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SENSITIVITY OF HOPF BIFURCATIONS TO POWER SYSTEM PARAMETERS

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Abstract: Hopf bifurcation of a power system leads to oscillatory instabilities and it is desirable to design system parameters to ensure a sufficiently large loading margin to Hopf bifurcation. We present formulas for the sensitivity of the Hopf loading margin with respect to any power system parameter. These first order sensitivities determine an optimum direction in parameter space to change parameters to increase the loading margin. We compute the Hopf bifurcation sensitivities of a simple power system with a voltage regulator and a dynamic load model. Parameter sensitivities of the Hopf and saddle node bifurcations are compared. An idea for eliminating some Hopf bifurcations is presented.

1 Introduction

Power systems require parameters or controls to be chosen so that oscillatory instabilities are avoided. This has previously been done by linearizing the power system model about an operating point and designing the linearized system to avoid instabilities [16]. More recently, starting with the work of Abed and Varaiya [1], the onset of oscillatory instability is studied in a nonlinear context as a Hopf bifurcation [20, 2, 3]. We formulate the design to avoid oscillatory instabilities in the nonlinear context as avoiding the Hopf bifurcation in the following manner [11]:

Suppose a stable operating equilibrium with a vector of nominal parameter values p_0 is given. If loads increase, then stability is lost in a Hopf bifurcation and the proximity of the base case to the Hopf bifurcation is measured by a loading margin M. M changes as the parameters p are varied from their nominal values p_0. We want to compute the first order sensitivity of M with respect to the power system parameters p in order to obtain an optimum direction of parameter change to increase M. Increasing M improves the system robustness to oscillatory instability caused by slow load increase. This paper derives and illustrates the computation of the sensitivity of M with respect to any power system parameters.

We review previous work [12] on avoiding of saddle node bifurcations since this is similar to the proposed method of avoiding Hopf bifurcations. Saddle node bifurcation is associated with voltage collapse of the power system [14] and always occurs for sufficently high loading. The dynamical consequences of saddle node bifurcation [9] seem consistent with some observed voltage collapses, in which voltage magnitudes decline monotonically. However, some simplified power system models become oscillatory unstable in a Hopf bifurcation before the saddle node bifurcation occurs [1,20,2,3]. The load power margin to a saddle node bifurcation is computed by continuation or direct methods by increasing the loading until saddle node bifurcation is first encountered [e.g. 21, 6]. (We assume throughout the paper that the distribution of load increase is specified.) The next step is to compute the normal vector to the saddle node bifurcation surface at the critical loading; the formula for the normal vector follows from one of the transversality conditions of bifurcation theory [10]. It turns out that the first order sensitivity of the load power margin to any power system parameters or controls is trivial to compute from the normal vector. This sensitivity determines (at least locally) the combination of parameters and controls to be varied in order to optimally increase the load power margin.

Since there are computations for the first Hopf bifurcation as loading increases [21, 5] and there is a formula for the normal vector to the Hopf bifurcation surface [13,7,15], the sensitivity to any power system parameters of the load power margin to Hopf instability can similarly be computed. In a Hopf bifurcation, a complex pair of eigenvalues of the linearized system crosses the imaginary axis and the normal vector essentially contains sensitivities with respect to parameters of the real parts of these eigenvalues. (The Hopf bifurcation hypersurface is determined by the vanishing of the real parts of these eigenvalues.)

The sensitivity of the margin M also shows which parameters couple most strongly with the Hopf bifurcation. For example, one expects the voltage regulator parameters to strongly influence the margin to Hopf bifurcation. The sensitivity of the load power margin to both the Hopf and the saddle node bifurcation are compared to determine the extent to which different sets of parameters affect both margins.

