

**Monitoring of single and multiple line outages with synchrophasors in areas of  
the power system**

by

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A Thesis submitted to the graduate faculty  
in partial fulfillment of the requirements for the degree of  
DOCTOR OF PHILOSOPHY

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2015

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## ABSTRACT

When power grids are heavily stressed with a bulk power transfer, it is useful to have a fast indication of the increased stress when multiple line outages occur. Reducing the bulk power transfer when the outages are severe could forestall further cascading of the outages. Phasor measurement units (PMUs) are vital elements for monitoring and control of these heavily stressed power system. This work presents a new approach to implement and utilize PMU information to monitor operational transfer capability and limits based on voltage phasor angles with respect to thermal limits of transmission lines. This work demonstrates an algorithm to obtain thresholds based on the angle and then quickly deploy PMU data to monitor stress changes due to single and multiple outages in real time to send fast notification of emergency situations. Area angle uses the topology and the synchronized measurements of angles across an area of power system to measure stress caused by outages within the area. The proposed algorithm is easy, quick and computationally suitable for real systems to capture bulk stress caused by outages and also identify local stress. This work first illustrates the idea of area angle in a Japanese test system and then explores the choice of the border buses. It further investigates the relation between area angle to area susceptance and supports the findings in two areas of the Western North American power system. Finally, this work develops a procedure to define thresholds for the area angle that relate to the maximum power that can be transferred through the area until a line limit is reached. The algorithm finding the area angle thresholds offline and then in real time monitoring the area angle and comparing it to the thresholds after multiple outages determines the urgency (or not) of actions to reduce the bulk transfer of power through the area. The procedure also identifies exceptional cases in which separate actions to resolve local power distribution problems are needed. The findings are supported by testing on a 1553 bus reduced model of the Western interconnection power system.

## CHAPTER 1. OVERVIEW

### 1.1 Introduction

Transferring bulk power through a geographical area is a basic and economically important function of the power transmission system. But the maximum power flowing in individual transmission lines is limited, and this in turn limits in a complicated way the maximum power that can flow through the area. Another consideration is satisfying these limits when one or more transmission lines is outaged, and for severe outages it may be necessary to reduce the power flow through the area.

We are interested to monitor single and cascading outages using synchrophasor data and to determine online the maximum transfer capability of an area with respect to line limits after single and multiple line outages. Synchrophasor measurements of voltage angles at all the area tie lines can be used to indicate the severity of multiple outages. These synchrophasor measurements are readily combined into an “area angle” that can quickly track the severity of multiple outages after they occur. This thesis applies, analyzes, and tests the idea of area angle and shows that the area angle can track the maximum power transfer of the area with respect to thermal limit of lines after single and multiple outages. The area angle monitoring is a practical way to measure online the area stress caused by transferring power through the area.

The thesis also determines offline thresholds based on the area angle corresponding to the desired limits of the maximum power transfer, such as the maximum power transfer under any single contingency (known as the “N-1 limit”). These maximum power transfer thresholds are typically calculated offline, and are used later in real time to determine the severity of the multiple outages and emergency situations in the network. Since area stress can be quantified

in terms of the maximum power that could enter the area, first limits of the maximum power that could enter the area after the outages can be determined and then the corresponding area angle thresholds for them can be set. Then in real time comparing the area angle after outages with its thresholds quickly discriminates different stress situations inside the area, and indicates to the operators the urgency of actions that should or should not be taken to reduce the power transfer.

## 1.2 Contributions

This thesis shows that synchrophasor measurements around a power system area that are combined into an area angle can track bulk stress after multiple outages inside the area. One important advantage of this approach is that the effect of outages on the maximum power transfer through the area can be monitored. That is, our formulation in terms of area angle allows an emergency area angle threshold to be determined based on the maximum power transfers through the area. If the monitored area angle exceeds the emergency angle threshold, the area bulk power transfer should be reduced.

In industry, static feasibility boundaries such as those associated with transmission line limits can be determined from grid models with power flows based on SCADA and state estimation. Our work is different since we use synchrophasor measurements to monitor in real time the stress with regard to bulk power transfer through areas due to multiple outages inside the area. Methods based on the state estimator produce a much more detailed view of a representative power system condition averaged over the SCADA sampling period, and require some computation time for actionable information. Our method based on synchrophasors is approximate but faster, and will work under multiple outage conditions in which the state estimator may not readily converge. Also it needs fewer PMUs and does not require detailed observability of all of the detail of the state of the area. Once the border buses of an area are determined, finding the area angle from PMUs in the border buses is straightforward while in other methods the PMUs must be placed so that the system is observable.

Also, in contrast to other research that considers the worst operation scenarios predicted for static security, our method, getting the benefit of synchrophasors, finds the limits in the power

transfer based on angles and so make it possible to dynamically track the possible increase in power transfer and its changing margin using synchrophasor information online. Loadings on many overhead transmission lines in the U.S. are based on conservative criteria to avoid overloads. Online quick monitoring of area power transfer due to line limits after single or multiple outages may allow recapture of unused transfer capability and the lost opportunity costs in the dispatch process.

Finally, we summarize the contributions in terms of new formulation, analysis, testing and practical application as follows:

- In terms of formulation, this thesis introduces new formulation to find out stress across an area with respect to power transfer directions through the areas. The areal perspective contrasts with other work that is trying to monitor increased stress between two points, but which encounters difficulties in setting meaningful thresholds. Indeed our formulation quantifies stress with respect to the power transfer through areas and makes it possible to set thresholds and take actions to reduce power transfer that mitigates this stress if the thresholds are exceeded.
- In term of analysis, this thesis finds and establishes useful approximate relations between area angle and area impedance or susceptance and the maximum power transfer through the area.
- In term of testing, this thesis uses a real case of 1553 bus WECC system and tests the area angle method for both single and multiple outages. The results show that the method can be applied in large real power systems.
- In term of practical application, this thesis proposes an easy and quick way to find out stress in real time using area angle and so make it possible to take real time actions for emergency situations. The thresholds for the actions are straightforward to systematically compute offline.
- Several publications listed in [8.4](#) proposes the method, contributions and applications in real large power systems.

### 1.3 Dissertation Organization

Area angles are a way to quantify the stress across an area of a power system by combining synchrophasor measurements of angles at the border buses of the area. One use of the area angle is to quickly monitor stress changes due to line outages within the area. We are interested to obtain area angle thresholds corresponding to specific stress limits in the area and then observe the changes in the area angle caused by different outages inside the area in real time and get fast notification of different stress conditions in the area.

Chapter 2 provides reviews of previous work for wide area monitoring using PMUs and explores the possible directions that can be further investigated.

Chapter 3 explains the area angle, illustrates its use on a 30-bus Japanese test system, and discusses how to choose areas and border buses.

Chapter 4 develops the relationship of area angle with area susceptance after outages happen. It shows that the variation of the area angle for single line outages can be approximately related to the changes in the overall susceptance of the area after outages and the line outage severity. This chapter supports the finding using two areas of the WECC system.

Chapter 5 proceeds to multiple outages and shows the monitoring of several outages using PMU data and using area angle. It introduces the idea of setting thresholds so the outages can be classified and the emergency situation after outages can be monitored.

Chapter 6 investigates and develops methodology to find thresholds of angle off line. The overall strategy is to set thresholds based on the line limits in terms of the economically significant maximum power transfer through the area, and then converts the threshold on the maximum power transfer to an equivalent threshold on the angle between the buses. Then in real time monitoring the area angle and comparing it to the area angle threshold can detect safe, alarm or emergency situations to indicate the urgency of action that is needed to reduce the power transfer in order to maintain security.

Chapter 7 investigates the relationship between area angle and maximum power transfer and relates it to the generation shift factor of the outaged and congested line.

Chapter 8 discusses possible future work and some assumptions in the thesis. This chapter

also summarizes and concludes the thesis.

The chapters are written to be largely self contained.

## CHAPTER 2. REVIEW OF LITERATURE

Operation of the power system has become more complicated as load is increasing and additional market forces are also in play. With increasing and variable demands placed on the power transmission system, areas of the power grid are often stressed by bulk transfers through the area. It is important to be able to quickly determine the severity of the outages so that the appropriate remedial actions can be taken. Especially in the case of multiple outages, a quick response could prevent further cascading and a blackout. Many observed cascading blackouts start with a few outages occurring more slowly, which gives a possibility of quick action to forestall the subsequent, faster cascading processes that lead to a widespread blackout. It is well appreciated that major blackouts have occurred partly due to lack of comprehensive situational awareness of the power grid (1; 2).

It is essential to provide wide area monitoring and control in this stressed power system. Indeed, new technologies make it possible to monitor and control these stressed power systems quickly. Synchronized phasor measurements (3) are becoming more widespread and wide area monitoring and control using phasor measurement units (PMUs) make it possible (4; 5; 6) to monitor and manage system stresses in order to keep the whole system stable and reliable. Several real time operation tools for wide area monitoring have been used in the western interconnection (5) and in the eastern interconnection (7). Although we are trying to monitor line limits in this thesis, there are interesting works like (8) that are trying to monitor voltage stability using PMUs.

Our method focusses on measuring stress across a particular area of the power system using synchrophasor measurements around the border of the area; that is, synchrophasor measurements at all the tie lines of the area. These synchrophasor measurements around the border of the area are combined into a single angle across the area called the area angle. The area angle

obeys circuit laws and is derived from circuit theory in (9; 10; 11). The area angle concept is a generalization of the angle across a cutset area concept developed and proposed for stress monitoring in (12; 13). (14; 15; 16) presents the theory and (17) discusses the Kron reduction used to develop area angle.

