Although the function is no longer convex, it seems to find $\mu(M_a)$ reliably enough to greatly offset this disadvantage. In cases where local instead of global maxima occur, the algorithm simply reverts to (1).

The new method can easily be extended to the formulation of [8] by using the dependencies previously shown. Let $S \in \mathscr{S}$ be the scaling matrix obtained at a stationary point of the scaled singular value in (1) and denote by Y_1 the corresponding right singular vector. Then the optimal solution x^* to (16) is

$$x^* = \frac{S^{-1}Y_1}{\|S^{-1}Y_1\|}.$$

Replacing S and Y_1 with the relationships developed earlier and carrying out the multiplication, it is easily seen that

$$\begin{bmatrix} 1 \\ y_{21} \\ \vdots \\ y_{n1} \\ y_{n+1} \\ y_{21}y_{n+1} \\ \vdots \\ y_{n1}y_{n+1} \\ y_{2n+1} \\ y_{21}y_{2n+1} \\ \vdots \\ y_{n1}y_{(n-1)n+1} \end{bmatrix}. \tag{17}$$

Here, Y_1 has simply been scaled so that $y_{11} = 1$. As before, only $2(n-1)$ free variables remain making the optimization process much more efficient. Using the system of Example 1, the optimal $S^{-1}Y_1$ becomes

$$S^{-1}Y_1 = \begin{bmatrix} 1 \\ y_{21} \\ y_{31} \\ y_{41} \\ y_{21}y_{41} \\ y_{31}y_{41} \\ y_{71} \\ y_{21}y_{71} \\ y_{31}y_{71} \end{bmatrix} = \begin{bmatrix} 1 \\ 1.236 \\ 1.094 \\ 0.855 + 0.519i \\ 1.056 + 0.641i \\ 0.935 + 0.567i \\ 0.355 + 0.935i \\ 0.438 + 1.155i \\ 0.388 + 1.023i \end{bmatrix}$$

with corresponding

$$\|M_a x^*\| = 8.25 = \bar{\sigma}(DM_aD^{-1}).$$

VI. Conclusion

A new method that significantly reduces the number of optimization variables required in the calculation of the structured singular value for scalar or block structured uncertainties is presented. This approach takes advantage of the structure of the uncertainty matrix Δ to provide a reduction in the number of free optimization variables. For the case of an uncertainty matrix with n^2 nonzero 1×1 blocks, we have shown that the structured singular value may be computed with $2(n-1)$ rather than $n^2 - 1$ free variables. It should be noted, however, that for this class of uncertainties the nonsimilarity scaling method of [2] also requires only $2(n-1)$ optimization variables over matrices of dimension $n \times n$ (not $n^2 \times n^2$). Hence, nonsimilarity scaling still offers the most efficient method of calculating μ for this uncertainty class.

Examples 2 and 3 illustrate how the new scaling technique offers significant computational advantages for block structured uncertainties over previous methods by again reducing the number of optimization parameters. These reductions are also shown to hold for the vector optimization approach developed in [8].

Recent work [11] has revealed that the results presented in this note may also be used to derive explicit expressions for the elements of the nonsimilarity scaling matrices in terms of the similarity scaling variables and further research is proceeding to remove the stationarity assumptions and to extend the results to more general classes of uncertainties.

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Computing an Optimum Direction in Control Space to Avoid Saddle Node Bifurcation and Voltage Collapse in Electric Power Systems

Ian Dobson and Liming Lu

Abstract—**This note computes an optimum direction of controls to avoid power system voltage collapse. That is, given a load power forecast, we compute the sensitivity with respect to controls of a load power margin measuring proximity to voltage collapse. The computation is simple enough to contribute to the practical planning and control of a power system to avoid voltage collapse blackouts. The computation applies to avoiding saddle node bifurcation instability of a general dynamical system.**

I. Introduction

Voltage collapse is an instability of heavily loaded electric power systems which leads to declining voltages and blackouts. It is associated with bifurcation and reactive power limitations of

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the power system. Power systems are expected to become more heavily loaded in the next decade as the demand for electric power rises while economic and environmental concerns limit the construction of new transmission and generation capacity. Heavily loaded power systems are closer to their stability limits and voltage collapse blackouts will occur if suitable monitoring and control measures are not taken. Progress has been made in understanding voltage collapse and monitoring the proximity to voltage collapse with various indexes (most approaches are covered in [1], [2]). The sensitivity to controls of some of the indexes have been computed so that control action may be taken to optimally improve the index and the voltage stability of the system. For example, Tiranuchit and Thomas [3] computed the sensitivity of the minimum singular value of the system Jacobian and Overbye and DeMarco [4] computed the sensitivity of an energy function index. Another popular index is load power margin which may be computed by direct methods [5]–[7], optimization [8]–[11], continuation [6], [7], [12], and continuation with linear predictor and no corrector [13], [14]. This note computes the first-order sensitivity of a load power margin index to any controls or parameters appearing in the load flow equations.