2 Hopf parameter sensitivity

Consider a power system modeled by smooth parameterized differential equations

$$\dot{z} = f(z, \lambda), \qquad z \in \mathbf{R}^n, \;\; \lambda \in \mathbf{R}^{m+1} \qquad (1)$$

The parameter vector $\lambda = (\ell, p)$ consists of a real loading parameter ℓ and a vector p of m system design parameters. We write $x \in \mathbf{R}^n$ for a particular equilibrium of (1) and assume that x is asymptotically stable at the parameter vector $\lambda_0 = (\ell_0, p_0)$. p_0 is a nominal choice of design parameters. We assume that when the loading parameter ℓ slowly increases to some critical value ℓ_* and the design parameters are held fixed at p_0, the equilibrium x loses stability in a Hopf bifurcation. The loading margin to instability is then $M = \ell_* - \ell_0$. If we write $\lambda_* = (\ell_*, p_0)$, then the loading margin may also be expressed as $M = |\lambda_* - \lambda_0|$. The measure M of closeness to Hopf bifurcation takes full account of system nonlinearity.

The question we address is: What is the optimum direction for first order change in the design parameters p from p_0 in order to increase the load margin M? That is, we regard the load margin M as a function of the design parameters p and want to compute the gradient or sensitivity $M_p|_{p_0}$ so that the design may be incrementally improved by changing parameters in the direction $M_p|_{p_0}$.

The sensitivity $M_p|_{p_0}$ is essentially a scaled projection of a normal vector to the Hopf bifurcation hypersurface. The details follow: Write Σ^{hopf} for the set of $\lambda'_* \in \mathbf{R}^{m+1}$ for which equation (1) has a Hopf bifurcation at (x_*, λ'_*) with $f_x|_*$ having a simple pair of eigenvalues $\pm j\omega_*$, $\omega_* \neq 0$ and all other eigenvalues with nonzero real parts and satisfying the transversality condition (4) presented below. Since $f_x|_*$ is invertible, the implicit function theorem implies that there is a smooth function u defined in a neighborhood of λ_* with $u(\lambda_*) = x_*$, $u(\lambda) = x$ and $f(u(\lambda), \lambda) = 0$. $u(\lambda)$ specifies the position of the equilibrium of interest as a function of the parameters and its Jacobian u_λ is given by solving

$$f_x u_\lambda = -f_\lambda \qquad (2)$$

There is also a smooth function μ evaluating to an eigenvalue defined in a neighborhood of λ_* with $\mu(\lambda_*) = j\omega_*$ and $\mu(\lambda)$ an eigenvalue of $f_x|_{(u(\lambda),\lambda)}$. The real part of the eigenvalue μ is a function

$$\alpha(\lambda) = \mathrm{Re}\{\mu(\lambda)\} \qquad (3)$$

which is smooth near λ_* and, if the transversality condition

$$\alpha_\lambda \neq 0 \qquad (4)$$

is satisfied, then there is a neighborhood $U \ni \lambda_*$ in which Σ^{hopf} is a smooth hypersurface specified by the zero set of α:

$$\Sigma^{\mathrm{hopf}} \cap U = \{\lambda \in U \mid \alpha(\lambda) = 0\} \qquad (5)$$

That is, Σ^{hopf} is locally specified by the vanishing of the real part of the complex pair of eigenvalues associated with the Hopf bifurcation. It follows that a normal vector to Σ^{hopf} is given by the sensitivities of the real part with respect to the parameters:

$$N(\lambda_*) = D_\lambda\big(\mathrm{Re}\{\mu(\lambda)\}\big)|_{\lambda_*} = \alpha_\lambda|_{\lambda_*} \qquad (6)$$

Now we compute the gradient $\alpha_\lambda|_{\lambda_*}$ in (6) in terms of the equations f in (1). Write v_* and w_* for the right and left complex eigenvector of $f_x|_*$ corresponding to $j\omega_*$; these eigenvectors are normalized according to $|v| = 1$ and $wv = 1$ (It is convenient to regard w as a row vector). Using this normalization, it is easy to show that

$$\mu(\lambda) = w f_x v$$

Differentiate with respect to λ to obtain

$$\begin{aligned}\mu_\lambda &= w D_\lambda(f_x) v + w_\lambda f_x v + w f_x v_\lambda \\ &= w D_\lambda(f_x) v + \mu D_\lambda(wv) \\ &= w D_\lambda(f_x) v \\ &= w\,(f_{xx} u_\lambda + f_{x\lambda})\,v\end{aligned}$$