There has been some previous work that combines phasor measurements at several buses. A weighted average of voltage magnitudes or reactive powers derived from WECC phasor measurements is discussed in (18). The weighted averages provide robust control signals that are the basis for wide area control schemes for transient and voltage stability. The weights are established by location and sensitivity considerations. Reference (18) also discusses weighting phasor voltage angles to calculate a center of inertia angle for an area. Some previous works on monitoring power system stress with phasor measurements have focused on the angle difference between two buses. Simulations of the grid conditions before the August 2003 USA/Canada blackout show that increasing large angle differences could be a blackout precursor (19). Simulations of the New England grid (20) show that angle differences can discriminate alert and emergency states (21). A large angle difference between two buses does indicate, in a general sense, a stressed power system, but it is difficult to interpret changes in the angle difference or set thresholds.

The advantage of combining the synchrophasor measurements around the border of an area into an area angle is that one is then monitoring stress in that particular area. Then the additional stress due to line outages inside the area can be quickly monitored in real time just after the outages occur. Furthermore, we will show that our formulation in terms of area angle allows an emergency area angle threshold to be determined based on the maximum power transfers through the area. If the monitored area angle exceeds the emergency angle threshold, the area bulk power transfer should be reduced.

Given suitable synchrophasor measurements available at a control center (22), the calculation of area angle is quick and easy so that the computations can be practical for large real systems. We note that synchrophasor measurements around the border of an area can be also advantageous for other applications such as combining AC voltage measurements in a transmission corridor to monitor voltage collapse (23) or locating line outages in the area (24; 25)

or stress between areas (13).

More generally, synchrophasor measurements provide fast monitoring of bus voltages over a wide area. As more synchrophasors are deployed, one of the challenges is summarizing and understanding the new data. One advantageous approach is to use physical principles to combine together synchrophasor measurements into quantities that are more meaningful and actionable. We combine voltage angles around the border of an area of the power system into a bulk angle across the area.

In somewhat related work by other authors, static feasibility boundaries such as those associated with transmission line limits can be determined from grid models with power flows based on SCADA and state estimation. For example, (26; 27; 28) compute minimum security margins under operational uncertainty with respect to thermal overloads. Also (29) provides a tool for computation of transfer capability margins. The methodology proposed in this dissertation that is fully discussed in (30; 31; 32; 33) uses synchrophasor measurements to monitor in real time the stress with regard to bulk power transfer through areas due to multiple outages inside the area. Methods based on the state estimator produce a much more detailed view of a representative power system condition over the SCADA sampling period, and require some computation time for actionable information. This method proposed based on synchrophasors is approximate but faster, and will work under multiple outage conditions in which the state estimator may not readily converge. The proposed method applies the real time value of the PMUs in all the buses around an area to monitor the severity of the outages that happen in the area. It also further develops thresholds based on the angle so emergency situation after multiple outages can be monitored.

## CHAPTER 3. MONITORING SINGLE OUTAGES

### General Nomenclature

$\theta$	bus voltage angles
$\theta_{\text{area}}$	area voltage angle
$B$	susceptance matrix
$b_{\text{area}}$	area susceptance
$P$	real power
$\Lambda$	diagonal matrix of line susceptances
$A$	bus line incidence matrix
$\rho$	power transfer distribution factor

### 3.1 Introduction

Synchrophasor measurements provide fast monitoring of bus voltages over a wide area. As more synchrophasors are deployed, one of the challenges is summarizing and understanding the new data. One advantageous approach is to use physical principles to combine together synchrophasor measurements into quantities that are more usable. This chapter studies how voltage angles may be combined into angles across areas of the power system. The concept of the voltage angle across a power system area is new and is described in detail in (9; 10), including how it derives from circuit theory principles. We begin with a brief overview of the voltage angle across an area in the DC load flow case. The complex voltage difference across an area in the AC load flow case is explained in (9).

The voltage phasor angle across an area is formed by suitably combining voltage angles at all the buses on the border of the area to give a single number that is the angle across the area.

For example, to get the angle difference north-south across an area, a weighted combination of angles at buses on the southern tie lines is subtracted from a weighted combination of angles at buses on the northern tie lines. The angle across an area is useful because it summarizes the circuit behavior of the area. The angle across an area behaves similarly to the angle difference across a transmission line. In particular, the angle across the area satisfies the basic circuit laws so that the effective power flow through the area is the product of the angle across the area and the effective susceptance of the area. The area angle concept is a generalization of the angle across a cutset area concept developed and proposed for stress monitoring in (12; 11; 13). (The cutset area must be chosen to extend all the way across the power system whereas the area can, in principle, be any connected area.)

The angle across an area and its special case of the cutset area angle are promising for power system monitoring, and here we are most interested in further developing its application to quantify stress across an area that is caused by line outages inside the area.

### 3.1.1 Simple example of measuring stress with an angle.

The motivation for using area angles to measure stress can be illustrated with the simple example of a double line joining bus  $a$  to bus  $b$  shown in Fig. 3.1.

We assume lossless lines and a DC load flow and can compare two stress indices, the real power  $P_{ab}$  flowing from  $a$  to  $b$  and the angle  $\theta_{ab}$  between bus  $a$  and bus  $b$ . The DC load flow equation from Ohm's law is  $P_{ab} = b_{ab}\theta_{ab}$ , where  $b_{ab}$  is the total susceptance of the lines between  $a$  and  $b$ . If we regard the double line as an area and the buses  $a$  and  $b$  as the border buses of the area, then in this simple case  $\theta_{ab}$  is the area angle and  $b_{ab}$  is the area susceptance.

Under normal conditions,  $P_{ab}$  and  $\theta_{ab}$  are proportional and both indices indicate the stress on the lines. But the indices behave very differently if one of the lines outages as illustrated in Fig. 3.1. The power flow  $P_{ab}$  from bus  $a$  to bus  $b$  is unchanged, but the admittance  $b_{ab}$  is halved and the angle  $\theta_{ab}$  doubles. Thus the angle  $\theta_{ab}$  reacts to and indicates the increase in stress caused by the outage, whereas the power flow  $P_{ab}$  from bus  $a$  to bus  $b$  does not change and does not indicate the increase in stress.

We can also consider the limits on the indices that are determined by the thermal limits

(or other flow limits) of the lines. The line outage causes the maximum power flow  $P_{ab}^{\max}$  from bus  $a$  to bus  $b$  to halve, but the maximum angle  $\theta_{ab}^{\max}$  remains the same.

In summary, the  $\theta_{ab}$  index of stress is better than the power flow  $P_{ab}$  index of stress because it responds to a line outage, but its maximum value remains constant. One objective of the area angle is to try to get approximately similar benefits for a bulk measurement across an entire area.

## Motivation: Monitoring a double line

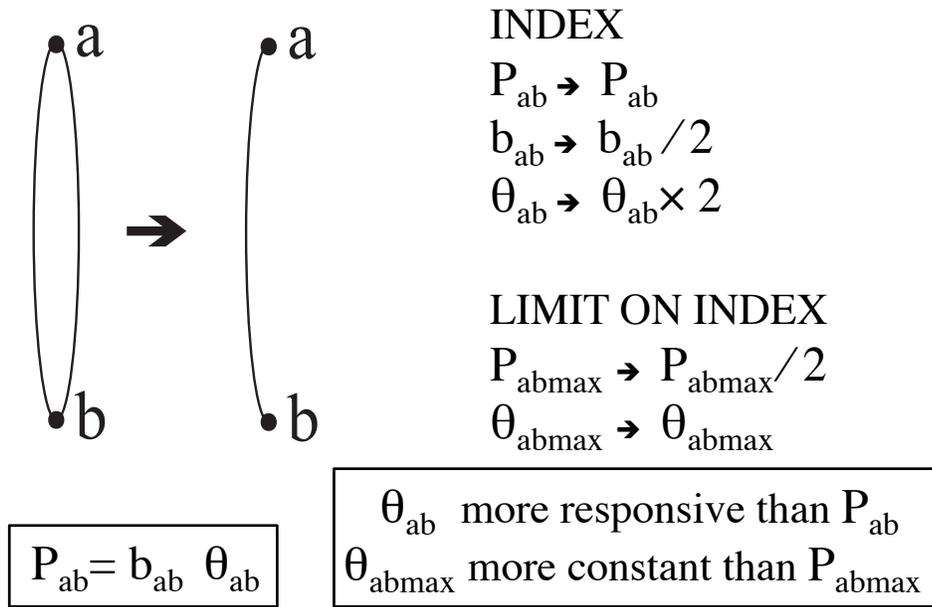


Figure 3.1 Comparing  $P_{ab}$  and  $\theta_{ab}$  for monitoring stress for an outage in a simple double line example.

### 3.1.2 Requirements for areas and their angles

There are some restrictions on the allowable areas and which synchrophasor measurements are needed in order to define an area angle (9):

1. The area must be connected. In other words, when all the tie lines of the area are tripped,

the area must form only a single island.

2. Synchrophasor measurements must be available at all the border buses of the area. We denote the border buses of the area by  $M$ . (Expressed in terms of network theory, the buses in  $M$  must form a nodal cutset, so that when the buses in  $M$  are removed the network is divided into two or more islands.) Each border bus corresponds to tie lines of the area to the rest of the system. This requirement prevents power flows entering or leaving the area without being tracked by the area angle. (It may be possible to relax this requirement in practice for border buses with high impedance, low voltage tie lines.)
3. Each border bus in  $M$  must be classified as either an “ $a$ ” bus or a “ $b$ ” bus. We write  $M = M_a \cup M_b$ . Then the area angle is defined across the area from the  $M_a$  buses to the  $M_b$  buses. For example,  $M_a$  can be the buses on the north border of the area and  $M_b$  can be the buses on the south border of the area, so that the area angle is defined from north to south. Given an area with border buses  $M$ , there are multiple ways to choose  $M_a$  and  $M_b$  and each choice gives a different area angle.
4. The weights used to calculate the area angle from the border bus angles are computed from a DC load flow model of the area. A recent base case of the DC load flow model is generally available (22). In our calculations, we use the base case DC load flow for the area angle weights, and do not, impractically, attempt to update the DC load flow model based on the outage we are trying to monitor.