If system parameters vary slowly and continuously, then voltage collapse can be explained as a dynamic consequence of a saddle node bifurcation instability in which the system operating equilibrium disappears [15]. Discrete contingencies such as loss of a power line can also weaken the system stability and contribute to voltage collapse; if this occurs, or is hypothesized to occur when assessing security, we use the saddle node bifurcation explanation *after* the discrete contingency. This note computes an optimum control direction which steers the system away from a saddle node bifurcation and the associated voltage collapse.

We exploit the geometry of a control space Λ, the vectors of which contain real and reactive powers of the loads and generators and other control parameters such as settings of tap changing transformers and shunt capacitor devices. (The settings of tap changing transformers and shunt capacitor devices are discrete but we compute the control assuming they are continuous and then approximate the control with the nearest discrete setting [16].) Note that the "controls" include load powers which vary freely in normal operation and are only controlled in emergencies by selectively shedding loads. It is also useful to include design parameters in Λ when planning changes to a power system. Define Σ to be the control values in Λ at which the stable operating equilibrium has a saddle node bifurcation. Σ typically consists of curved hypersurfaces and their intersections.[1] We denote the current control parameters by λ_0; the position of λ_0 relative to Σ is the key to monitoring and avoiding voltage collapse. We assume that a load power forecast is given in the form of a linear increase from the current load powers. This forecast is described by a ray in Λ based at λ_0; controls other than load powers are constant along the ray. The ray will intersect Σ at the point λ_* at which the system bifurcates and loses stability and the proximity to voltage collapse can be measured by the distance $|\lambda_0 - \lambda_*|$. Since the controls other than load powers were fixed in the load forecast ray, $\lambda_0 - \lambda_*$ lies in the load power subspace of Λ and $|\lambda_0 - \lambda_*|$ is a load power margin. If $|\lambda_0 - \lambda_*|$ is too small, then control action should be taken to increase $|\lambda_0 - \lambda_*|$ in order to avoid or reduce the risk of voltage collapse. Control action is represented by a displacement of λ from λ_0 in Λ. This note computes $\nabla_\lambda(|\lambda - \lambda_*|)|_{\lambda_0}$, the gradient with respect to λ of the index $|\lambda - \lambda_*|$ evaluated at λ_0. $\nabla_\lambda(|\lambda - \lambda_*|)|_{\lambda_0}$ is the sensitivity of the load power margin with respect to control and its direction gives the optimum direction for control. We prove that $\nabla_\lambda(|\lambda - \lambda_*|)|_{\lambda_0}$ is parallel to the normal vector n_* to Σ at λ_* using generic assumptions and a dynamic power system model and show that it is straightforward to compute n_* and $\nabla_\lambda(|\lambda - \lambda_*|)|_{\lambda_0}$ from the load flow equations.

This note adopts the objectives of Kumano, Yokoyama, and Sekine [17] and we are indebted to these authors for the geometric idea of computing an optimum direction in a load power space which is augmented with other controls. Our computation of the optimum direction is based on the left eigenvector formula in Dobson [18] for the normal vector n_* to Σ. The formula for the normal vector in [17], which uses a right eigenvector, is only valid for power systems with symmetric Jacobians so that the left and right eigenvectors coincide. The computation in this note is much simpler and more direct than that of [17]. The idea of using a normal vector to determine which loads to shed appeared in [18].

II. Power System Static and Dynamic Models and the Normal Vector n_*

Consider the load flow equations

$$0 = f(x, \lambda) \quad \text{or} \quad \begin{array}{l} 0 = f_1(x, \lambda) \\ 0 = f_2(x, \lambda) \end{array} \tag{1}$$

where $x \in \boldsymbol{R}^n$ is a state vector describing the bus voltage phasors and $\lambda \in \Lambda$ is a control vector. f_1 describes real power balance at the generators and f_2 describes real and reactive power balance at the loads.