Take the real part and use $D_\lambda\big(\mathrm{Re}\{\mu(\lambda)\}\big)|_{\lambda_*} = \mathrm{Re}\{\mu_\lambda\}|_{\lambda_*}$ and (6) to obtain

$$N(\lambda_*) = \alpha_\lambda|_{\lambda_*} = \mathrm{Re}\{w(f_{xx}u_\lambda + f_{x\lambda})v\}\big|_* \qquad (7)$$

where u_λ is given by solving equation (2). (To exemplify the notation, note that $f_{xx}u_\lambda$ is an

$n \times n \times (m+1)$ tensor; contraction with w and v yields an $(m+1)$ vector.) A different scaling of formula (7) and similar computations appear in [15,13]. If the power system equations (1) are linearized *before* the sensitivity of the real parts of the critical eigenvalues are computed, then the term $\mathrm{Re}\{w(f_{xx}u_\lambda)v\}\big|_*$ does not appear. This omission arises because the linearization effectively fixes the equilibrium at the origin. Since this term has often been overlooked in the past, it would be interesting to know whether this term is significant in typical power system applications.

The loading increase from $\lambda_0 = (\ell_0, p_0)$ to $\lambda_* = (\ell_*, p_0)$ can be written as

$$\lambda(\ell) = \lambda_0 + (\ell - \ell_0)\, e_1 \tag{8}$$

where $e_1 = (1, 0, 0, ..., 0)$. We assume a transversality condition

$$D_\ell \mathrm{Re}\{\mu(\lambda(\ell))\}\,|_{\ell_*} = \alpha_\lambda|_{\lambda_*} e_1 \neq 0 \tag{9}$$

so that the critical eigenvalues pass the imaginary axis with nonzero speed as the Hopf bifurcation occurs. (This condition is generically satisfied.) We prove below that the gradient of M with respect to λ is

$$M_\lambda\big|_{\lambda_0} = -(N(\lambda_*)e_1)^{-1} N(\lambda_*) \tag{10}$$

Then it is clear from $M_\lambda = (M_\ell, M_p)$ that $M_p|_{p_0}$ is the projection π of $M_\lambda\big|_{\lambda_0}$ onto the m dimensional design parameter space:

$$M_p|_{p_0} = \pi M_\lambda\big|_{\lambda_0} \tag{11}$$

That is, if

$$N(\lambda_*) = (n_0^\ell, n_1^p, n_2^p, n_2^p, ..., n_m^p)$$

then (10) and (11) yield

$$M_p|_{p_0} = -(n_0^\ell)^{-1}(n_1^p, n_2^p, n_2^p, ..., n_m^p) \tag{12}$$

Our assumption (9) implies that $n_0^\ell \neq 0$.

The proof of (10) is adapted from the proof in [12] by substituting the Hopf bifurcation hypersurface Σ^{hopf} for a saddle node bifurcation hypersurface. $\lambda_*(\lambda)$ is a well defined smooth function of the parameters λ near λ_0 because Σ^{hopf} is a smooth hypersurface near λ_* and (9) implies that $N(\lambda_*)e_1 \neq 0$ so that the direction of load increase e_1 intersects Σ^{hopf} transversally at λ_*. It follows that $M(\lambda) = |\lambda_*(\lambda) - \lambda|$ is a well defined smooth function of the parameters λ near λ_0. Then

$$0 = \alpha(\lambda_*) = \alpha(\lambda + M(\lambda)e_1)$$

and differentiating with respect to λ and evaluating at λ_0 yields

$$\begin{aligned} 0 &= \alpha_\lambda|_{\lambda_*}(I + e_1 M_\lambda)\big|_{\lambda_0} \\ &= N(\lambda_*) + N(\lambda_*)e_1 M_\lambda\big|_{\lambda_0} \end{aligned}$$

and the result (10) follows by rearranging terms. The geometric content is clear: the optimum direction to increase the distance in a given direction e_1 of a point λ_0 to a hypersurface Σ^{hopf} is antiparallel to the outward normal to Σ^{hopf}.