It is not enough to choose an area and define a valid area angle according to these requirements; it is also important to choose an area angle that is meaningful and useful for power systems operation. In this chapter, we choose an area of the transmission system between major generation and major load to try to describe with the area angle the stress resulting from the transfer of power through the area and how the stress varies with line outages inside the area.

We note that synchrophasor measurements around the border of an area can be advantageous for other applications such as locating line outages in the area (24). (In particular, the measurements at the border can be augmented with synchrophasor measurements inside the

area and processed using a DC load flow model of the area. The processed measurements do not respond to line trips or power redispatches outside the area. The method extends previous methods that locate line trips in an entire network (22) so that they work in a particular area.) More generally, the border measurements can be used to effectively decouple the area from the rest of the interconnection (25). These methods will be particularly useful when utilities or ISOs in large interconnections restrict their attention to network models and phasor measurements for only their own area.

## 3.2 Stress Monitoring with Angles and Powers

### 3.2.1 Quantities for stress monitoring

Each line in the area has a limit on its real power flow that corresponds to the line thermal limit or is a proxy for other system limits. As the generation and load increase, there is increased stress on the transmission system, and lines may approach or reach their limits, especially under contingency conditions in which a line outages.

Our goal is to monitor a single quantity for the area that summarizes or captures well enough the degree to which the lines in the area are near their thermal limits. The single quantities that we consider are the real power into the area  $P_a^{\text{into}}$  and the area angle  $\theta_{\text{area}}$ . The real power into the area  $P_a^{\text{into}}$  is the sum of the real powers flowing into the area along the tie lines connected to the border buses  $M_a$ . (In practical power systems, flows in tie lines, or groups of tie lines, are monitored, and  $P_a^{\text{into}}$  is the corresponding combined flow for the tie lines connected to the “ $a$ ” border buses of an area.) Ideally, the monitored quantity changes from its base case value if a line outages, and the amount of change should indicate the severity of the outage in some sense. (It turns out that  $\theta_{\text{area}}$  is generally much more responsive to line outages than  $P_a^{\text{into}}$ .)

To determine the limits on the monitored quantity, we stress the power system by assuming a particular pattern of load and generation increase that increases the power flowing through the area. This stress is increased until the first line in the area reaches its thermal limit. The value of the monitored quantity in this stressed condition is its limiting value. For example,

the limiting value of  $P_a^{\text{into}}$  is written as  $P_a^{\text{intomax}}$ .

This limiting value of a monitored quantity can be determined either in the base case or in the contingency condition in which a particular line is outaged. Limits on the power flowing through the area have significant economic consequences when the limit is reached. Therefore we rank outage severity according to the corresponding limiting value of  $P_a^{\text{intomax}}$ . It is of interest to find out how much monitoring  $\theta_{\text{area}}$  gives some indication of the outage severity.

### 3.2.2 Formulas for voltage angle and power through the area

We summarize from (9) formulas related to the area angle and power entering the area. We consider an area  $R$  of the power system with border buses  $M$  and interior buses  $N$ . (The interior buses  $N$  have no incident lines joining them to buses outside  $R$ .) The susceptance matrix from the base case DC power flow is written as  $B$  or  $B^{(0)}$ , with subscripts indicating submatrices or elements of  $B$ . The following notation is used for column vectors of voltage angles and powers:

$\theta_m$  voltage angles at border buses  $M$

$P_m$  power injected at border buses  $M$

$\theta_n$  voltage angles at interior buses  $N$

$P_n$  power injected at interior buses  $N$

The vector of powers entering  $R$  into border buses  $M$  along tie lines not in  $R$  is

$$P_r^{\text{into}} = \sum_{j \notin R} (-B_{mj})(\theta_j - \theta_m) \quad (3.1)$$

The vector of powers  $P_m^R$  entering the border buses of  $R$  is the sum of the power  $P_m$  injected directly at the border buses and the power  $P_m^{\text{into}}$  flowing into the area along the tie lines:

$$P_m^R = P_m + P_m^{\text{into}}. \quad (3.2)$$

The susceptance matrix of the area  $R$ , considered as an isolated area without its tie lines, is  $B_{mm}^R$ . Retaining the border buses  $M$  and applying to  $R$  a standard Ward or Kron reduction

to eliminate the interior buses  $N$ , we get

$$P_m^{\text{Rred}} = P_m^{\text{R}} - B_{mn}B_{nn}^{-1}P_n, \quad (3.3)$$

$$B_{mm}^{\text{Rred}} = B_{mm}^{\text{R}} - B_{mn}B_{nn}^{-1}B_{nm}. \quad (3.4)$$

The reduced subnetwork Rred is electrically equivalent to R from the perspective of the border buses. Ohm's law is valid:

$$P_m^{\text{Rred}} = B_{mm}^{\text{Rred}}\theta_m. \quad (3.5)$$

We indicate the partition of the border buses into two sets  $M_a$  and  $M_b$  by specifying the row vector

$$(\sigma_a)_i = \begin{cases} 1 & \text{bus } i \text{ in } M_a \\ 0 & \text{otherwise.} \end{cases} \quad (3.6)$$

$\sigma_a$  corresponds to a new process of contracting the nodes of  $M_a$  as explained in (9).

Now we can define our main quantities in terms of (3.1)–(3.4) and (3.6). The power into the area through  $M_a$  is

$$P_a^{\text{into}} = \sigma_a P_m^{\text{into}}. \quad (3.7)$$

The susceptance of the area  $b_{\text{area}}$  is

$$b_{\text{area}} = \sigma_a B_{mm}^{\text{Rred}} \sigma_a^T. \quad (3.8)$$

The area angle  $\theta_{\text{area}}$  is

$$\theta_{\text{area}} = \frac{\sigma_a B_{mm}^{\text{Rred}} \theta_m}{b_{ab}}. \quad (3.9)$$

The equivalent power that flows from  $M_a$  to  $M_b$  through R is

$$P_{\text{area}} = \sigma_a P_m^{\text{Rred}}. \quad (3.10)$$

Ohm's law remains valid throughout the reduction and contraction so that

$$P_{\text{area}} = b_{\text{area}}\theta_{\text{area}}. \quad (3.11)$$

### 3.2.3 Problem setup

Now we use formulas (3.7)–(3.9), in the base case and just after each outage to determine area angle, area susceptance, and the power entering into the area. Furthermore, after we estimate the extra power that can enter into the area in the base case and after each outage, we can define the maximum amount of the area voltage angle and the maximum power entering into the area that are possible without violating any line limits.

We consider both the base case and single, non-islanding outages inside the area. These outages will cause the susceptance of the area and the area angle to change. For a general area that has parallel paths around the area that are parallel to the power flow through the area, an outage inside the area will cause some change in the power into the area tie lines. But if there are no such parallel paths around the area, as is the case for a cutset area, the power in the tie lines does not change for a non-islanding outage, and the power entering into the area will remain constant. (Note that a line outage that is non-islanding implies that all generation and load continues to be connected to all of the grid.)

In the next step, to determine the maximum area voltage angle or limit area voltage angle in the base case and after each outage, we stress the system until the first line violates its limit power flow which is the maximum power flow on that line. To do so, we determine the power transfer distribution factor for all lines considering the specified set of buses as injection buses. Then, for each line, considering its maximum amount of the power flow, the maximum value of injection which satisfies the power flow on that line can be estimated and then the minimum of these injection for all lines are considered as the maximum amount of the possible injection or stress that system can withstand in the base case or in that particular contingency before violating the limit power flow in lines. For this injection, the limit area voltage angle and limit power entering to the area can also be computed.

We use the following notation:

$P$	vector of net active power injected at buses
$P_{\text{line}k}$	power flow through line $k$
$P_{\text{line}}$	vector of power flows through lines
$P_{\text{area}}$	equivalent power flow through area
$P_{\text{stress}}$	amount of power injected to stress the system
$\theta_j$	voltage angle at bus $j$
$\theta$	vector of voltage angles at buses
$\theta_{\text{area}}$	voltage angle across the area
$\theta_{\text{line}}$	voltage angle in each line
$B$	susceptance matrix
$b_{\text{area}}$	area susceptance
$\Lambda$	diagonal matrix of line susceptances
$A$	bus line incidence matrix
$\rho_k^{rs}$	power transfer distribution factor for line $k$ with respect to injections in buses $r$ and $s$

It is convenient to evaluate some of the variables above in different cases and notate this as follows:

$X$	generic variable
$X^{(i)}$	$X$ evaluated for contingency number $i$ . The base case is contingency number 0.
$X^{k\text{max}}$	$X$ evaluated at the maximum stressed case obtained by applying stress until line $k$ reaches its maximum power flow rating.
$X^{(i)\text{max}}$	$X$ evaluated for the maximum stressed case obtained under contingency number $i$
$X^{\text{limit}}$	operating limit established for $X$

Line outage  $i$  is the  $i$ th outage inside the area that does not island the area.  $i = 0$  indicates the base case. The following calculation is done assuming the outage of line  $i$ , or the base case if  $i = 0$ .

From the DC load flow with line  $i$  outaged, we have

$$P^{(i)} = B^{(i)}.\theta^{(i)} \quad (3.12)$$

where  $P^{(i)}$  is the net active power injected at buses,  $B^{(i)}$  is the susceptance matrix and  $\theta^{(i)}$  is the bus voltage angles, all of them assuming the base case power injections. The area susceptance  $b_{\text{area}}^{(i)}$  after line outage  $i$  is computed from  $B^{(i)}$  similarly to the computation of  $b_{\text{area}}$  from  $B$  in (3.4) and (3.8).

