utilities are interested in number of failures such as transmission line failures because these are operational data that can be monitored in a control center and can sometimes be prevented or mitigated. Customers, industry, regulators and politicians are interested in quantities that directly affect them such as load shed or energy not served.

For an extensive listing and short description of previous work by other authors in cascading failure blackouts we refer the reader to [18] (particularly for cascading failure in power systems) and [22] (cascading failure in general). Much of the authors' previous work in cascading failure blackouts ([8], [4], [7], [22], [6], [17]) is summarized in [18].

We now briefly summarize the most immediate technical background for this paper. Branching processes [26], [2], [24] are shown to approximate an abstract model of cascading failure called CASCADE in [17]. CASCADE is compared to a power systems model of cascading line outages in order to estimate failure propagation in [6], [20]. Initial work fitting supercritical branching processes in discrete and continuous time to observed blackout data is in [21].

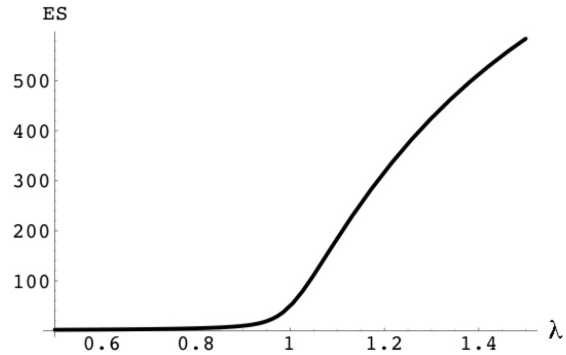

Fig. 1. Average number of failures in branching process model with $n = 1000$ as λ increases. Critical loading occurs at kink in curve at $\lambda = 1$ where the average number of failures sharply increases.

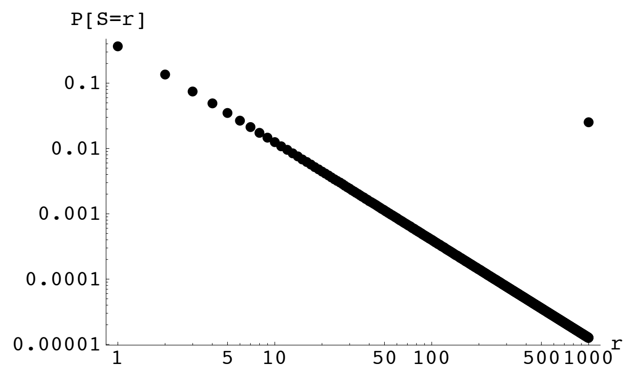

Fig. 2. Log-log plot of PDF of total number of failures in branching process model at criticality.

II. Criticality and blackout risk

As load increases, it is clear that cascading failure becomes more likely, but exactly how does it become more likely? Our previous work shows that the cascading failure does not gradually and uniformly become more likely; instead there is a transition point at which the cascading failure becomes increasingly more likely. This transition point has some of the properties of a critical transition or a phase transition.

In complex systems and statistical physics, a critical point for a type 2 phase transition is characterized by a discontinuity of the gradient in some measured quantity. At this point fluctuations of this quantity can be of any size and their correlation length becomes of the order of the system size. As a consequence, the probability distribution of the fluctuations has a power tail. Figures 1 and 2 show the criticality phenomenon in the branching process cascading failure model that is introduced in section III. At criticality Figure 2 shows a power dependence with exponent -1.5 before saturation. (A power dependence with exponent -1 implies that doubling the blackout size only halves the probability and appears on a log-log plot as a straight line of slope -1. An exponent of -1.5 as shown by the slope -1.5 in the log-log plot of Figure 2 implies that doubling the blackout size divides the probability by $2^{1.5}$.)

A similar form of critical transition has been observed in blackout simulations [4], [11] and abstract models of cascading failure [22], [17]. A power law distribution of blackout size with exponent between -1 and -2 is also consistent with the empirical probability distribution of energy unserved in North American blackouts from 1984 to 1998 [8], [9]. This suggests that the North American power system has been operated near criticality. The power tails are of course limited in extent in a practical power system by a finite cutoff near system size corresponding to the largest possible blackout. The distribution of the number of elements lost in North American contingencies from 1965 to 1985 [1] also has a heavy tail distribution [13].