We seek to compute the sensitivity $\nabla_\lambda(|\lambda - \lambda_*|)|_{\lambda_0}$ from the static equations (1) but want the results to be consistent with the sensitivity for a general dynamic model extending (1) to which the voltage collapse theory of [15] applies. A dynamic model [18] which extends (1) by including generator swing dynamics and general load dynamics is

$$\dot{z} = h(z, \lambda) \quad \text{or} \quad \begin{cases} \dot{\omega} = f_1(\delta_G, y, \lambda) - \Delta\omega \\ \dot{\delta}_G = \omega \\ \dot{y} = g(f_2(\delta_G, y, \lambda), \omega). \end{cases} \tag{2}$$

The state vector $z = (\omega, x) = (\omega, \delta_G, y)$ where δ_G are the generator voltage angles in x and Δ is a diagonal matrix of generator dampings. g defines any smooth dynamic load model which depends on frequency ω and the real and reactive power balance at each load (see [15] for an example). Little is known about the large signal behavior of such load dynamics and a convincing function g is hard to obtain. For our purposes, we require the existence of g and the assumptions that g has a unique zero at $(0, 0)$ and that Dg, the Jacobian of g with respect to its first argument, is globally invertible. We also assume the technical condition that the operating region of (2) is contained in a compact positively invariant set; see [15].

[1] The set of all points of Λ at which there is a bifurcation is called the bifurcation set and it generally contains points other than Σ. These points correspond to saddle node bifurcations not involving the stable equilibrium point, Hopf bifurcations, and other bifurcations not generic in one parameter systems. We do not consider these bifurcations here: Saddle node bifurcations not involving the stable equilibrium do not affect the local stability of the stable operating equilibrium. Hopf bifurcations cause oscillatory instabilities, but are probably not important in voltage collapse because voltage collapses are not observed to be oscillatory. The nongeneric bifurcations are not expected to occur in practice.

We assume a load forecast in which the load powers increase as a linear function of a loading parameter t:

$$\lambda(t) = \lambda_0 + t\mu \tag{3}$$

where μ is a fixed vector of unit length defining the direction of the load forecast ray in Λ. We write $\lambda_* = \lambda(t_*)$ for the first intersection of the load forecast ray with Σ and t_* for the loading parameter at the bifurcation. (The disappearance of the operating equilibrium and the existence of λ_* is guaranteed by the absence of any equilibrium for absurdly high loadings.) If the load increase is slow enough to be modeled as a quasistatic variation of (2) and the resulting system $\dot{z} = h(z, \lambda(t))$ parameterized by t is assumed to be a generic one parameter system, then the theory of [15], [19] applies and we obtain the following conclusions about the structure of the bifurcation at (z_*, λ_*): If we write $h_z|_*$ for the Jacobian of h with respect to z evaluated at (z_*, λ_*), then $h_z|_*$ has a single, simple zero eigenvalue and corresponding right and left eigenvectors v'_*, w'_* so that $h_z|_* v'_* = w'_* h_z|_* = 0$. In addition, the bifurcation is a saddle node bifurcation satisfying the transversality conditions [20], [21], [19]

$$w'_* h_t|_* = w'_* h_\lambda|_* \mu \neq 0 \tag{4.1}$$

$$w'_* h_{zz}|_*(v'_*, v'_*) \neq 0. \tag{4.2}$$

Moreover, Σ is a smooth hypersurface near λ_* and has a well defined normal vector n_* at λ_*. Note that we write w'_*, n_*, and $\nabla_\lambda(|\lambda - \lambda_*|)|_{\lambda_0}$ as row vectors.

The formula from [18] for the normal vector n_* to Σ at λ_* is simply

$$n_* = w'_* h_\lambda|_* \tag{5}$$

where $h_\lambda|_*$ is the Jacobian of h with respect to λ evaluated at the bifurcation.

