torics, Lecture Notes in Mathematics, 406, New York, pp. 247–261, 1974.
[7] E. Seneta, *Nonnegative Matrices and Markov Chains*. New York: Springer Verlag, 1980.
[8] M. Marcus and H. Minc, *A Survey of Matric Theory and Matrix Inequalities*. Boston, MA: Allyn and Bacon, 1964.

Voltage Collapse Precipitated by the Immediate Change in Stability When Generator Reactive Power Limits are Encountered

Ian Dobson and Liming Lu

***Abstract*—When a generator of a heavily loaded electric power system reaches a reactive power limit, the system can become immediately unstable and a dynamic voltage collapse leading to blackout may follow. We study the statics and dynamics of this mechanism for voltage collapse by example and by the generic theory of saddle node and transcritical bifurcations. Load power margin calculations can be misleading if the immediate instability phenomenon is neglected.**

I. Introduction

Voltage collapse is an instability of heavily loaded electric power systems that leads to declining voltages and blackout. It is associated with bifurcation and reactive power limitations of the power system. Power systems are expected to become more heavily loaded in the next decade as the demand for electric power rises while economic and environmental concerns limit the construction of new transmission and generation capacity. Heavily loaded power systems are closer to their stability limits and voltage collapse blackout will occur if suitable monitoring and control measures are not taken. It is important to understand mechanisms of voltage collapse so that voltage collapse blackouts may be effectively prevented. Most of the current approaches to analyzing voltage collapse are represented in [1] and [2].

One aspect of voltage collapse is that power systems become more vulnerable to voltage collapse when generator reactive power limits are encountered [3]–[11]. The effect of a generator reactive power limit is to immediately change the system equations. For example, the effect of a generator excitation current limit may be simply modeled by replacing the equation describing a constant output voltage magnitude by an equation describing a constant excitation current. Although the system state is unchanged, the immediate change in the system equations causes a discontinuous change in the stability margin of the system. The case in which the stability margin decreases when the reactive power limit is encountered but the system remains stable is familiar [3], [6]–[8], [10]. We study the case in which the system becomes immediately unstable when the reactive power limit is encountered. This possibility was mentioned by Borremans *et al.* [5] but otherwise appears to have been overlooked. The immediate instability of the system can lead to voltage collapse and the main purpose of this paper is to study the statics and dynamics of this mechanism for voltage collapse.

Manuscript received July 17, 1991; revised August 25, 1992. This work was supported in part by NSF Research Initiation Grant ECS-9009079. This paper was recommended by Associate Editor M. Hasler.

I. Dobson is with the Department of Electrical and Computer Engineering University of Wisconsin, Madison, WI 53706.

L. Lu is with Harris-Syndeco, Belleville, MI 48111.

IEEE Log Number 9204225.

We analyze a power system example from [12] and demonstrate that at lesser loadings, encountering the reactive power limit is expected to decrease, but not destroy stability. At sufficiently high loadings, encountering the reactive power limit will immediately destabilize the system and can precipitate a voltage collapse along a trajectory which is one part of the unstable manifold of an unstable equilibrium. The movement along this trajectory is a new model for the dynamics of voltage collapse. Moreover, we can argue using the theory of saddle node and transcritical bifurcations that the immediate instability and subsequent dynamic voltage collapse are likely to be typical for a general, heavily loaded power system. The results have an important implication for correctly measuring the proximity to voltage collapse using load margins; it seems that the load power margin to bifurcation can be a misleading indication of system stability unless the possibility of immediate voltage collapse is taken into account. A more detailed version of these results appeared in [13] and [14].

II. Power System and Reactive Power Limit Model

We briefly describe a 3-bus power system example from [12] and the modeling of a reactive power limit of a generator [13], [14]. Details of the equations and parameters are given in the Appendix. The 3-bus power system example shown in Fig. 1 consists of two generators (one is slack bus) and a dynamic load with capacitative support. The parameter values used in this paper are identical to those of [12] except that the generator damping D has been increased to 0.12. This eliminates the Hopf bifurcations and other oscillatory phenomena discovered in [15] and [16] at high loadings. A well-designed power system stabilizer would suppress these oscillations. Note that the value of D has no effect on the loading at which saddle node bifurcation occurs.