Blackout risk is the product of blackout probability and blackout cost. Here we conservatively assume that blackout cost is roughly proportional to blackout size, although larger blackouts may well have costs (especially indirect costs) that increase faster than linearly [3]. The importance of the power law tail in the distribution of blackout size is that larger blackouts become rarer at a similar rate as costs increase, so that the risk of large blackouts is comparable to, or even exceeding, the risk of small blackouts [5]. For example, if the power law tail for the blackout size has exponent -1, then doubling blackout size halves the probability and doubles the cost and the risk is constant with respect to blackout size. A little less approximately, consider in Figure 3 the variation of blackout risk with blackout size computed from the branching process model at criticality. The pdf power law exponent of -1.5 is combined with the assumed linear increase in costs to give

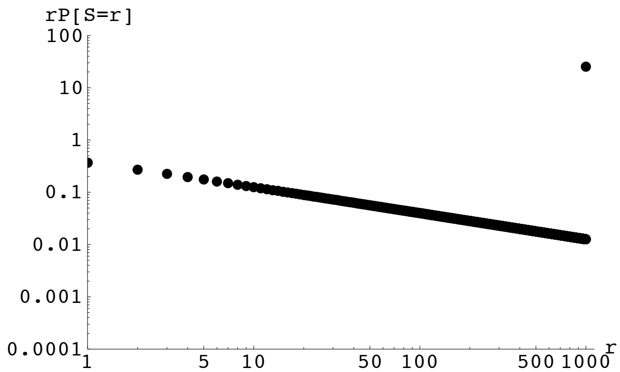

Fig. 3. Blackout risk $rP[S=r]$ as a function of number of failures r. Cost is assumed to be proportional to the number of failures and is measured in arbitrary units.

a modest -0.5 power law decrease in risk before saturation. The risk of the saturated case of all 1000 components failing is substantial. We conclude that the power law tails in both the NERC data and the blackout simulation results imply that large blackouts cannot be dismissed as so unlikely that their risk is negligible. On the contrary, the risk of large blackouts is substantial near criticality. Standard probabilistic techniques that assume independence between events imply exponential tails and are not applicable to blackout risk.

The terminology of "criticality" comes from statistical physics and it is of course extremely useful to use the standard scientific terminology. However, while the power tails at critical loading indicate a substantial risk of large blackouts, it is premature at this stage of knowledge to automatically presume that operation at criticality is bad simply because it entails some substantial risks. There is also economic gain from an increased loading of the power transmission system.

III. Branching process model

One approach models the growth of blackout failures using a branching process and then estimates the branching process parameter λ that measures both the extent to which failures propagate after they are started and the margin to criticality. We first summarize a basic branching process model. Branching process models are an obvious choice of stochastic model to capture the gross features of cascading blackouts because they have been developed and applied to other cascading processes such as genealogy, epidemics and cosmic rays [26]. The first suggestion to apply branching processes to blackouts appears to be in [17].

There are more specific arguments justifying branching processes as useful approximations to some of the gross features of cascading blackouts. Our idealized probabilistic model of cascading failure [22] describes with analytic formulas the statistics of a cascading process in which component failures weaken and further load the system so that subsequent failures are more likely. We have shown that this cascade model and variants of it can be well approximated by a Galton-Watson branching process with each failure giving rise to a Poisson distribution of failures in the next stage [17], [19]. Moreover, some features of this cascade model are consistent with results from cascading failure simulations [6], [20]. All of these models can show criticality and power law regions in the distribution of failure sizes or blackout sizes consistent with NERC data [8]. While our main motivation is large blackouts, these models are sufficiently simple and general that they could be applied to cascading failure of other large, interconnected infrastructures.

The Galton-Watson branching process model [26], [2] gives a way to quantify the propagation of cascading failures with a parameter λ. In the Galton-Watson branching process the failures are produced in stages. The process starts with M_0 failures at stage zero to represent the initial disturbance. The failures in each stage independently produce further failures in the next stage according to an probability distribution with mean λ. The failures "produced" by one of the failures in the previous stage can be thought of that failure's children or offspring and the distribution of failures produced by one of the failures in the previous stage is sometimes called the offspring distribution.