3 Saddle node parameter sensitivity

We summarize formulas from [12] for the sensitivity of loading margin to saddle node bifurcation to any power system parameters to establish notation used in the example and to compare with the corresponding results for Hopf bifurcation.

The saddle node bifurcation occurs at loading ℓ_{SN} and the loading margin is $M^{SN} = \ell_{SN} - \ell_0$. At the saddle node bifurcation f_x is singular and has left eigenvector w_{sn} corresponding to the zero eigenvalue of $f_x|_{sn}$. Under suitable transversality assumptions the saddle node hypersurface has at λ_{sn} a normal vector

$$N(\lambda_{sn}) = w_{sn} f_\lambda\big|_{sn} \tag{13}$$

The normal vector formula (13) is simpler than the corresponding formula (7) for Hopf.

The sensitivity of the loading margin follows from the normal vector in the same way as developed for the Hopf bifurcation in section 2:

$$\begin{aligned} M_p^{SN}|_{p_0} &= \pi M_\lambda\big|_{\lambda_0} = -\pi(N(\lambda_{sn})e_1)^{-1} N(\lambda_{sn}) \\ &= (w_{sn} f_\ell|_{\ell_{SN}})^{-1} w_{sn} f_p|_{p_0} \end{aligned}$$

4 Illustrative Example

We illustrate the computation of the sensitivity of the Hopf load power margin in a simple power system example.

Chow and Gebreselassie [8] compute a Hopf bifurcation in a power system model consisting of single machine with a voltage regulator supplying a constant power load through a single line. Our example (see Fig. 1) is based on this model and we refer to [8] for most of the model equations and their description. We simplify the treatment of the voltage regulator set points in [8] by setting the reference voltage setpoint E_{ref} = 1.1 pu and computing the transformer high side voltage E_s in terms of other state variables.

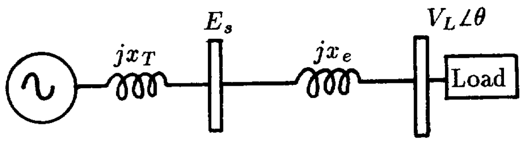

Fig. 1 One machine system with dynamic load

One problem in computing Hopf bifurcations is that the Hopf bifurcation depends on dynamical details of the models such as time constants and little reliable information is known about the dynamics of loads. (In contrast, saddle node bifurcations are somewhat independent of the details of load dynamics as argued in [Dobsonw].) We address the problem of poorly known but possibly significant load dynamics by assuming a crude form of dynamic load model and roughly estimating parameter values of an appropriate order of magnitude and then computing the sensitivity to the estimated parameters to assess the validity of the results.

The dynamic load model represents an aggregate load and replaces the constant real and reactive power loads of [8] by

$$\ell\ \mathrm{PF} + D\,\dot\theta + a\,\dot V_L$$
$$\ell\sqrt{1-\mathrm{PF}^2} + b\,\dot\theta + k\,\dot V_L$$

respectively, where ℓ parameterizes the increase of the constant power part of the load, PF stands for power factor and D, a, k, b are time constants of the load dynamics. Induction load models with similar terms are discussed in [19,9]. The nominal load parameters are PF $= 0.95$, $D = 0.05$, $a = 0$, $b = 0$, $k = 0.1$. The order of magnitude of D and k is consistent with power system tests in [17,18,4].