The load model of [12] includes a dynamic induction motor model with a constant PQ load in parallel. The combined model for the motor and the PQ load is specified by load powers

$$P_l = P_0 + P_1 + K_{pw}\dot{\delta} + K_{pv}(V + T\dot{V}) \tag{2.1}$$

$$Q_l = Q_0 + Q + K_{qw}\dot{\delta} + K_{qv}V + K_{qv2}V^2 \tag{2.2}$$

where $V \angle \delta$ is the load voltage phasor and Q is a parameter that varies with the load reactive power demand. These load dynamics can be rearranged and combined with standard swing equation dynamics to obtain the system differential equations with state vector $x = (\delta_m, \omega, \delta, V)$ where δ_m, ω are the generator angle and frequency [12].

The two main causes of the reactive power output Q_g reaching a limit in a generator are the excitation current limit and the stator thermal limit [17], [18]. The two limits have similar overall effects on the system, and we only consider the excitation current limit.

Because the generator internal voltage E is proportional to the excitation current, the excitation current limit may be modeled by E encountering a limit $E^{\lim}$. Before the limit is encountered, $E < E^{\lim}$ and the generator terminal voltage E_m is controlled so that $E_m = E_m^{\text{imp}}$, the terminal voltage imposed by the voltage regulator [6]. (The dynamics of the voltage regulator are not modeled.) When the limit is encountered $E = E^{\lim}$ and E varies with $E_m < E_m^{\text{imp}}$. Encountering the limit may be thought of as changing from the constraint of constant E_m to the constraint of constant E. The effect of the limit on the system

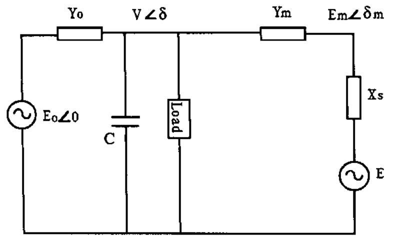

Fig. 1. Three-bus example power system.

differential equations is to replace E_m by a lengthy expression $h(x, E^{\text{lim}})$ involving the system state x and the constant E^{lim} (see Appendix).

III. Immediate Voltage Collapse When a Limit is Encountered

We present two cases of the example power system encountering excitation current limits. It is convenient to write J for the Jacobian of the unlimited system evaluated at the operating equilibrium x_0 and J^{lim} for the Jacobian of the limited system evaluated at x_0. The first case is well known and the bifurcation diagrams of the unlimited and limited systems are shown in Fig. 2(a). The excitation limit is encountered at $Q = Q^{\text{lim}} = 11.0$. When the limit is encountered the system changes structure and J changes to J^{lim}. The eigenvalues of J^{lim} differ from the eigenvalues of J but their real parts remain negative (see Table I, Case (a)). The equilibrium at x_0 of the limited system has reduced stability but remains stable. The system reactive power margin $Q^* - Q^{\text{lim}}$ is reduced since the reactive power Q^* at which saddle node bifurcation occurs is reduced. Note that the upper portion of each of the bifurcation diagrams is stable and the lower portion is unstable; in this case, x_0 is on the upper and stable portion of both the unlimited and limited system bifurcation curves.

In the second case, the limit is encountered at the higher system loading $Q = Q^{\text{lim}} = 11.4$. The system changes so that J^{lim} has an eigenvalue with positive real part (see Table I, Case (b)). In this case, x_0 is on the upper and stable portion of unlimited system bifurcation curve and the lower and unstable portion of limited system bifurcation curve (see Fig. 2(b)). The operating equilibrium x_0 becomes immediately unstable when the limit is encountered and the system dynamics will move the system state away from x_0. References [13] and [14] show analytically that this immediate instability is typical for sufficiently high loadings of this example by approximating the system equations. Now we describe the dynamic consequences of the instability; see [19] and [20] for background in dynamical systems and bifurcations.