Now we demonstrate that the saddle node bifurcation of the dynamic model (2) at $(z_*, \lambda_*) = (0, x_*, \lambda_*)$ induces a bifurcation of the solutions of the static equations (1) at (x_*, λ_*) and the Jacobian $f_x|_*$ has a zero eigenvalue with corresponding left eigenvector w_*. Moreover the normal vector n_* of (5) is also given by

$$n_* = w_* f_\lambda|_*. \tag{6}$$

This allows n_* to be calculated from the static equations (1) assuming only the general form, but not the details, of the dynamical model (2). To derive (6), first note that the unique zero of g at $(0,0)$ implies that (1) has an equilibrium at (x_0, λ_0) if and only if (2) has an equilibrium at $(0, x_0, \lambda_0)$. The respective Jacobians of (1) and (2) are

$$f_x = \begin{pmatrix} f_{1x} \\ f_{2x} \end{pmatrix} \quad \text{and} \quad h_z = \begin{pmatrix} -\Delta & f_{1x} \\ I & 0 \\ g_\omega & Dgf_{2x} \end{pmatrix}.$$

Writing $w'_* = (w'_{1*}, w'_{2*}, w'_{3*})$, observe that $(w'_{1*}, w'_{2*}, w'_{3*})h_z|_* = 0$ implies that

$$0 = (w'_{1*}, w'_{3*})\begin{pmatrix} f_{1x} \\ Dgf_{2x} \end{pmatrix} = (w'_{1*}, w'_{3*}Dg)\begin{pmatrix} f_{1x} \\ f_{2x} \end{pmatrix}$$

so that $w_* = (w'_{1*}, w'_{3*}Dg)$ is the left eigenvector of f_x corresponding to a zero eigenvalue. (w_* is unique since if $w_* f_x = (w_{1*}, w_{3*})f_x = 0$ then $(w_{1*}, w_{1*}\Delta - w_{3*}(Dg)^{-1}g_\omega, w_{3*}(Dg)^{-1})h_z|_* = 0$ and $w_{1*} = w'_{1*}$ and $w_{3*} = w'_{3*}Dg$ since w'_* is unique.) Moreover

$$n_* = w'_* h_\lambda|_* = (w'_{1*}, w'_{2*}, w'_{3*})\begin{pmatrix} f_{1\lambda}|_* \\ 0 \\ Dgf_{2\lambda}|_* \end{pmatrix}$$

$$= (w_{1*}, w_{3*})\begin{pmatrix} f_{1\lambda}|_* \\ f_{2\lambda}|_* \end{pmatrix} = w_* f_\lambda|_*. \tag{7}$$

It is straightforward to check that $w'_* h_{zz}|_*(v'_*, v'_*) = w_* f_{xx}|_*(v_*, v_*)$ where $v'_* = (0, v_*)$ and v_* is the right eigenvector of $f_x|_*$ corresponding to the zero eigenvalue. This, together with (7), shows that the bifurcation of (1) inherits from (2) the transversality conditions (4).

III. Computation of n_* and the Load Power Margin Sensitivity to Control

Given a load forecast (3), the system will bifurcate and collapse at $\lambda_* = \lambda_0 + t_* \mu$. We compute λ_* by solving a variant of the extended system equations [7], [5], [6]:

$$\left.\begin{aligned} f(x_*, \lambda_*) &= 0 \\ w_* f_x|_* &= 0 \\ w_* u - 1 &= 0 \end{aligned}\right\}. \tag{8}$$

(The last equation of (8) ensures that w_* is nonzero; in practice u can be chosen to be any nonzero constant vector.) Equations (8) can be solved for (x_*, w_*, t_*) using Newton–Raphson iteration. Then $\lambda_* = \lambda_0 + t_* \mu$. Note that equations (8) are a variant of the extended system equations in that they use left eigenvectors in place of the usual right eigenvectors. This has the advantage of computing the left eigenvector w_* required in the formula (6) for n_* as a by-product of the computation of (x_*, λ_*). The optimization methods of computing the load power margin also share this advantage since w_* is the Lagrange multiplier vector [8]. Continuation methods [7], [12], [13], [6] could also be used but would require a further computation of w_* from $f_x|_*$.

There is a sign ambiguity in the formulas (5) and (6) for n_* since both n_* and $-n_*$ are normal vectors to Σ. It is convenient to assume that the normal n_* is pointing "outward" so that changing λ in the direction n_* leads to the disappearance of the equilibrium x_*. Formula (6) becomes

$$n_* = \pm w_* f_\lambda|_*. \tag{9}$$

Strictly speaking, the sign of n_* should be determined by checking that the equilibrium x_* disappears when $\lambda = \lambda_* + \epsilon n_*$ for small positive ϵ. In practice, the sign of n_* should be apparent from the expectation that n_* should correspond to an increase in power for most loads. f_λ is easy to compute and evaluate at the bifurcation.