The branching process is a transient discrete time Markov process and its behavior is governed by the parameter λ. In the subcritical case of $\lambda < 1$, the failures will die out (i.e., reach and remain at zero failures at some stage) and the mean number of failures in each stage decreases exponentially. In the supercritical case of $\lambda > 1$, although it possible for the process to die out, often the failures increase exponentially without bound.

There are obviously a finite number of components that can fail in a blackout, so it must be recognized that the cascading process will saturate when most of the components have failed. Moreover, many observed cascading blackouts do not proceed to the entire interconnection blacking out. The reasons for this may well include inhibition effects such as load shedding relieving system stress, or successful islanding, that apply in addition to the stochastic variation that will limit some cascading sequences. Understanding and modeling these inhibition or saturation effects is important. However, in some parts of this paper such as estimating λ, we avoid this issue by analyzing the cascading process before saturation occurs.

Analytic formulas for the total number of components failed can be obtained in some cases. For example, assume that there are M_0 initial failures, the offspring distribution is Poisson with mean λ, and the process saturates when n components fail. Then the total number of failures S is distributed according to a saturating Borel-Tanner distribution:

$$P[S=r] = \begin{cases} M_0\lambda(r\lambda)^{r-M_0-1}\dfrac{e^{-r\lambda}}{(r-M_0)!}; & M_0 \le r < n \\ 1-\displaystyle\sum_{s=M_0}^{n-1} M_0\lambda(s\lambda)^{s-M_0-1}\dfrac{e^{-s\lambda}}{(s-M_0)!}; & r=n \end{cases} \tag{1}$$

Forms of saturation different than that in (1) are described in [17], [20].

Approximation of (1) for large $r < n$ using Stirling's formula and a limiting expression for an exponential yields

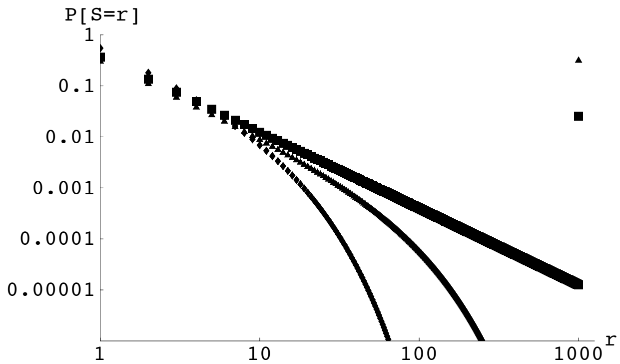

Fig. 4. Log-log plot of PDF of total number of failures in branching process model for three values of λ. $\lambda = 0.6$ is indicated by the diamonds. $\lambda = 1.0$ (criticality) is indicated by the boxes. $\lambda = 1.2$ is indicated by the triangles.

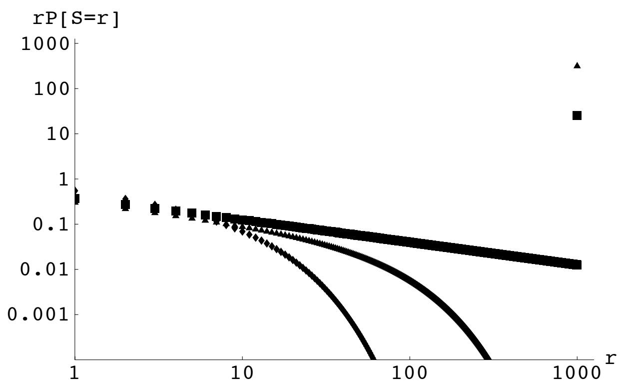

Fig. 5. Blackout risk $rP[S = r]$ as a function of number of failures r for three values of λ. $\lambda = 0.6$ is indicated by the diamonds. $\lambda = 1.0$ (criticality) is indicated by the boxes. $\lambda = 1.2$ is indicated by the triangles. Cost is assumed to be proportional to the number of failures and is measured in arbitrary units.