The unstable equilibrium x_0 has a one-dimensional unstable manifold W^u that consists of two trajectories W^u_- and W^u_+ leaving x_0 and x_0 itself (see the idealized sketch of Fig. 3). W^u is a smooth curve passing through x_0 which is tangent at x_0 to the eigenvector v of J^{lim} associated with the positive eigenvalue. x_0 also has a three-dimensional stable manifold W^s that divides the four-dimensional state space near x_0 into two parts. Note, for example, that W^s has an additional dimension that is not shown in Fig. 3.

The system state is initially at x_0 but cannot remain there because of the inevitable small perturbations on the state. If the state is perturbed from x_0 to one side of W^s, it will be attracted towards W^u, and we can simply approximate the dynamics by motion along W^u_-. Similarly, if the state is perturbed from x_0 to the other side of W^s, it will be attracted towards W^u_+ and we can

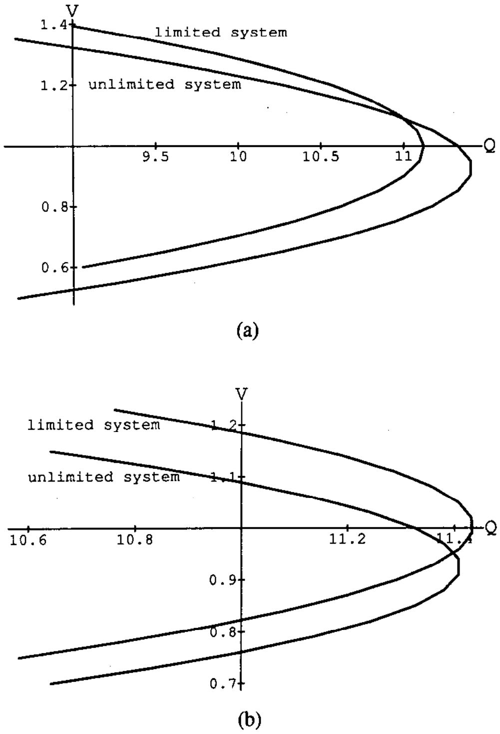

Fig. 2. (a) Limit encountered at a lesser loading. (b) Immediate instability when limit is encountered.

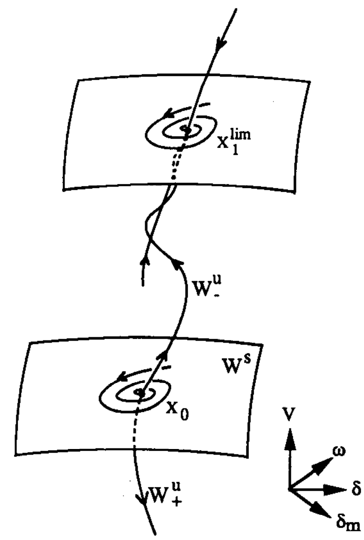

Fig. 3. Idealized sketch of limited system dynamics.

simply approximate the dynamics by motion along W^u_+. In short, the dynamical consequences of the immediate instability are motion along the trajectory W^u_- or the trajectory W^u_+. Either outcome is possible and there seems no reason to regard W^u_- or W^u_+ as more likely.

We integrated the differential equations of the limited system starting near x_0 to find the outcome of motion along W^u_- or W^u_+. (The initial conditions were chosen to be $x_0 \pm 0.001v$ and Gear's integration method [21] was used since differential equations are stiff near a saddle node bifurcation.) The corresponding time histories of V are shown in Fig. 4. The trajectory W^u_- tends to the nearby stable equilibrium x_1^{lim} and the initial portion of the slow, oscillatory convergence of V to x_1^{lim} is

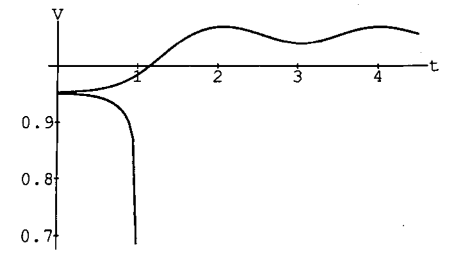

Fig. 4. Possible dynamics caused by immediate instability.

shown is the upper graph of Fig. 4. (This may not occur in practice because the voltage control system could prevent the voltage from rising [10].) The trajectory W_+^u diverges so that V decreases and the motion along W_+^u shown in the lower graph of Fig. 4 is a voltage collapse. Thus we describe a model of a new mechanism for voltage collapse:

The operating equilibrium x_0 becomes immediately unstable when a reactive power limit is encountered, and one of the possible dynamical consequences is voltage collapse along part of the unstable manifold of x_0.