Based on (3.9), the area angle after each outage is computed using

$$\theta_{\text{area}}^{(i)} = \frac{\sigma_a.B_{eq}.\theta_m^{(i)}}{b_{\text{area}}}. \quad (3.13)$$

Note that (3.13) uses the susceptance matrix and area susceptance evaluated *before* the outage of line  $i$ .  $B_{eq}$  is the susceptance of the equivalent reduced network.

Now we determine how much more power can enter into R when the area is stressed until a line limit is reached. It is convenient to first consider an area stress caused by injecting power at bus  $r$  (in the “generating side” outside or on the border of the area) and decreasing power at bus  $s$  (in the “load side” outside or on the border of the area). The voltage angles across the lines are

$$\theta_{\text{line}}^{(i)} = A^T.\theta^{(i)}, \quad (3.14)$$

and the power flows in lines are

$$P_{\text{line}}^{(i)} = \Lambda^{(i)}.\theta_{\text{line}}^{(i)}, \quad (3.15)$$

where  $\Lambda^{(i)}$  is the diagonal matrix of the susceptances with line  $i$  outaged. The maximum amount of the increase in the power flow of line  $k$  until its limit is reached is

$$\Delta P_{\text{line}k}^{(i)} = P_{\text{line}k}^{\text{limit}} - P_{\text{line}k}^{(i)}, \quad (3.16)$$

where  $P_{\text{line}k}^{\text{limit}}$  is the power flow limit of line  $k$ . Suppose that line  $i$  is outaged and that line  $k$  joins bus  $u$  to bus  $v$ . Then the power transfer distribution factor for line  $k$  is the amount of the increase in the power flow in line  $k$  due to a unit injection of power in bus  $r$  and a unit decrement of power in bus  $s$ :

$$\begin{aligned}\rho_k^{rs(i)} &= e_k^T \Lambda^{(i)} A^T (B^{(i)})^{-1} (e_r - e_s) \\ &= b_k (e_u^T - e_v^T) (B^{(i)})^{-1} (e_r - e_s)\end{aligned}\quad (3.17)$$

Here  $e_r$  denotes a vector with 1 at entry  $r$  and all other entries zero. Now, the maximum amount of injection in bus  $r$  and decrement from  $s$  until line  $k$  reaches its line limit is

$$\Delta P^{rs(i)k\text{max}} = \frac{\Delta P_k^{\text{limit}(i)}}{\rho_k^{rs(i)}}\quad (3.18)$$

Then the maximum possible injection  $P_{\text{stress}}^{(i)}$  at the  $r$  and  $s$  buses which satisfies all the line limits is the minimum amount of the maximum stress for each line:

$$P_{\text{stress}}^{(i)} = \text{Min}\{\Delta P^{\text{rs}(i)1\text{max}}, \Delta P^{\text{rs}(i)2\text{max}}, \dots, \Delta P^{\text{rs}(i)n\text{max}}\},\quad (3.19)$$

where  $n$  is the total number of lines inside the area. Adding the amount of injection  $\pm P_{\text{stress}}^{(i)}$  to the power injected at buses  $r$  and  $s$ , the voltage angle  $\theta^{(i)\text{max}}$  corresponding to the maximum stress for the outage  $i$ , can be calculated from (3.12). Then the border bus components of  $\theta^{(i)\text{max}}$  can be extracted and written as  $\theta_m^{(i)\text{max}}$ .

Using (3.19), the maximum power  $P_r^{\text{into}(i)\text{max}}$  entering into the area corresponding to the maximum stress for the outage  $i$ , can be calculated as well: We add the extra injection into bus  $r$  to its entering base case power to calculate  $P_r^{\text{into}(i)\text{max}}$ , the maximum power that could enter bus  $r$  after contingency  $i$ :

$$P_r^{\text{into}(i)\text{max}} = P_r^{\text{into}} + P_{\text{stress}}^{(i)}\quad (3.20)$$

The maximum area angle  $\theta_{\text{area}}^{(i)\text{max}}$  corresponding to the maximum stress case for the outage  $i$ , is a weighted combination of the border bus angles at the maximum stress case:

$$\theta_{\text{area}}^{(i)\text{max}} = \frac{\sigma_a \cdot B_{eq} \cdot \theta_m^{(i)\text{max}}}{b_{\text{area}}}\quad (3.21)$$

Furthermore, the calculation given above for area stress caused by injections at bus  $r$  and  $s$  can be extended to more general patterns of stress that distribute the injections with specific weights among two groups of buses outside or at the border of the area.

### 3.3 Results

The first test area for a 30 generator model of the eastern Japan is shown with the dark colored buses in Figure 3.2. The north-east border buses of the area are shown in red and the south-west border buses are shown in blue. This area was selected based on the position of the major generation and load in the network so that the transfer of power through the area captured a major power transfer of the system.

Using the base case DC load flow, the formula (3.9) for the area angle as a weighted combination of the border bus angles for this system evaluates to

$$\begin{aligned} \theta_{\text{area}} = & 0.0010 \theta_{45} + 0.2424 \theta_{54} + 0.1631 \theta_{60} \\ & + 0.1192 \theta_{59} + 0.0793 \theta_{72} + 0.3494 \theta_{73} + 0.0464 \theta_{39} \\ & - 0.2359 \theta_{77} - 0.5898 \theta_{71} - 0.1741 \theta_{78} \end{aligned}$$

The system data does not include line limits, so we obtained artificial line limits by coordinating them so that the N-1 criterion was minimally satisfied and then increasing each line limit by 20%.

We assume the system stress to be the pattern of power injection at each border bus of the area that is proportional to the base case tie line flow for each border bus.

The results in Figs. 3.3 and 3.4 show the area angle  $\theta_{\text{area}}$  and other quantities for all the non-islanding line outages inside the area. The base case is indicated by line number 0. The results are ordered by decreasing severity of line outage, and this can be verified by noting that the value of the maximum power  $P_a^{\text{into}}$  that can enter the area increases from left to right. It can be seen from Figs. 3.3 and 3.4 that the area angle  $\theta_{\text{area}}$  in most case decreases from left to right and so mostly responds to the severity of the outage. Since all the line outages are non-islanding and there are no parallel paths for power to flow around the area, the power  $P_a^{\text{into}}$  entering the area is constant and is not shown in the Figs. 3.3 and 3.4. That is, in this case

monitoring  $P_a^{\text{into}}$  gives no indication of the area stress changing when one of the line outages occurs. The area angle only depends on the line susceptance and the base case power flows. Very roughly and imprecisely speaking, it seems that the area angle changes when the line outages because the susceptance of the area changes whereas the power flow entering into the area remains constant. The results show that  $\theta_{\text{area}}^{(i)}$  is approximately inversely related to the area susceptance  $b_{\text{area}}^{(i)}$ , and this effect seems to correspond to some approximated version of Ohm's law.

There are some exceptions to the overall pattern of behavior such as the outages of lines 29 and 51. The outage of line 51 causes a disproportionately large decrease of the area admittance, and hence a disproportionately large increase in the area angle. This arises from the special configuration in which line 49 has a low admittance and line 51 has a high admittance. In the case of the outage of line 29 we can see that outage severity and area susceptance don't track each other. It seems that this effect can be traced to the load at bus 99. The load at bus 99 causes line 37 to have a smaller line limit and so in the case of outage of line 29, line 37 congests at an unexpectedly low stress level.

It should be noted that the area angle does not depend on the line limits. However, the maximum area angle and the severity of the outage measured by the maximum power into the area both depend on the line limits. Thus the assumed line limits do affect the outage severity and thus the extent to which the area angle indicates outage severity. We note the limitations of the simple scheme used to coordinate and obtain the artificial line limits used in our test case. Our experience so far on another test system is that more realistic and coordinated line limits can significantly improve the results.

The angle monitoring can work to some extent for an arbitrarily chosen area, but choosing a better area can give area angles that better summarize the effect of line outages as we now discuss.

One method to choose a good area is to select the  $M_a$  border near large generation and the  $M_b$  border near load in the network. Then the area angle from  $M_a$  to  $M_b$  reflects dominant power flows in the network and we get a better indicator for outages inside the area. We used this principle to choose the first test area of Fig. 3.2.

In the first test area, border bus 54 excludes two generators from the area. Removing border bus 54 from the first test area yields a second test area that includes the two generators inside the area. The results for the second test area are shown in Fig. 3.5, and it can be seen that area angle still responds, but tracks the severity of the line outage more imperfectly.

It seems better to avoid large load or generation inside the area. The anomaly of line outage 29 in figure 3.2 was attributed to load inside the area in the previous discussion. Another example is the third test area shown with the loads inside the area in Fig. 3.6. The resulting area angles are shown in Fig. 3.7. For the outage of lines 37 and 38 we get the same susceptance for the area, but the maximum power that can enter the area after the outage, which indicates the severity of the outage, is much greater for line 38 than for line 37. The difference in severity seems to be related to the load at bus 99. The third test area also includes a small part of the network, namely bus 73 and generator bus 26 and line 100, which is not in the main power flow from  $M_a$  to  $M_b$  when either line 50 or line 51 is outaged. The outage of either line 50 or line 51 has a significant effect on the area admittance but little impact on the severity.

### 3.4 Conclusion

We explore monitoring of area stress due to non-islanding line outages with area angles in a 30-bus Japanese test system. The area angle is easy to compute from synchrophasor measurements at the border buses of the area and it satisfies circuit laws. The area angle responds to the line outages by increasing. Given a suitable choice of area that separates the main generation and load, the amount of the increase in the area angle approximates the outage severity in most cases. In contrast, the power entering the area does not indicate these line outages. These first results suggest that real-time monitoring of angles across areas could be a promising way to help operators quickly detect stress due to line outages. Issues that are addressed in other chapters of the thesis include the effects of multiple and islanding outages and setting actionable thresholds to distinguish the severe outages. Issues to be resolved in future work include further guidelines for good choices of area and the possible use of multiple angles across an area.