The model of [8] could be written as 5 differential equations and 2 algebraic equations, together with a procedure for determining settings for E_{ref} at different loadings that yield a specified value of E_s. Our modifications to the model of [8] can be summarized as including a dynamic load model with a lower power factor and fixing $E_{ref} = 1.1$ pu for all loading levels. We use the nominal generator, machine and voltage regulator parameters of [8] except that the stabilizer gain $K_f = 0.1$. These modifications to the model of [8] allow us to write the model as 7 differential equations with loading parameter ℓ. The state vector is $(E'_d, E'_q, V_R, E_{FD}, R_f, \theta, V_L)$ where E'_d and E'_q are machine voltages, V_R is the voltage regulator output voltage, E_{FD} is the field voltage, R_f is the state of the stabilizer, and $V_L\angle\theta$ is the load voltage phasor. Specifying the load dynamics resolves the singularity of the load algebraic equations encountered in [8].

The first instability encountered by the stable equilibrium as the loading is increased from $\ell_0 = 0$ is a Hopf bifurcation at $\ell_H = 0.370$ so that the loading margin $M = 0.370$. The power system parameters are $p = (D, a, b, k, \mathrm{PF}, E_{ref}, K_A, T_A, T_E, K_f, T_f, x_T, x_e, x_d, x_q, x'_d, T'_{d0}, T'_{q0})$ where K_A and T_A are the gain and time constant of the voltage regulator, T_E is the exciter time constant, K_f and T_f are the gain and time constant of the stabilizer, x_T and x_e are the reactances of the step up transformer and the transmission line, x_d and x_q are the machine synchronous reactances, x'_d is the machine transient reactance, and T'_{d0} and T'_{q0} are the open circuit machine time constants. The sensitivities M_p are shown in the second row of Table 1. We verified the results by increasing K_f by 0.01 and recomputing M. M increased by 0.020 whereas the sensitivity predicts M increasing by 0.022. Increasing T_f by 0.1 caused M to decrease by 0.015 whereas the sensitivity predicts M decreasing by 0.015.

In our example, the dynamic load parameters b and k are moderately sensitive, but since their base values are small, the effect on the loading margin of, say, letting a, b, k, D tend to 0 is small. This is of interest since setting $a = b = k = D = 0$ effectively makes the load differential equations into algebraic equations. Further modeling and experiments along these lines are required to obtain a general conclusion about the relative importance of the dynamic load model.

The saddle node bifurcation occurs at the loading margin $M^{SN} = 1.03$. The sensitivities of M^{SN} to the power system parameters were computed according to the formulas of section 3 and are shown in the third row of Table 1. As expected, the voltage regulator parameters do not affect M^{SN} (The slight dependence of M^{SN} on K_A can be attributed to our simplified modeling of E_{ref} as a constant). This suggests that it might be desirable to design the voltage regulator system of this example to avoid Hopf bifurcations and system oscillations before addressing the avoidance of saddle node bifurcations and voltage collapse. Increasing the power factor PF or the reference voltage E_{ref} increases the margins to both the Hopf and saddle node bifurcations.

5 Eliminating Hopf bifurcations

In cases in which a further increase in loading past the Hopf bifurcation yields a "reverse" Hopf bifurcation which restores the stability of the equilibrium, we suggest that parameters be optimally changed to eliminate the Hopf bifurcation by making it coalesce with the "reverse" Hopf bifurcation.

Suppose the Hopf bifurcation occurs at a loading ℓ_H and the reverse Hopf bifurcation occurs at a higher loading $\ell_{RH} > \ell_H$. The inevitable saddle node bifurcation occurs at a loading $\ell_{SN} > \ell_{RH}$. Write x for the stable equilibrium at low loading. One of the possible situations as loading increases is that the Hopf bifurcation at ℓ_H makes x unstable and creates a stable periodic orbit γ which persists until it coalesces with the unstable equilibrium x at the reverse Hopf bifurcation at ℓ_{RH}. The reverse Hopf bifurcation restores the stability of x and the stability of x persists as the loading further increases until x disappears in the saddle node bifurcation at ℓ_{SN}. One possibility is that the stable periodic orbit γ can period double to chaos and reverse period double back to a stable periodic orbit in the interval (ℓ_H, ℓ_{RH}).