$$P[S=r] \approx \frac{M_0}{\sqrt{2\pi}}\,\lambda^{-M_0}\,r^{-1.5}e^{-r/r_0}; \quad 1 \ll r < n \quad (2)$$
$$\text{where } r_0 = (\lambda - 1 - \ln\lambda)^{-1}$$

In approximation (2), the term $r^{-1.5}$ dominates for $r \lessapprox r_0$ and the exponential term e^{-r/r_0} dominates for $r_0 \lessapprox r < n$. Thus (2) reveals that the distribution of the number of failures has an approximate power law region of exponent -1.5 for $1 \ll r \lessapprox r_0$ and an exponential tail for $r_0 \lessapprox r < n$.

The qualitative behavior of the distribution of blackout size as λ is increased can now be described. This behavior is illustrated in Figure 4. For subcritical λ well below 1, r_0 is well below n and the exponential tail for $r_0 \lessapprox r < n$ implies that the probability of large blackouts of size near n is exponentially small. The probability of large blackouts of size exactly n is also very small. As λ increases in the subcritical range $\lambda < 1$, the mechanism by which there develops a significant probability of large blackouts of size near n is that r_0 increases with λ so that the power law region extends to the large blackouts. For near critical $\lambda \approx 1$, r_0 becomes large and exceeds n so that power law region extends up to $r = n$. For supercritical λ well above 1, r_0 is again well below n and there is an exponential tail for $r_0 \lessapprox r < n$. This again implies that the probability of large blackouts of size near n is exponentially small. However there is a significant probability of large blackouts of size exactly n and this probability of total blackout increases with λ.

Figure 5 shows the distribution of risk with respect to the number of failures for the same values of λ considered in Figure 4. The essential point is that, given an assumption about the blackout cost as a function of blackout size, the branching process model gives a way to compute blackout risk in terms of λ. Both the expected risk of Figure 1 and the distribution of that risk over blackout size of Figure 5 can be computed.

A variant of the branching process produces potential failures at each stage according to the offspring distribution. Then the potential failures fail independently with probability p. For example, if one thinks of each failure as overloading other components according to the offspring distribution, then this corresponds to either the failure overloading and failing only a fraction of the components [19] or only a fraction of the overloaded components failing [20]. This is a simple form of emigration added to the branching process in the sense that the potential failures leave the process [2, page 266]. If the offspring distribution without emigration has generating function $f(s)$ and propagation λ, then the process with emigration is a branching process with generating function $g(s) = f(1 - p + ps)$. It follows that

$$\lambda_{\text{emigration}} = g'(1) = pf'(1) = p\lambda \quad (3)$$

IV. Detecting criticality in blackout models

We suggest and outline methods of detecting subcriticality or supercriticality and the closeness to criticality from a generic blackout simulation model.

A. Blackout model assumptions

For a given initial failure and a given loading or stress level L, the model produces

1) A sequence of failures. The failures correspond to the internal cascading processes such as transmission line outages. Often models will naturally produce failures in stages in an iterative manner. If not, then the failures need to be grouped into stages. In run j, the model produces failures M_{j0}, M_{j1}, M_{j2}, ... where M_{jk} is the number of failures in stage k.
2) A blackout size such as load shed or energy unserved. In run j, the model produces blackout size B_j.

There is a means of randomizing the initial failure and the system initial conditions so that different sequences of failures at the loading level L are generated for each run. There are a number of different blackout models that satisfy these generic assumptions [4], [11], [23], [25], [27].

Although L may often be chosen as an overall system loading such as total system load or total mean of random loads, there are other important ways of parameterizing the overall system stress. L could measure the overall system margin or reserves, as for example in [6], where the system "loading" is measured by the ratio of generator reserve to load variability or the average ratio of transmission line power flow to line maximum power rating. L could also be the amount of a power transfer across a system. In the sequel we will refer to L as "loading" for convenience while retaining its expansive interpretation as a measure of overall system stress.