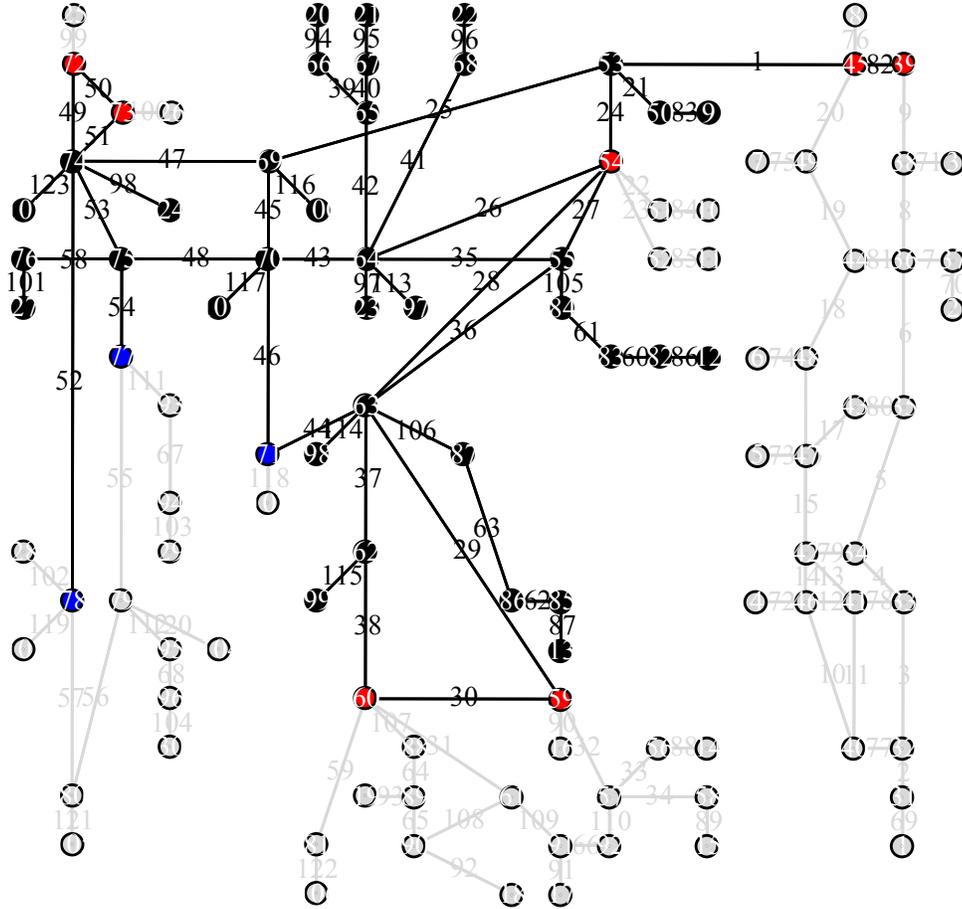


Figure 3.2 First test area of 30 generator Japanese system. Buses inside the area are black, east-north border buses are red, and south-west border buses are blue.

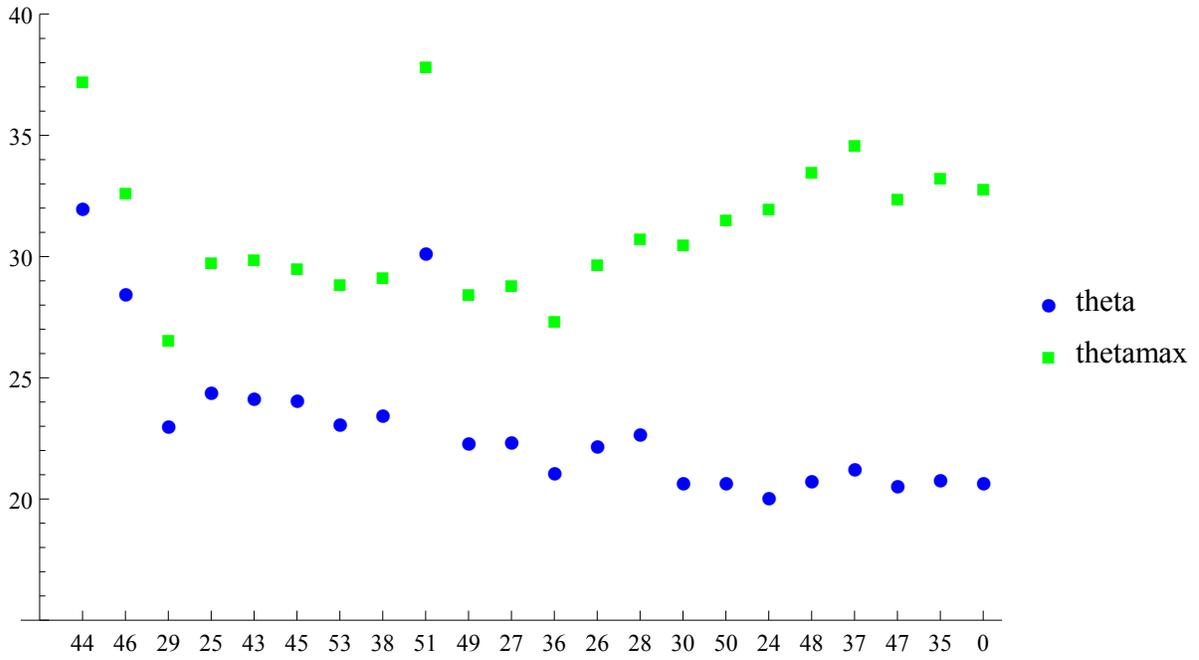


Figure 3.3 Area angle  $\theta_{\text{area}}^{(i)}$  and the maximum area angle  $\theta_{\text{area}}^{(i)\text{max}}$  after line outage  $i$  as  $i$  varies for the first test area.

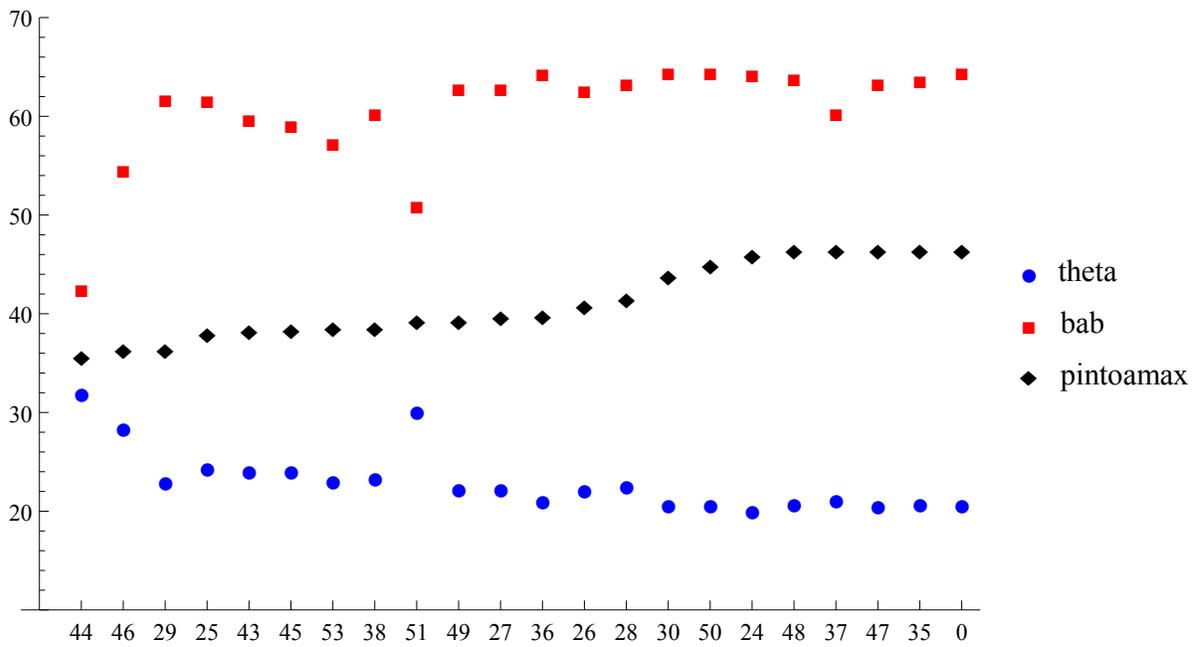


Figure 3.4 Area angle  $\theta_{\text{area}}^{(i)}$ , area susceptance  $b_{\text{area}}^{(i)}$ , and maximum power entering into the area  $P_a^{\text{into}(i)\text{max}}$  after each line outage  $i$  as  $i$  varies for the first test area.