We measure the extent to which Hopf bifurcation is present in the system by the extent of the interval or "window" over which the Hopf bifurcation destabilizes the system. That is, we define the index $W = \ell_{RH} - \ell_H$ and suggest that decreasing W to zero will eliminate the Hopf bifurcation by causing the Hopf and "reverse" Hopf bifurcations to coalesce and disappear. We compute the sensitivity of W with respect to power system parameters. This sensitivity could be used to obtain the optimum direction in which to change the power system parameters so that W is decreased. Driving W to zero eliminates the Hopf bifurcations so that the stability of the equilibrium x is only limited by the saddle node bifurcation.

The sensitivity of W with respect to power system parameters is easy to obtain from the previous sensitivity results. Write $M^H = \ell_H - \ell_0$ and $M^{RH} = \ell_{RH} - \ell_0$ for the respective loading margins of the Hopf and reverse Hopf bifurcations. The index

$$W = \ell_H - \ell_{RH} = M^H - M^{RH}$$

so that the gradient of W is now easy to compute by applying formula (11):

$$W_p|_{p_0} = \pi(M_\lambda^H|_{\lambda_0} - M_\lambda^{RH}|_{\lambda_0}) \qquad (14)$$

That is, using (12), the ith element of $W_p|_{p_0}$ is

$$[W_p|_{p_0}]_i = n_i^H/n_0^H - n_i^{RH}/n_0^{RH} \quad i = 1, ..., m.$$

6 Conclusions

Exact formulas for the first order sensitivity of the loading margin to Hopf bifurcation to any power system parameters have been obtained. The formulas are illustrated using a small power system example and verified by numerically computing some of the sensitivities. These sensitivities could be used to optimally increase the loading margin to Hopf bifurcation. The loading margin and the sensitivity computation take full account of the system nonlinearities. The sensitivity results follow easily from computing a normal vector to a Hopf bifurcation hypersurface in parameter space. The normal vector contains the sensitivities of the real part of the critical pair of eigenvalues associated with the Hopf bifurcation. The formulas include a term associated with movement of the equilibrium which has been neglected in eigenvalue sensitivity studies of linearized power system models. Our results involve eigenvalue sensitivities but are exact first order sensitivities of loading margins. We also compare the sensitivities of the Hopf and saddle node bifurcations in our example.

The Hopf bifurcation depends on dynamic aspects of the load models and these are not well known. We approach this problem by choosing a crude dynamic load model with roughly estimated parameters and then computing the sensitivity of our margins to the estimated parameters. The dynamic load model allowed the model to be differential equations rather than differential-algebraic equations.

We have suggested a method of computing first order parameter changes which in some cases would tend to make two Hopf bifurcations coalesce and disappear.

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	D	a	b	k	PF	E_{ref}	K_A	T_A	T_E	K_f	T_f
p	0.05	0.00	0.00	0.10	0.95	1.10	30.0	0.40	0.56	0.10	1.30
M_p	−0.05	−0.01	−0.33	−0.31	0.48	0.70	0.00	−0.27	−0.18	2.20	−0.15
M_p^{SN}	0.00	0.00	0.00	0.00	2.20	1.88	0.01	0.00	0.00	0.00	0.00
	x_T	x_e	x_d	x_q	x'_d	T'_{d0}	T'_{q0}				
p	1.00	0.3406	1.00	1.00	0.18	5.00	1.50				
M_p	−0.25	−0.31	0.00	0.00	−0.26	0.04	0.00				
M_p^{SN}	−0.10	−2.45	−0.10	0.00	0.00	0.00	0.00				

Table 1. Sensitivities of Hopf and saddle node loading margins to parameters