This model of immediate voltage collapse has some similarities with the center manifold collapse model [12]; the voltage collapse dynamics can be modelled by movement along a particular trajectory. However, the trajectory is part of an unstable manifold instead of the unstable part of a center manifold and there is also the possibility of convergence to a nearby stable equilibrium along the other part of the unstable manifold.

IV. Immediate Instability and Voltage Collapse in a General Power System Model

The particular example presented above shows that a sufficiently heavily loaded but stable system can become immediately unstable when a reactive power limit is encountered. The dynamical consequences of this instability are either collapse along the unstable manifold trajectory W_+^u or convergence to a nearby stable equilibrium along the unstable manifold trajectory W_-^u. Now we argue that this description is expected to apply in the general case. Our main assumptions are that applying a reactive power limit does not increase the margin of system stability, the phenomena occurring are generic and a simplification that only one bifurcation occurs.

Consider a general power system modeled by smooth parameterized differential equations $\dot{x} = f(x, \lambda)$, where $x \in \boldsymbol{R}^n$ is the system state and $\lambda \in \boldsymbol{R}^m$ are slowly changing system parameters such as real and reactive load powers [12]. We suppose the system is operated at a stable equilibrium x_0 when the parameters are λ_0. When a reactive power limit is encountered the system equations immediately change to $\dot{x} = f^{\lim}(x, \lambda)$ but the position of the equilibrium x_0 is unchanged. That is, $0 = f(x_0, \lambda_0) = f^{\lim}(x_0, \lambda_0)$.

Now suppose that the equations $\dot{x} = f(x, \lambda)$ are gradually changed into the equations $\dot{x} = f^{\lim}(x, \lambda)$. This can be done by combining f and $f^{\lim}$ into new equations

$$\dot{x} = g(x, \lambda_0, k) \tag{4.1}$$

with a parameter k so that $g(x, \lambda_0, 0) = f(x, \lambda_0)$ and $g(x, \lambda_0, 1) = f^{\lim}(x, \lambda_0)$. We also require that

$$g(x_0, \lambda_0, k) = 0 \qquad \text{for } k \in [0, 1]. \tag{4.2}$$

In short, we construct a homotopy joining f and $f^{\lim}$ that preserves the equilibrium at x_0. Note that λ is fixed at λ_0 as the parameter k is varied. k gradually increasing from 0 to 1 has the effect of gradually applying the reactive power limit to the system. This allows the change in structure between f and $f^{\lim}$ to be studied using bifurcation theory. However, we do not seek to represent the manner in which the reactive power limit is applied in practice by the gradual increase in k.

Equation (4.1) is a one-parameter system of differential equations with the restriction (4.2) of an equilibrium at x_0. If we assume that this a generic one-parameter system of differential equations, then the only bifurcations which can occur are the transcritical bifurcation and the Hopf bifurcation [19]. (In the set of all smooth one parameter differential equations without restrictions or symmetries, the generic bifurcations are the saddle node bifurcation and the Hopf bifurcation. An equilibrium disappears in a saddle node bifurcation and therefore the restriction (4.2) precludes saddle node bifurcations [20].) In a generic transcritical bifurcation two equilibria approach each other, coalesce, and then separate with an exchange of stability. The Jacobian has a single, simple zero eigenvalue at the bifurcation. If one of the equilibria is stable before the bifurcation, then the other is type one unstable. After the bifurcation the equilibria are also stable and type one unstable but each equilibrium has changed its stability. Fig. 5 shows a typical bifurcation diagram for the transcritical bifurcation in which the solid line indicates stable equilibria and the dashed line denote unstable equilibria. In the generic Hopf bifurcation, the equilibrium changes stability by interacting with a limit cycle and the Jacobian has a single, simple pair of imaginary eigenvalues at the bifurcation. Of course it is also generic for there to be no bifurcation for $k \in [0, 1]$.