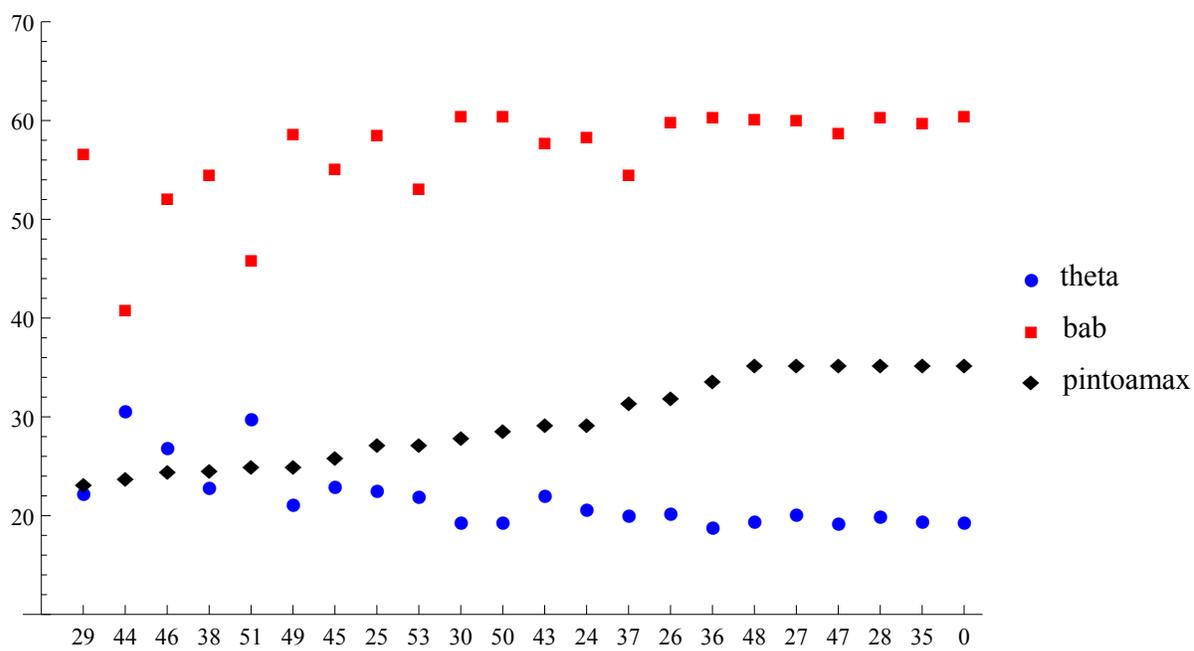


Figure 3.5 Area angle  $\theta_{\text{area}}^{(i)}$ , area susceptance  $b_{\text{area}}^{(i)}$ , and maximum power entering into the area  $P_a^{\text{into}(i)\text{max}}$  after each line outage  $i$  for the second test area.



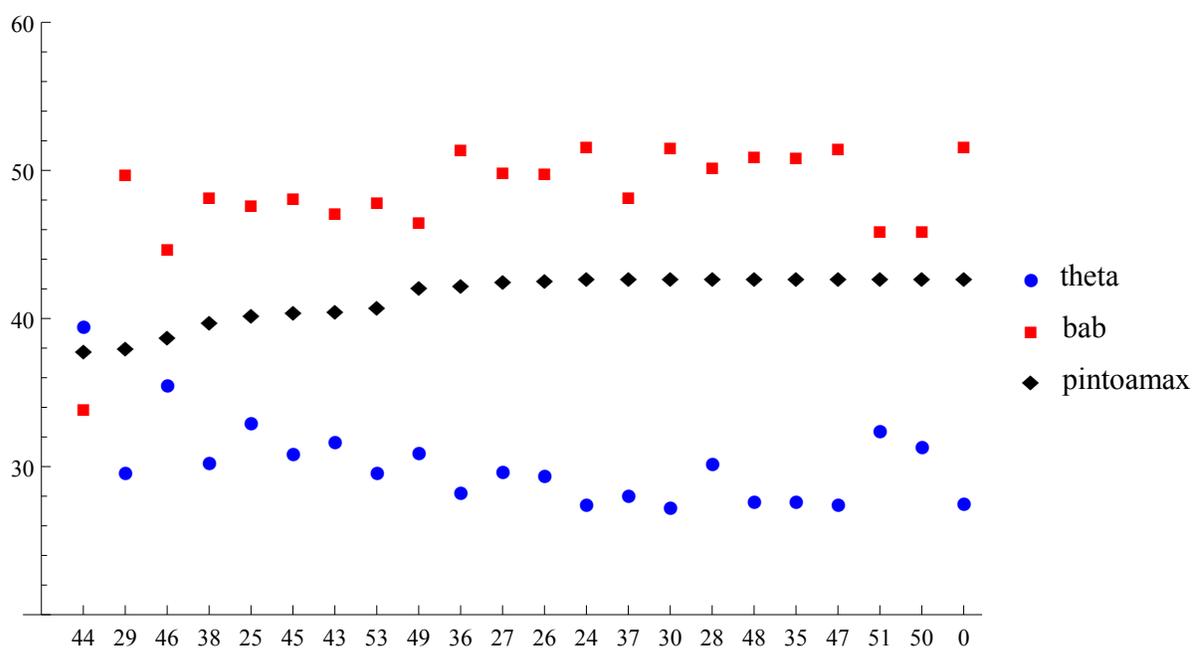


Figure 3.7 Area angle  $\theta_{\text{area}}^{(i)}$ , area susceptance  $b_{\text{area}}^{(i)}$ , and maximum power entering into the area  $P_a^{\text{into}(i)\text{max}}$  after each line outage  $i$  for the third test area.

## CHAPTER 4. TRACKING OF SEVERITY AND AREA SUSCEPTANCE BY AREA ANGLE

### 4.1 Introduction

The angle across an area of a power system is a weighted combination of synchrophasor measurements of voltage phasor angles around the border of the area (9; 10). The weights are calculated from a DC load flow model of the area in such a way that the area angle satisfies circuit laws. Area angles were first developed for the special case of areas called cutset areas that extend all the way across the power system (12; 11; 13). We previously showed how area angle responded to single line outages inside the area in some Japanese test cases in Chapter 3 and (31). The increase in area angle largely reflected outage severity and ways to choose the area were discussed.

Area angles are easy to calculate from synchrophasor measurements, and their general value is in giving a fast and meaningful bulk measure related to stress in a specific area of the power system. Area angle monitoring would complement slower monitoring via state estimation. The approximate relation of changes in area angle to outage severity suggests that it could be easier to set alarm thresholds using area angles. Another measure of stress, the voltage angle between two synchrophasor locations, responds to events throughout the power system, and is not easy to relate to a particular area. This work seeks to quantify outage severity with bulk area monitoring; to identify the line outage in the area see (24; 25; 22).

The area angle is measured across an area from one “side” of the area such as the north to the other side of the area such as the south. The area susceptance across the area can also be defined, and, according to Ohm’s law in a DC power flow context, the equivalent power flow through the area is the product of the area susceptance and the area angle. The power

flow through the area is often approximately constant, so it is intuitively plausible that when a line outages, the area susceptance decreases and the area angle increases. In this work, we explain and examine this approximate relationship between area angle and area susceptance in detail, including testing on two areas of the WECC. We choose these areas of the transmission system between major generation and major load to try to describe with the area angle the stress resulting from the transfer of power through the area from generation to load. There are some arts to choose a good area to be meaningful with respect to power flow direction. The testing on the WECC areas also shows how changes in area angle can usually distinguish the single line outage severity. This chapter is limited to single line outages that, for simplicity, do not island the system.<sup>1</sup>

## 4.2 Area Angle and Area Susceptance Formulas and Relations

### 4.2.1 Formulas for voltage angle and power through the area

We summarize from (9) formulas related to the area angle and power entering the area. We consider a connected area  $R$  of the power system with border buses  $M$  and interior buses  $N$ . The susceptance matrix from the base case DC power flow is written as  $B$ , with subscripts indicating submatrices or elements of  $B$ . The following notation is used for column vectors of voltage angles and powers:

$\theta_n$	voltage angles at interior buses $N$
$P_n$	power injected at interior buses $N$
$\theta_m$	voltage angles at border buses $M$
$P_m$	power injected at border buses $M$
$P_m^{\text{into}}$	power entering $R$ at border buses $M$
	along tie lines

The vector of powers  $P_m^R$  entering the border buses of  $R$  is the sum of the power  $P_m$  injected directly at the border buses and the power  $P_m^{\text{into}}$  flowing into the area along the tie lines:

$$P_m^R = P_m + P_m^{\text{into}}. \quad (4.1)$$

---

<sup>1</sup>Islanding line outages require assumptions about generator redispatch

The susceptance matrix of the area  $R$ , considered as an isolated area without its tie lines, is  $B_{mm}^R$ . Retaining the border buses  $M$  and applying to  $R$  a standard Ward or Kron reduction to eliminate the interior buses  $N$ , we get

$$P_m^{\text{Rred}} = P_m^R - B_{mn}B_{nn}^{-1}P_n, \quad (4.2)$$

$$B_{mm}^{\text{Rred}} = B_{mm}^R - B_{mn}B_{nn}^{-1}B_{nm} \quad (4.3)$$

We indicate the partition of the border buses into two sets  $M_a$  and  $M_b$  by specifying the row vector  $\sigma_a$ , whose  $i$ th component is one if bus  $i$  is in  $M_a$ , and is zero otherwise.

Now we can define our main quantities. An equivalent power (9) that flows from  $M_a$  to  $M_b$  through R is

$$P_{\text{area}} = \sigma_a P_m^{\text{Rred}}. \quad (4.4)$$

The susceptance of the area  $b_{\text{area}}$  is

$$b_{\text{area}} = \sigma_a B_{mm}^{\text{Rred}} \sigma_a^T. \quad (4.5)$$

The area angle  $\theta_{\text{area}}$  is the scalar quantity

$$\begin{aligned} \theta_{\text{area}} &= \frac{\sigma_a B_{mm}^{\text{Rred}} \theta_m}{b_{\text{area}}} \\ &= w \theta_m = w[1]\theta_m[1] + w[2]\theta_m[2] + \dots + w[k]\theta_m[k] \end{aligned} \quad (4.6)$$

where  $w$  is a row vector of weights  $w = (w[1], w[2], \dots, w[k])$  that depend only on the area topology and the susceptances of lines in the area.  $k$  is the number of border buses. To monitor the area angle with (4.6), we use the synchrophasor measurements of  $\theta_m$  at the border buses and recent base case susceptances and topology of a DC load flow<sup>2</sup> of the area R to calculate the weights  $w$ . If an outage of line  $i$  occurs, then the synchrophasor measurements at the border buses change to  $\theta_m^{(i)}$  but we continue to use the weights computed *before* the outage to compute the area angle as

$$\theta_{\text{area}}^{(i)} = \frac{\sigma_a B_{mm}^{\text{Rred}} \theta_m^{(i)}}{b_{\text{area}}} = w \theta_m^{(i)}. \quad (4.7)$$

---

<sup>2</sup>Such DC load flows are generally available (22).