One important issue is that instead of regarding all the failures as equivalent and counting them equally, one can weight them according to their importance. For example, the relative impact of a transmission line failure on the system is roughly proportional to the power flowing on it, so that an appropriate weight is the maximum power rating. If the maximum power ratings for individual lines are not available, then the nominal voltage squared (proportional to the surge impedance loading) could be used for the weight.

B. Distribution of blackout size

The model is run to accumulate statistics of the pdf of blackout size. Inspection of the probability of a large blackout at saturation and the extent to which there is a power law region reveals whether the pdf is subcritical or supercritical. This method has been applied to several power system blackout models [4], [11] and was also used to process observed blackout data from NERC [8]. The method does not quantify the closeness to criticality and it is very time consuming to approximate the pdf accurately, especially for the rare large blackouts near criticality. For example, in [4] 60 000 runs were used to estimate the pdf of blackout size of a 382 bus network.

C. Mean blackout size

The mean blackout size $\mu(L)$ at the loading level L can be estimated by J runs using

$$\mu(L) = \frac{1}{J}\sum_{j=1}^{J} B_j \tag{4}$$

Then the sharp change in the slope of the expected blackout size at criticality can be exploited to test for subcriticality or supercriticality (this assumes a type 2 phase transition at criticality). Suppose it is known from previous computations that the slope of the mean blackout size with respect to loading L is approximately $\text{slope}_{\text{sub}}$ below the critical value of L and approximately $\text{slope}_{\text{super}}$ above the critical value of L. Define the average slope

$$\text{slope}_{\text{average}} = \frac{1}{2}(\text{slope}_{\text{sub}} + \text{slope}_{\text{super}}) \tag{5}$$

Estimate the local slope by evaluating with the model $\mu(L + \Delta L)$ and $\mu(L)$ for small ΔL and using

$$\text{slope}\mu(L) = \frac{\mu(L+\Delta L) - \mu(L)}{\Delta L} \tag{6}$$

Then

$$\text{stress } L \text{ is} \begin{cases} \text{subcritical if } \text{slope}\mu(L) < \text{slope}_{\text{average}} \\ \text{supercritical if } \text{slope}\mu(L) > \text{slope}_{\text{average}} \end{cases} \tag{7}$$

This approach gives as a useful byproduct the slope of the mean blackout size with respect to loading.

Now the critical loading and hence the margin to critical loading can be found with further computations of $\mu(L)$ at different values of L. Since (7) gives a way to test whether L is less than or above the critical loading, it is straightforward to approximate the critical loading by first finding an interval containing the critical loading and then interval halving. The interval containing the critical loading is found by increasing L until supercriticality if the first tested L is subcritical and decreasing L until subcriticality if the first tested L is supercritical.

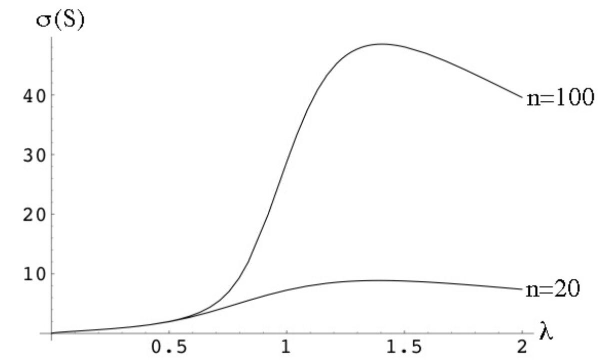

Fig. 6. Standard deviation of the total number of failures S as a function of λ for saturation at $n = 20$ failures and $n = 100$ failures.