We first consider the special case of encountering a reactive power limit at the point of voltage collapse; that is, λ_0 is chosen so that $\dot{x} = f(x, \lambda)$ with parameter λ has a generic saddle node bifurcation at (x_0, λ_0). Then x_0 is a degenerate equilibrium formed by the coalescence of a stable equilibrium and a type one unstable equilibrium and the Jacobian of f evaluated at (x_0, λ_0) is singular [12], [20]. Since the Jacobian of f evaluated at (x_0, λ_0) and the Jacobian of g evaluated at $(x_0, \lambda_0, 0)$ are identical, the Jacobian of g evaluated at $(x_0, \lambda_0, 0)$ is also singular. Therefore, $(x_0, \lambda_0, 0)$ is also a bifurcation point of system (4.1) with k as parameter. If we consider only generic phenomena, than the bifurcation of (4.1) at $(x_0, \lambda_0, 0)$ is a transcritical bifurcation. The Hopf bifurcation is precluded by the single zero eigenvalue and absence of nonzero imaginary eigenvalues of the Jacobian at the generic saddle node bifurcation of $\dot{x} = f(x, \lambda)$ with λ as parameter.

In a generically occuring transcritical bifurcation the bifurcation diagram (suitably reduced to the center manifold of the suspended system [20]) is as shown in Fig. 5. There are two possible "directions" through the bifurcation as k is increased from zero so that the equilibrium x_0 is either stable or unstable for small positive k. Since reactive power limits are generally observed to limit system performance, it seems likely that partially applying a reactive power limit (imposing small positive k) would destabilize rather than stabilize the system. Thus the more usual case to consider should be x_0 unstable for small positive k. For small positive k there is a type one unstable equilibrium x_1 close to x_0 and part of the unstable manifold W_-^u of x_0 is a trajectory tending to x_1. In this case, if we further assume for simplicity that there are no further bifurcations as k increases from a small positive value to one, then we can conclude that at $k = 1$, the system $\dot{x} = g(x, \lambda_0, 1) = f^{\lim}(x, \lambda_0)$ has x_0 unstable and type one. Moreover, there is a stable

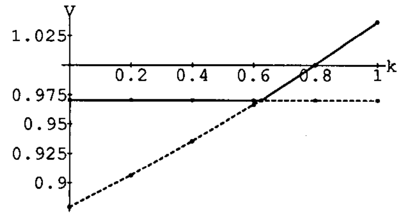

Fig. 5. Transcritical bifurcation diagram when limit is gradually applied.

equilibrium x_1 in the vicinity and the part W^u_- of the unstable manifold of x_0 is a trajectory tending to x_1.

The assumption of genericity of the one parameter differential equations (4.1) implies that the occurence of the transcritical bifurcation is robust to small changes in (4.1). In particular, if we consider a sufficiently heavily loaded but still stable system $f(x, \lambda'_0)$ with λ'_0 chosen close to λ_0 so that $f(x, \lambda'_0)$ has a stable equilibrium x'_0, then the corresponding homotopy $g(x, \lambda'_0, k)$ will have a transcritical bifurcation as k increases from zero to one and we can extend the conclusions for the case of a reactive power limit encountered at the saddle node bifurcation to the case of a reactive power limit encountered just before the saddle node bifurcation. The only difference is that x_0 is stable for $k = 0$ and the transcritical bifurcation will occur at a positive value of k. Fig. 5 shows the bifurcation diagram for the change in stability of the 3-bus example system in case (b) (the black circles are the data we computed). (For these results $g(x, \lambda, k)$ was obtained from the system differential equations by replacing E_m with $(1-k)E_m^{\text{imp}} + kh(x, E^{\text{lim}})$. Note that (4.2) is satisfied since $h(x_0, E^{\text{lim}}) = E_m^{\text{imp}}$.)

V. Discussion and Conclusions

The instantaneous change in the system equations when a generator reactive power limit is encountered causes the system dynamics and structure to instantaneously change, although the system state is unchanged. In particular, the Jacobian at the operating equilibrium and the closest unstable equilibrium change discontinuously. It follows that most of the voltage collapse indexes proposed in the literature that are functions only of the system *before* a reactive power limit is encountered are *discontinuous* when the reactive power limit is encountered. We emphasize this point because the literature often neglects this somewhat unpalatable discontinuity or misleadingly describes it as a discontinuity in the derivative of the index. Exceptions are indexes such as the total generated reactive power index of Begovic and Phadke [7] that are a functions of the system operating point only and hence continuous when a reactive power limit is encountered. Two other exceptions are the energy function index of Overbye and DeMarco [11] and the load power margin index when proper account is taken of the reactive power limits as, for example, in Van Cutsem [10].