Now we prove that the load power margin sensitivity

$$\nabla_\lambda(|\lambda - \lambda_*|)|_{\lambda_0} = -(n_* \mu)^{-1} n_* \tag{10}$$

it follows from (10) that the optimum direction for control is $-n_*$. λ_* and t_* are well defined smooth functions of the current control parameters λ near λ_0 because Σ is a smooth hypersurface near λ_* and (4.1) implies that $n_* \mu \neq 0$ so that the load forecast ray intersects Σ transversally at λ_*. $\lambda_* = \lambda + t_* \mu$ and $|\mu| = 1$ imply that $\nabla_\lambda(|\lambda - \lambda_*|)|_{\lambda_0} = \nabla_\lambda t_*|_{\lambda_0}$. Specify Σ near λ_* as the zero set of a smooth real function α. Then $\alpha(\lambda_*) = 0$ and t_* satisfies $0 = \alpha(\lambda + t_* \mu)$ and $0 = (\nabla_\lambda \alpha)|_{\lambda_*}(I + \mu \nabla_\lambda t_*)|_{\lambda_0}$. Hence $0 = n_*(I + \mu \nabla_\lambda t_*|_{\lambda_0}) = n_* +$

$(n_* \mu)\nabla_\lambda t_*|_{\lambda_0}$ and $\nabla_\lambda t_*|_{\lambda_0} = -(n_* \mu)^{-1} n_*$, yielding (10). The geometric content is clear: the optimum direction to increase the distance in a given direction μ of a point λ_0 to a hypersurface Σ is antiparallel to the outward normal n_* to Σ.

It may not be desirable to use all the possible controls to improve voltage stability. For example, load powers can be controlled by shedding loads but this is only done in an emergency. In less extreme cases, the objective is to move further from voltage collapse using controls other than load shedding. The optimum control direction is then the projection of $-n_*$ onto the subspace of Λ containing the controls other than load powers.

Reactive power limitations of generators have a strong effect on the the load power margin and should be accounted for when the load power margin is computed [8], [13], [9], [10]. Since a generator reaching a reactive power limit changes the system (for example, generator voltage magnitude and reactive power output may interchange their roles as state and control variables), care must be taken that λ_* is computed for the system consistent with the generator reactive power limits. This system will often differ from the more lightly loaded system at λ_0.

IV. Example

We illustrate the computation of the sensitivity of the load power margin to controls with the Ward & Hale 6 bus system with a modification to bus 6 as shown in Fig. 1 (see [22] for a similar modification). The state vector x contains the voltage angles and magnitudes of buses 3, 4–6, 2; bus 1 is the slack bus. The control vector $\lambda = (P_3, Q_3, P_4, Q_4, P_5, Q_5, P_6, Q_6, P_2, V_2, n_4, n_7, C_4, C_6)$ contains real and reactive bus powers, bus 2 generator voltage magnitude, transformer tap ratios, and the capacitances at bus 4 and 6. λ_0 at the operating point x_0 is given by the values in Fig. 1. The load forecast is given by linearly increasing the load powers at buses 3, 5, 6 so that

$$\lambda(t) = \lambda_0 + t\mu = \lambda_0 + t(0.3207, 0.0758, 0, 0, 0.1749, 0.1049, 0.2915, 0.8746, 0, 0, 0, 0, 0, 0)^T. \quad (11)$$

We solve (8) for (x_*, w_*, t_*) (see Table I) and compute $\lambda_* = \lambda(t_*)$ from (11). The load power margin $|\lambda_* - \lambda_0| = t_* = 0.162$. The first nine columns of the f_λ matrix correspond to load powers and are a 9×9 identity matrix. The remaining columns correspond to the other controls. It is straightforward to compute the normal vector n_* using formula (9) (see Table II).

The optimum direction for parameter control is $-n_*$. For example, inspection of the components of $-n_*$ shows that Q_6 is the load bus power must influential on the load power margin and bus 6 is the load which can most effectively be shed. Bus 6 can also be interpreted as the weakest system bus [13]. If no loads are to be shed, then the optimum control direction is given by the negative of the second row of n_* in Table II, showing, as might be expected, that increasing the capacitor values and decreasing the tap ratios steers the system away from voltage collapse. The relative sizes of the components of $-n_*$ give the relative proportions of each control to most effectively increase the load margin.