### 4.2.2 Approximate inverse relation between area angle and area susceptance

We informally explain why the monitored area angle  $\theta_{\text{area}}^{(i)}$  varies approximately inversely to the area susceptance  $b_{\text{area}}^{(i)}$ .

It turns out<sup>3</sup> that the monitored area angle  $\theta_{\text{area}}^{(i)}$  of (4.7) is close to the following area angle (note the square brackets in the superscript  $[i]$ ):

$$\theta_{\text{area}}^{[i]} = \frac{\sigma_a B_{mm}^{\text{Rred}(i)} \theta_m^{(i)}}{b_{\text{area}}^{(i)}} = \frac{\sigma_a B_{mm}^{\text{Rred}(i)} \theta_m^{(i)}}{\sigma_a B_{mm}^{\text{Rred}(i)} \sigma_a^T}, \quad (4.8)$$

which is the area angle that would be computed after the outage of line  $i$  if the outage of line  $i$  were accounted for in the weights. (The difference between (4.8) and (4.7) is that the susceptance matrix  $B_{mm}^{\text{Rred}(i)}$  that accounts for the outage of line  $i$  replaces  $B_{mm}^{\text{Rred}}$  in both the numerator and denominator of (4.8).) The results in section 4.4 show numerical evidence that  $\theta_{\text{area}}^{(i)}$  and  $\theta_{\text{area}}^{[i]}$  are close; that is,

$$\theta_{\text{area}}^{(i)} \approx \theta_{\text{area}}^{[i]}. \quad (4.9)$$

It is the case (9) that Ohm's law applies to area angles so that

$$P_{\text{area}} = b_{\text{area}} \theta_{\text{area}}. \quad (4.10)$$

In particular, when line  $i$  outages, we have

$$P_{\text{area}}^{(i)} = b_{\text{area}}^{(i)} \theta_{\text{area}}^{[i]}, \quad (4.11)$$

and, from (4.1), (4.2), and (4.4), we have

$$P_{\text{area}}^{(i)} = \sigma_a (P_m + P_m^{\text{into}(i)} - B_{mn}^{(i)} (B_{nn}^{(i)})^{-1} P_n). \quad (4.12)$$

Since line  $i$  is assumed to be a non-islanding outage, and there are assumed to be no losses in the DC load flow approximation, there is no redispatch or load shedding and  $P_m$  and  $P_n$  do not change when the line outages. The term  $B_{mn}^{(i)} (B_{nn}^{(i)})^{-1} P_n$  describes how the injected powers  $P_n$  redistribute to equivalent injections at the border buses after line  $i$  outages, and is usually close to the equivalent injections  $B_{mn} B_{nn}^{-1} P_n$  before the outage.<sup>3</sup> Now we consider the effect of

<sup>3</sup>This approximation will be established with more rigor in a future chapter.

the line outage on the power  $P_m^{\text{into}}$  entering the area R along the tie lines. There are two cases. In the first case, there is no alternative path for power to flow around the area (that is, the area is a cutset area (12; 11) in that removing the area disconnects the network), and the power entering the area along the tie lines does not change so that  $P_m^{\text{into}(i)} = P_m^{\text{into}}$ . In the second case, there is an alternative path for the power to flow around the area, and  $P_m^{\text{into}(i)}$  will be different than  $P_m^{\text{into}}$ . However, in the practical cases considered in this chapter, the alternative paths have fairly high impedance so that the difference between  $P_m^{\text{into}(i)}$  and  $P_m^{\text{into}}$  is small. The conclusion is that in this chapter,  $P_{\text{area}}^{(i)} \approx P_{\text{area}}$ .

Gathering these relationships and approximations together we obtain

$$\theta_{\text{area}}^{(i)} \approx \theta_{\text{area}}^{[i]} = \frac{P_{\text{area}}^{(i)}}{b_{\text{area}}^{(i)}} \approx \frac{P_{\text{area}}}{b_{\text{area}}^{(i)}} \quad (4.13)$$

Also a numerical example of approximation (4.13) is given at the end of Section IV. Thus  $\theta_{\text{area}}^{(i)}$  and  $b_{\text{area}}^{(i)}$  are approximately inversely related.

### 4.3 Simple Examples

To better understand the relationship between the susceptance of the area and the area voltage angle, we first consider a very simple case of 3 parallel lines connecting bus  $a$  to bus  $b$  with respective susceptances  $b_1$ ,  $b_2$ , and  $b_3$ . Power  $P_a$  is generated at bus  $a$  and consumed at bus  $b$ . In this simple case, the area susceptance  $b_{\text{area}} = b_1 + b_2 + b_3$  is the sum of the line susceptances and the area angle  $\theta_{\text{area}} = \theta_a - \theta_b$  is the angle difference between the voltages at bus  $a$  and  $b$ , and the equivalent power through the area  $P_{\text{area}} = P_a$ . In the base case,

$$\theta_{\text{area}} = \frac{P_{\text{area}}}{b_{\text{area}}} = \frac{P_a}{b_1 + b_2 + b_3} \quad (4.14)$$

If line 1 outages, the power flowing through the area  $P_{\text{area}} = P_a$  remains constant, the area susceptance decreases to  $b_{\text{area}}^{(1)} = b_2 + b_3$ , and the area angle increases to

$$\theta_{\text{area}}^{(1)} = \theta_{\text{area}}^{[1]} = \theta_a^{(1)} - \theta_b^{(1)} = \frac{P_{\text{area}}^{(1)}}{b_{\text{area}}^{(1)}} = \frac{P_a}{b_{\text{area}}^{(1)}} = \frac{P_a}{b_2 + b_3} \quad (4.15)$$

The voltage angle increase reflects the decreased susceptance in the network and the increased area stress. We also have  $\theta_{\text{area}}^{(2)} = P_a/b_{\text{area}}^{(2)}$  and  $\theta_{\text{area}}^{(3)} = P_a/b_{\text{area}}^{(3)}$ , and it can be seen that outaging the line with the largest susceptance gives the largest increase in area angle.

To observe the same effects in an example in which multiple voltage angles are combined to form the area angle, consider the simple symmetric network shown in Figure 4.1. Buses 1 and 2 are north border buses and buses 4 and 5 are south border buses. The susceptance of each of the four lines connected to the north border is 30 pu, the susceptance of each of the two lines between bus 3 and bus 4 is 20 pu, and the susceptance of each of the two lines between bus 3 and bus 5 is 40 pu. The power generation at the north border and the loads at the south border are shown in per unit in Figure 4.1. The larger susceptance lines 7 and 8 have a larger power flow of 40 pu.

We are interested in the voltage angle across the area from the north border to the south border, which is the following weighted combination of the border voltage angles:

$$\theta_{\text{area5bus}} = 0.5\theta_1 + 0.5\theta_2 - 0.33\theta_4 - 0.67\theta_5 \quad (4.16)$$

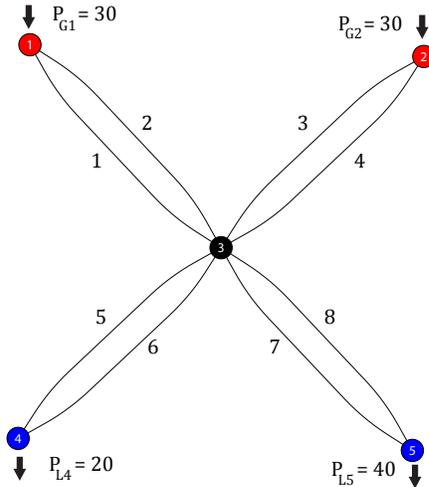


Figure 4.1 5 bus example network with north border buses 1 and 2 in red and south border buses 4 and 5 in blue.

We take out each line in the system in turn and calculate the area susceptance  $b_{\text{area5bus}}^{(i)}$  and the monitored area angle  $\theta_{\text{area5bus}}^{(i)}$  in each case. The results in Figure 4.2 show that the area voltage angle responds to and changes inversely with the area susceptance. Moreover, the changes are largest for most severe line outages. For example, lines 7 and 8 have the largest susceptances and power flows, and when either line 7 or line 8 outages, the area angle increases

the most and the susceptance decreases the most. Lines 5 and 6 have the smallest susceptances and power flows, and when either line 5 or line 6 outages, the area angle increases the least and the susceptance decreases the least.