One important consequence of a generator reactive power limit causing an immediate instability is that the load power margin can be misinterpreted to give an incorrect measure of system stability. A load power margin measures the load increase that the system can sustain before bifurcation but is only valid when the system is operated at a stable equilibrium. If the operating equilibrium becomes immediately unstable, the load power margin is incorrect because the system is (at least momentarily) at an *unstable* equilibrium. (If the system were operated at the nearby stable equilibrium, then the load power margin would be a valid measure of system stability. We note that the system state might converge to the nearby stable equilibrium as a result of the immediate instability but the possibility of voltage collapse as a result of the immediate instability is at least as likely.)

In case (a) the example power system retains stability when the reactive power limit is encountered and the reactive power margin $Q^* - Q^{\text{lim}}$ does measure the system stability (see Fig. 2(a)). In the more heavily loaded case (b), the system becomes immediately unstable when the reactive power limit is encountered and the system may immediately collapse whereas the reactive power margin $Q^* - Q^{\text{lim}}$ can be misinterpreted as a positive margin of stability (see Fig. 2(b)). We conclude that the stability of the limited system equilibrium should be checked when load power margins are computed.

Borremans *et al.* [5] recognized that a generator reactive power limit could precipitate a voltage collapse in the manner of Fig. 2(b), but regarded this possibility as more theoretical than the case of Fig. 2(a). We show by an example and general arguments that immediate voltage collapse is likely to be typical for a sufficiently highly loaded system encountering a generator reactive power limit. However, our results have not established that immediate voltage collapse occurs over a significant range of loadings up to the bifurcation. That is, we have not excluded the possibility that the immediate voltage collapse might only exist for a small interval of loadings before bifurcation. (Our power system example is not conclusive in this regard because of its small size and the lack of validation of the load model.) Cañizares and Alvarado [22] observed the immediate voltage collapse in a large power system model very near the bifurcation. We conclude that immediate instability when a generator reactor power limit is encountered is a plausible cause of voltage collapse whose relative importance is not yet established.

We also study the simplest and most likely dynamical consequences of the immediate instability in a general power system using the theory of generic saddle node and transcritical bifurcations. The dynamical consequences are either convergence to a nearby stable equilibrium or a voltage collapse. The voltage collapse dynamics may be modeled by movement along a specific trajectory which is part of the unstable manifold of the unstable equilibrium. This is a new model for voltage collapse dynamics with some similarities with the center manifold model of voltage collapse [12]. The dynamics of voltage collapse are easy to simulate by numerical integration along the unstable manifold and were illustrated using the example power system.

We hope our example and general analysis will encourage more study of the interaction of system limits and voltage collapse. In particular, we note the usefulness of the transcritical bifurcation in understanding how encountering a system limit can affect the system behavior.

VI. Appendix: Details of Model and Parameters

The example power system can be described by following differential equations [12]:

$$\dot{\delta}_m = \omega \tag{7.1}$$

$$M\dot{\omega} = -D\omega + P_m + E_m Y_m V \sin(\delta - \delta_m - \theta_m) + E_m^2 Y_m \sin\theta_m \tag{7.2}$$

$$K_{qw}\dot{\delta} = -K_{qv2}V^2 - K_{qv}V + Q_l - Q_0 - Q \tag{7.3}$$

$$TK_{qw}K_{pv}\dot{V} = K_{pw}P_{qv2}V^2 + (K_{pw}K_{qv} - K_{qw}K_{pv})V + K_{qw}(P_l - P_0 - P_1) - K_{pw}(Q_l - Q_0 - Q) \tag{7.4}$$