We now roughly estimate the number of runs J needed to accurately obtain $\mu(L)$ at a single loading level L. We assume that the runs correspond to independent samples, each starting from one initial failure, and that the failures are generated by a branching process with a Poisson offspring distribution with mean λ and saturation at n failures. Then in run j, the total number of failures S_j is distributed according to the Borel-Tanner distribution (1) with $M_0 = 1$. We also make the simple assumption that the blackout size B_j is proportional to the total number of failures S_j. The standard deviation of $\mu(L)$ is then proportional to $\sigma(S)/\sqrt{J}$, so that the number of runs depends on the standard deviation $\sigma(S)$ of S. If saturation is neglected, $\sigma(S) = \sqrt{\lambda/(1-\lambda)^3}$ becomes infinite as λ increases to criticality at $\lambda = 1$. The saturation makes $\sigma(S)$ larger but finite near criticality as shown in Figure 6. (To obtain Figure 6, the variance of S was obtained via evaluating $D_t^2 E t^S$ at $t = 1$ with computer algebra.) For example, if saturation is at 100 components and $\lambda = 1.3$, then $\sigma(S) = 48$ and a mean blackout size standard deviation corresponding to 0.5 failures requires $(48/0.5)^2 = 9200$ runs. If saturation is instead at 20 components then $\sigma(S) = 9$ and the same accuracy can be achieved with $(9/0.5)^2 = 320$ runs. The number of runs depends greatly on λ, the accuracy required and the saturation.

The mean blackout size $\mu(L)$ was computed for a range of system loadings for several different power system cascading failure models in [4], [11], [27].

D. Propagation λ

We would like to estimate the average propagation λ over a stages. The a stages are limited to the period before saturation effects apply, because the branching process model assumed for the estimation is a branching process model without saturation that only applies to the propagation of failures before saturation. Define the total number of failures in each stage by summing over the J runs

$$M_k = M_{1k} + M_{2k} + ... + M_{Jk}, \qquad k = 1, 2, ..., a \tag{8}$$

Define the cumulative number of failures up to and including stage k to be

$$S_k = M_0 + M_1 + M_2 + ... + M_k \tag{9}$$

Then an estimator for λ is [24], [15]

$$\hat{\lambda} = \frac{M_1 + M_2 + ... + M_a}{M_0 + M_1 + ... + M_{a-1}} = \frac{S_a - M_0}{S_{a-1}} = \frac{S_a - M_0}{S_a - M_a} \tag{10}$$

$\hat{\lambda}$ is a maximum likelihood estimator when observing numbers of failures in each stage for a wide class of offspring distributions, including the exponential family. $\hat{\lambda}$ is biased and its mean underestimates λ, but the bias is inversely proportional to the number of runs J [24, pp. 37-39]. In the special case of $a = 1$, $\hat{\lambda} = M_1/M_0$.

The first stage is usually comprised of the initiating failures. The number of stages a could be limited by one of several methods. For example, to avoid the saturation effects the number of stages could be limited so that the fraction of components failed was below a threshold.

If grouping failures into stages is needed, then, since (10) only requires S_a, M_0, and M_a, it is only necessary to group failures into the first stage to obtain M_1 and into the last stage to obtain M_a. To group failures into stages, the failure data will be assumed to include the time of each failure and perhaps some additional data explaining the causes of the failure and specifying the type and location of the failure. Factors that would tend to group several failures into the same stage could be their closeness in time or location, or being caused by failures in the previous stage.

We now roughly estimate the number of runs J needed to accurately obtain $\hat{\lambda}$. We assume that the runs correspond to independent samples, each starting from one initial failure, and that the failures are generated by a branching process with a Poisson offspring distribution with mean λ. Then as J tends to infinity, the standard deviation of $\hat{\lambda}$ is asymptotically [24, p. 53]

$$\sigma(\hat{\lambda}) \sim \frac{\sigma_{S_{1a}}(\lambda, a)}{\sqrt{J}} = \frac{1}{\sqrt{J}} \sqrt{\frac{\sum_{j=0}^{2a} \lambda^{j+1} - (2a+1)\lambda^{a+1}}{(\lambda - 1)^2}} \tag{11}$$

where $\sigma_{S_{1a}}(\lambda, a)$ is the standard deviation of the total number of failures S_{1a} produced by one initial failure $M_{10} = 1$. That is, $S_{1a} = M_{10} + M_{11} + \ldots + M_{1a}$. Note that $\sigma_{S_{1a}}(1, a) = \sqrt{(a + 3a^2 + 2a^3)/6}$. Figure 7 shows $\sigma_{S_{1a}}(\lambda, a)$. For example, if $\lambda = 1.3$ and the number of stages $a = 5$, then $\sigma_{S_{1a}}(1.3, 5) = 15$ and $\sigma(\hat{\lambda}) = 0.05$ requires $(15/0.05)^2 = 90\,000$ runs. If instead the number of stages $a = 2$ then $\sigma_{S_{1a}}(1.3, 2) = 3$ and the same accuracy can be achieved with $(3/0.05)^2 = 3600$ runs. The number of runs depends greatly on λ, the accuracy required, and the number of stages a.