V. Discussion and Conclusions

When the load power margin is computed by a constrained optimization technique, the sensitivity of the load power margin to the controls such as load powers on the right-hand side of the constraints is given by the Lagrange multiplier vector at the bifurcation [8]–[10]. This is useful because weak buses or areas may be identified as those with large voltage magnitude sensitivities to generator power variations [14], [9], [10]. The Lagrange multiplier result follows from our computation because the left eigenvector w_* of the Jacobian is the Lagrange multiplier vector at the bifurcation and the f_λ matrix is simply the identity matrix in this case so that the Lagrange multiplier vector is n_*. Our computation confirms the Lagrange multiplier results and generalizes them to any controls or parameters appearing in the load flow equations.

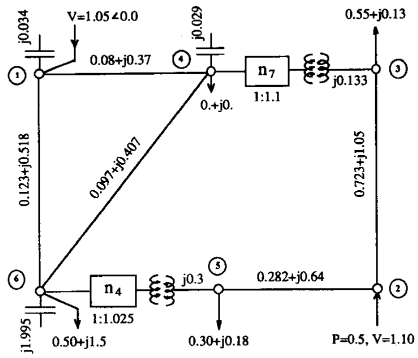

Fig. 1. Modified Ward & Hale 6 bus system.

Overbye [4] and Kumano [17] suggest using the optimum control direction for real time avoidance of voltage collapse: Once the optimum control direction $-n_*$ is computed, a term can be included in the optimal power flow which has minimum cost when selected controls are changed in the direction $-n_*$. In an emergency, it would be appropriate to neglect cost and use all available controls according to the direction $-n_*$, including shedding selected loads. Note that the optimum combination of loads to be shed that we compute is strictly valid only for small load changes.

Liu and Vu [23] have demonstrated examples in which it can be advantageous to lock tapchangers to improve voltage stability; we suggest that the best direction and relative proportions of tapchanges to avoid voltage collapse may be deduced from the sensitivities of the load power margin to the tap changer settings.

This note uses an index of voltage collapse which assumes a load forecast. If the forecast is unavailable, or a measure of system robustness independent of load variation is wanted, then the sensitivity computation also works with the load power margin index of [24], [25] which measures the minimum distance (worst case load power variation) from λ_0 to Σ.

Basic to our computation is the smooth slow variation of the controls λ which allows quasistatic analysis of (2). This is an idealization; in practice, the system state and controls are subject to disturbances and variations which may be discrete or not slow compared to the dynamics of (2). A consequence is that the system state will not exactly track the stable equilibrium of (2) and may be displaced outside the region of attraction of the stable equilibrium just before bifurcation (see [4] for further discussion). Nevertheless, we suggest that it is still sensible to control the real system with the controls described above because steering the idealized system away from the bifurcation is a precondition for stability of the real system and will tend to increase the minimum size of disturbance which can destabilize the real system.

We have computed the sensitivity to controls and parameters of a load power margin index of voltage collapse. The computa-

TABLE I

	α_3	V_3	α_4	V_4	α_5	V_5	α_6	V_6	α_2
x_0	−0.216	1.106	−0.173	1.035	−0.225	1.101	−0.226	1.170	−0.019
x_*	−0.263	0.979	−0.203	0.910	−0.266	0.895	−0.253	0.892	−0.114
ω_*	0.181	0.258	0.145	0.275	0.303	0.473	0.298	0.632	−0.017

TABLE II

	P_3	Q_3	P_4	Q_4	P_5	Q_5	P_6	Q_6
n_*	0.114	0.162	0.091	0.172	0.190	0.296	0.187	0.395
	P_2	V_2	C_4	C_6	n_4	n_7		
	−0.010	−0.616	−0.314	−0.143	0.309	0.117		

tion applies to a dynamic load model with a general form of load dynamics but may be conveniently carried out on the load flow equations. The controls and parameters may appear anywhere in the load flow equations. The computation is simple, with the main computational burden being the computation of a left eigenvector at the bifurcation. However, two of the methods of computing the load power margin produce the required eigenvector as a by product. The computation has several applications to power system planning and security.

The computation applies to control or parameter changes to steer a general parameterized dynamical system away from a saddle node bifurcation if the formula (5) is used to calculate the normal vector n_*.

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Robust Discrete Controllers Guaranteeing l_2 and l_∞ Performances

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Abstract—**This note presents an algorithm to design discrete time controllers which are robust with respect to parameter variations and**

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