TABLE I

Case	$Q^{\lim}$	Eigenvalues of J			Eigenvalues of $J^{\lim}$		
(a)	11.000	−14.686	−0.110 ± 3.721j	−126.001	−7.602	−0.017 ± 3.379j	−142.961
(b)	11.400	−3.734	−0.040 ± 3.040j	−94.627	3.977	−0.904 ± 3.388j	−106.645

where the real and reactive powers supplied to the load by the network are

$$P_l = -E_0'Y_0'V\sin(\delta+\theta_0') - E_mY_mV\sin(\delta-\delta_m+\theta_m) + (Y_0'\sin\theta_0' + Y_m\sin\theta_m)V^2 \quad (7.5)$$

$$Q_l = E_0'Y_0'V\cos(\delta+\theta_0') + E_mY_mV\cos(\delta-\delta_m+\theta_m) - (Y_0'\cos\theta_0' + Y_m\cos\theta_m)V^2. \quad (7.6)$$

The state vector is $x = (\delta_m, \omega, \delta, V)$ and M, D, and P_m are the generator inertia, damping, and mechanical power, respectively. The capacitor at the load is accounted for by adjusting E_0, Y_0, and θ_0 to E_0', Y_0', and θ_0' to give the Thevenin equivalent of the circuit with the capacitor. The load parameter values are $K_{pw} = 0.4$, $K_{pv} = 0.3$, $K_{qw} = -0.03$, $K_{qv} = -2.8$, $K_{qv2} = 2.1$, $T = 8.5$, $P_0 = 0.6$, $Q_0 = 1.3$, $P_1 = 0.0$, and the network and generator parameter values are $Y_0 = 20.0$, $\theta_0 = -5.0$, $E_0 = 1.0$, $C = 12.0$, $Y_0' = 8.0$, $\theta_0' = -12.0$, $E_0' = 2.5$, $Y_m = 5.0$, $\theta_m = -5.0$, $E_m = 1.0$, $P_m = 1.0$, $D = 0.12$, $M = 0.3$, and $X_s = 0.15$.

The effect of the limit on the system differential equations (7.1)–(7.4) is to replace the constant E_m by a lengthy expression $h(x, E^{\lim})$ involving the system state and $E^{\lim}$. We now derive the equations for this replacement. Write the powers delivered to the network by the generator as

$$P_g = EE_mY_s\sin\delta_s \quad (7.7)$$

$$Q_g = EE_mY_s\cos\delta_s - E_m^2Y_s \quad (7.8)$$

where δ_s is the difference of the angle between E and E_m and Y_s is $1/X_s$. Squaring and adding (7.7) and (7.8) yields

$$\left(Q_g + E_m^2Y_s\right)^2 + P_g^2 = (EE_mY_s)^2. \quad (7.9)$$

Real and reactive power balance at the generator gives

$$P_g = -E_m^2Y_m\sin\theta_m + E_mY_mV\sin(\delta_m-\delta+\theta_m) \quad (7.10)$$

$$Q_g = E_m^2Y_m\cos\theta_m - E_mY_mV\cos(\delta_m-\delta+\theta_m). \quad (7.11)$$

A quadratic equation in E_m can be obtained from (7.9) to (7.11):

$$\begin{aligned} &E_m^2(Y_m\cos\theta_m + Y_s)^2 \\ &\quad - 2E_mY_mV(Y_m\cos\theta_m + Y_s)\cos(\delta_m-\delta+\theta_m) \\ &\quad + Y_m^2V^2\cos^2(\delta_m-\delta+\theta_m) + E_m^2Y_m^2\sin^2\theta_m \\ &\quad - 2E_mY_m^2V\sin\theta_m\sin(\delta_m-\delta+\theta_m) \\ &\quad + Y_m^2V^2\sin^2(\delta_m-\delta+\theta_m) - E^2Y_s^2 = 0. \end{aligned} \quad (7.12)$$

The lengthy expression $h(x, E^{\lim})$ for E_m is obtained by replacing E in (7.12) by $E^{\lim}$ and then solving the resulting quadratic equation for E_m (choose the positive solution).

In this paper it is convenient to specify the occurence of the excitation current limit in terms of the reactive power loading parameter Q and calculate the corresponding value of $E^{\lim}$ used in the lengthy expression for E_m. Suppose the limit is reached at $Q = Q^{\lim}$. Then, $E^{\lim}$ is the value of E computed from equations (7.9)–(7.11) when E_m is set to its constant value and δ_m, δ, V are equilibrium solutions of (7.1)–(7.4) when $Q = Q^{\lim}$.