To illustrate the choice of the number of stages a to avoid saturation, suppose that the failures saturate at $n = 100$ and that we can assume that $\lambda \leq 1.5$. Then in the most rapidly saturating case of $\lambda = 1.5$, the mean number of failures in stage k is 1.5^k. The mean total number of failures in stage 6 is 32 and the standard deviation of the total number of failures is $\sigma_{S_{1a}}(1.5, 6) = 38$. Therefore to avoid saturation we can choose the number of stages a in the computation of $\hat{\lambda}$ in the range $1 \leq a \leq 6$.

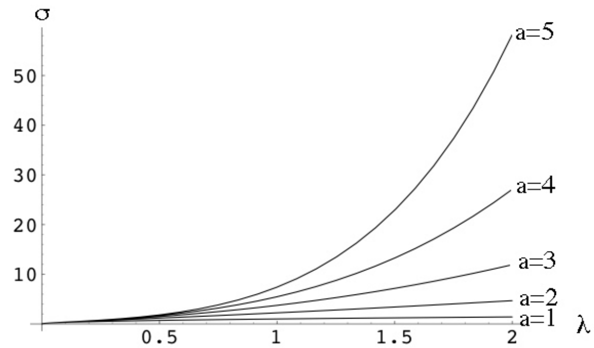

Fig. 7. $\sigma(\hat{\lambda})\sqrt{J} = \sigma_{S_{1a}}(\lambda, a)$ as a function of λ for number of stages $a = 1, 2, 3, 4, 5$.

V. Conclusions and research agenda

This paper discusses branching process models for cascading failure and shows how assuming these models gives a way to roughly estimate expected blackout risk and risk of blackouts of various sizes as a function of the branching process parameter λ. λ describes the average extent to which failures propagate and measures the closeness to criticality. At criticality $\lambda = 1$ and the branching process models show a power tail in the distribution of blackout size and a sharp rise in expected blackout size. The way in which the power law region extends as criticality is approached is described. Then we suggest approaches to determining the closeness to criticality via the expected blackout size or λ from runs of a generic cascading failure blackout model. Some rough estimates of computational effort are made. The approaches in this paper augment previous work relating branching models and other abstract models of cascading failure to power system blackout models and power system data [6], [20], [21]. Further development and testing of measures of closeness to criticality is needed. In particular, estimating λ and assuming a branching process model can yield the distribution of the risk of blackouts of various sizes as well as the average risk.

We now expand our focus and address more generally the research needed to further explore and develop the possibilities of bulk statistical analysis of blackout risk. We consider key research issues for two aspects. In the first aspect the power transmission system is assumed to be fixed and the main objective is to determine how close the system is to a critical loading at which the expected blackout size rises sharply and there is a substantial risk of large blackouts. In the second aspect, the power transmission system slowly evolves subject to the forces of rising demand and the upgrade of the transmission system in response to the blackouts. These dynamics of transmission system evolution can be seen as a form of self-organization in a complex system [7], [5].

A. Measuring proximity to criticality in a fixed network

Some research issues are:

Research access to blackout data. To develop models and methods based on reality, it is essential for blackout data to be collected and for researchers to have access to the data. Although the precise data needs have not yet evolved and will require iteration, it is clear that bulk statistical analysis of blackouts will neglect much of the blackout detail, so that concerns about confidentiality and homeland security can be addressed by only releasing a suitably and substantially filtered record of the blackout events. Discussion about which filters succeed in resolving confidentiality and homeland security concerns would be helpful. One specific goal is to gain research access to the data from the August 2003 blackout of Northeastern America that was collected for the blackout report [38].

Blackout costs. To estimate blackout risk, blackout cost needs to be approximated as a function of blackout size and, while there is considerable information available for smaller blackouts, the direct and indirect costs of large blackouts seem to be poorly known.