REFERENCES

[1] L. H. Fink, Ed., "Proceedings: Bulk power system voltage phenomena-voltage stability and security," *EPRI Rep. EL-6183*, Jan. 1989.

[2] ——, "Proceedings: Bulk power system voltage phenomena, voltage stability and security," *ECC/NSF Workshop*, ECC Inc., Aug. 1991.

[3] C. Barbier and J.-P. Barret, "Analyse des phénomènes d'écroulement de tension sur un réseau de transport" (An analysis of phenomena of voltage collapse on a transmission system), *Revue Generale de L'Électricite*, Tome 89, pp. 672–690, Oct. 1980.

[4] W. R. Lachs, "System reactive power limitations," *IEEE 1979 Power Engineering Society Winter Meeting*, 79CH1417-5 PWR, A79 015-9.

[5] P. Borremans *et al.*, "Stabilité de tension: Aspects fondamentaux et comparaison de critère pratiques," *CIGRÉ Report 38-11*, 1984.

[6] Th. Van Cutsem, "Network optimization-based reactive power margin calculation," in *Power Systems Modeling and Control Applications*, A. J. Calvaer, Ed. IFAC Proceedings Series, 1989, pp. 195–201.

[7] M. M. Begovic and A. G. Phadke, "Analysis of voltage collapse by simulation," *Int. Symp. Circuits and Systems*, May 1989.

[8] M. A. Pai and M. G. O'Grady, "Voltage collapse analysis with reactive generation and voltage dependent load constraints," *Electric Machines and Power Systems*, vol. 17, pp. 379–390, 1989.

[9] K. T. Vu and C. C. Liu, "Dynamic mechanisms of voltage collapse," *Systems and Control Letters*, vol. 15, pp. 329–338, 1990.

[10] T. Van Cutsem, "A method to compute reactive power margins with respect to voltage collapse," *IEEE Trans. Power Systems*, vol. 6, pp. 145–156, Feb. 1991.

[11] T. J. Overbye and C. L. DeMarco, "Voltage security enhancement using energy based sensitivities," *IEEE Trans. Power Syst.*, vol. 6, pp. 1196–1202, Aug. 1991.

[12] I. Dobson and H.-D. Chiang, "Towards a theory of voltage collapse in electric power systems," *Systems and Control Letters*, vol. 13, pp. 253–262, 1989.

[13] I. Dobson and L. Lu, "Immediate change in stability and voltage collapse when generator reactive power limits are encountered," in [2].

[14] L. Lu, "Immediate change in stability and voltage collapse when power system limits are encountered," M.S. thesis, Univ. Wisconsin, Madison, WI 1991.

[15] V. Ajjarapu and B. Lee, "Bifurcation theory and its application to nonlinear dynamical phenomena in an electrical power system," *IEEE Trans. Power Syst.*, vol. 7, pp. 424–431, Feb. 1992.

[16] E. H. Abed, *et al.*, "On bifurcations in power system models and voltage collapse," in *Proc. 29th Conf. Decision and Control*, pp. 3014–3015, 1990.

[17] B. M. Weedy, *Electric Power Systems*. Great Britain: Wiley, 1972, pp. 55–60.

[18] A. Capasso and E. Mariani, "Influence of generator capability curves representation on system voltage and reactive power control studies," *IEEE Trans. PAS*, vol. PAS-97 pp. 1036–1041 July/Aug 1978.

[19] J. M. T. Thompson and H. B. Stewart, *Nonlinear Dynamics and Chaos*. Britain: Wiley, 1987.

[20] J. Gukenheimer and P. Holmes, *Nonlinear Oscillations, Dynamical Systems and Bifurcations of Vector Fields*. New York: Springer-Verlag, 1986.

[21] NAG Fortran Library Manual Mark 13, vol. 2, D02EBF, July 1988.

[22] C. A. Cañizares and F. L. Alvarado, "Computational experience with the point of collapse method on very large AC/DC systems," in [2].