Confirm criticality phenomenon. While criticality has been observed in several power system blackout models [4], [11], it needs to be confirmed in power system blackout models representing different interactions and with varying levels of detail in order to be able to conclude that it is a universal feature of cascading failure blackouts. If no criticality or a different sort of criticality is observed, this needs to be understood.

Power system blackout models. The main issues are the tradeoffs between what interactions to model and in what detail to model them, test system size and computational speed.

Abstract cascading failure models. These models presently include branching process models in discrete and continuous time and CASCADE models. These models require substantial refinement and further comparison and validation with real and simulated blackout data to ensure that the main features of blackouts are represented. In particular, blackouts being inhibited and saturating at a fraction of the system size needs to be understood and better modeled.

Monitoring closeness to criticality. Suggested initial approaches are described in this paper and [6], [20], [21]. Much more needs to be done to establish practical statistical methods for monitoring closeness to criticality. Processing of failure data into stages and the appropriate scalings need to be investigated.

The critical loading as a power system limit. The critical loading essentially provides an additional system limit that guides power system planning and operation with respect to the risk of cascading failure. In contrast to an indirect way of limiting cascading failure such as the n-1 criterion, the critical loading directly relates to the risk of cascading failure. The appropriate operating margin to this limit should be based on risk computations and is not yet known. Little is known about the properties of the critical loading as power system conditions change. It would be very useful to be able to identify some easily monitored quantities that are strongly correlated to the critical loading [6], because this would open up the possibility of monitoring the closeness to criticality via these quantities. It would also be useful to evaluate the performance of the n-1 criterion when used as a surrogate for the critical loading limit.

Progression from understanding phenomena to offline models to online monitoring. The research questions above focus on understanding phenomena, developing and validating models and measuring closeness to criticality in power system models and in past blackouts. Once these questions start to be resolved, there is a natural progression to consider the feasibility of schemes to practically monitor closeness to criticality of power systems online.

B. Complex systems dynamics of power systems.

The complex systems dynamics of transmission network upgrade can explain the power tails and apparent near-criticality in the NERC data [8]. The complex system studied here includes the engineering and economic forces that drive network upgrade as well as the cascading failure dynamics. As a rough explanation, below criticality increasing load demand and economic pressures tend to increasingly stress the system. But when the system is above the critical loading, blackout risk rises and the response to real or simulated blackouts is to upgrade the system and relieve the system stress. Thus the system will tend to vary near criticality in a complex systems equilibrium. The system can be said to self-organize to near criticality. A power systems model that incorporates slow load growth and a simple form of transmission upgrade at lines involved in cascading blackouts converges to such a complex systems equilibrium [7]. Moreover, as might be expected in a complex system, simple forms of blackout mitigation can have the desired effect of decreasing small blackouts but also the somewhat counterintuitive effect of ultimately increasing large blackouts [5]. Other theories that can generate power laws or similar behavior include the influence model [34], highly optimized tolerance [35], graph-theoretic network analysis [40] and cluster models for line outages [13].

Some research issues are:

Reframing the problem of blackouts. Instead of simply avoiding all blackouts, the problem is to manage blackout risk both by manipulating the probability distribution of blackout size [5] and by finding ways to minimize blackout costs [36]. Blackout mitigation should take into account complex systems dynamics by which the power system and society slowly readjust themselves to any changes made.

Models for complex system dynamics. For theories such as the influence model, highly optimized tolerance, or graph-theoretic network analysis the challenge is to construct models of power systems and their evolution with an explicit correspondence to the abstract model and study their properties. For the self-organizing complex systems theory, such a model already exists and the challenge is to improve its representation of the engineering and economic forces, and particularly the transmission upgrade, economic investment and human factor aspects. Part of the challenge is understanding cascading failure and complex systems dynamics across several interacting or coupled complex systems [31]. It is necessary to balance the requirements for computational speed and accessibility of data against the requirements of a detailed model. It may be necessary to develop a hierarchy of models of varying detail to accommodate varying emphases on speed versus model detail.

Analysis tools. Diagnostics for monitoring and studying complex systems dynamics need to be developed.

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