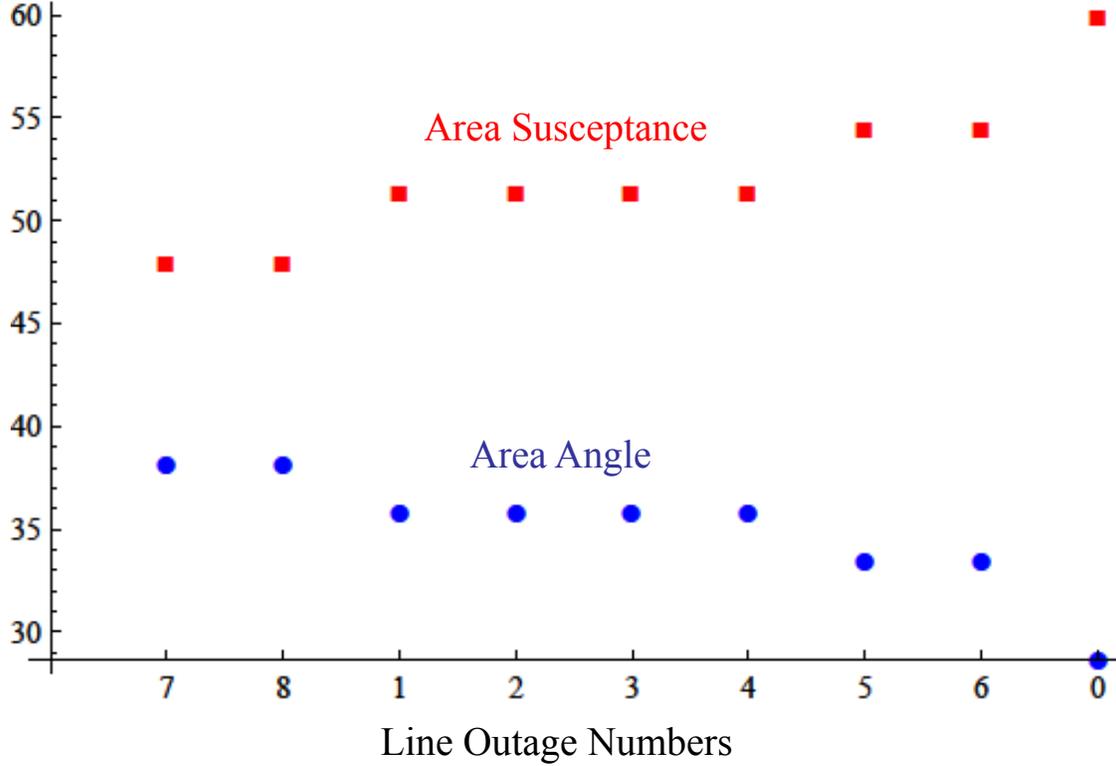


Figure 4.2 Area angle  $\theta_{\text{area5bus}}^{(i)}$  in degrees and area susceptance  $b_{\text{area5bus}}^{(i)}$  in pu for each line outage of 5 bus system. Base case is indicated as line 0.

A 9 bus example of an asymmetric network with lines of equal susceptance is shown in Figure 4.3. Buses 1 and 2 are north border buses and bus 3 is the south border bus. Buses 8 and 9 have generators each providing 8 pu and bus 10 has load of 16 pu, so the total power into the area at the north border is 16 pu. The north to south area angle is

$$\theta_{\text{area9bus}} = 0.44\theta_1 + 0.56\theta_2 - \theta_3 \quad (4.17)$$

The results in Figure 4.4 show that the area angle  $\theta_{\text{area9bus}}^{(i)}$  responds to and changes inversely with the area susceptance  $b_{\text{area9bus}}^{(i)}$ . In this example, although all the lines have the same

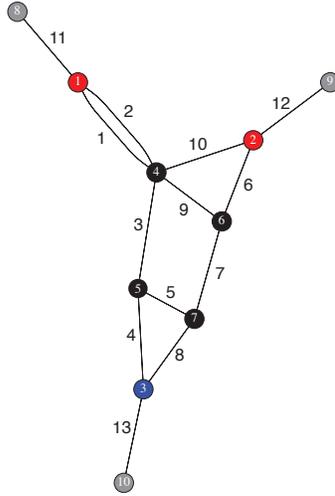


Figure 4.3 9 bus example network with north border buses 1 and 2 in red and the south border bus 3 in blue. The buses inside the area are black.

susceptance, they participate differently in transferring power north to south through the area. Therefore their outages have different severities and different impacts on the area susceptance and area angle. For example, after the line outages 3, 4, 7, 8, which have the largest power flow since they are in the main path of transferring power from north to south, we get the largest decrease in the susceptance and also the largest increase in the area angle, which correctly indicates that these are severe outages. In contrast, after the line outages 5 and 9, which have the smallest power flow since they are not in the main path of transferring power from north to south but instead run from east to west, we get the smallest change in area susceptance and area angle, which correctly indicates that these are less severe outages.

#### 4.4 Results for Angles Across Areas of WECC

We illustrate the use of area angles to monitor single, non-islanding line outages inside two areas of the WECC system.

The first area, for which the network, border buses, and weights are shown in Figure 4.5, covers roughly Washington, Oregon, Idaho, Montana, and Wyoming and contains over 700 lines. The north border is near the Canadian border and the south border is near the Oregon-

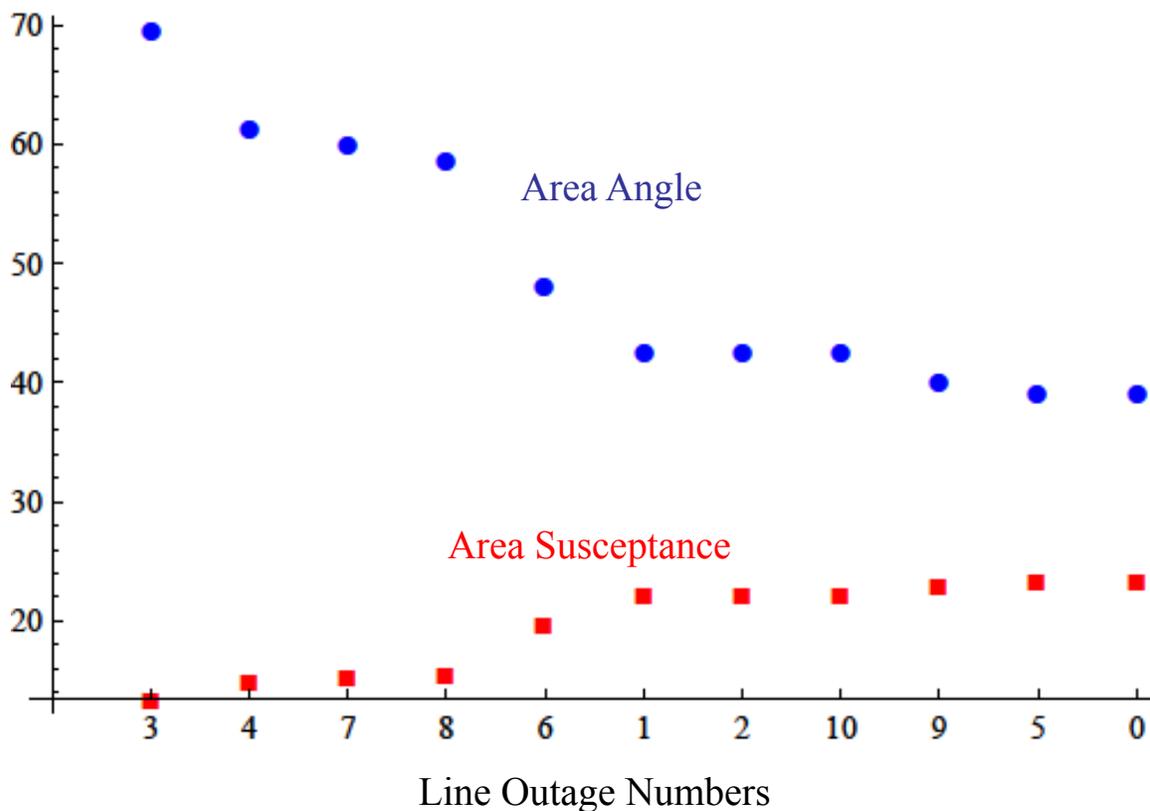


Figure 4.4 Area angle  $\theta_{\text{area9bus}}^{(i)}$  in degrees and area susceptance  $b_{\text{area9bus}}^{(i)}$  in pu for each line outage for 9 bus system. Base case is indicated as outage 0.

California border and its extension eastwards. The area angle is the following weighted combination of the border bus angles:

$$\begin{aligned} \theta_{\text{area1}} = & 0.79 \theta_1 + 0.21 \theta_2 - 0.42 \theta_3 - 0.46 \theta_4 \\ & - 0.02 \theta_5 - 0.05 \theta_6 - 0.04 \theta_7 - 0.01 \theta_8 \end{aligned}$$

The second and smaller area shown in Figure 4.6 covers roughly Washington and Oregon. The northern (and western) border is near the borders of Canada-Washington, Washington-Montana and Oregon-Idaho, and the south border is near the Oregon-California border. The

area angle is

$$\begin{aligned} \theta_{\text{area2}} = & 0.223 \theta_1 + 0.006 \theta_2 \\ & + 0.008 \theta_3 + 0.01 \theta_4 + 0.02 \theta_5 + 0.18 \theta_6 + 0.59 \theta_7 \\ & - 0.39 \theta_8 - 0.41 \theta_9 - 0.004 \theta_{10} - 0.03 \theta_{11} - 0.18 \theta_{12} \end{aligned}$$

In practice the measurements with very small weights could be omitted.

For both areas, we are interested in monitoring the north-south area stress with the area angle when there are single non-islanding line outages, and relating changes in the area angle to the area susceptance and the outage severity. We take out each line in the system in turn and calculate the monitored area angle  $\theta_{\text{area}}^{(i)}$  and the area susceptance  $b_{\text{area}}^{(i)}$  in each case.

To quantify the severity of each outage, we compute the maximum power that can enter the area after the outage of each line; for more detail see (31). The real power through the area is increased by increasing the power entering at each border bus proportionally. (Generally power enters the area at the northern border buses and leaves the area from the south border buses.) The maximum power entering the area through the north border occurs when the first line limit inside the area is encountered. The idea is that the more severe line outages will more strictly limit the maximum power that can be transferred north to south through the area. This definition of outage severity can be related to the economic effect of limiting the north-south transfer.

The area angle and the area susceptance for each line outage are shown in Figure 4.7 for area 1 and in Figure 4.8 for area 2. The similar patterns of changes in the area angles and area susceptances confirm that the inverse relationship between area angle and area susceptance usually applies.

Figures 4.7 and 4.8 also show the outage severity computed as the maximum power into the area. Note that the line outages are sorted according to increasing maximum power into the area (decreasing severity). The most severe line outages are on the left hand sides of Figures 4.7 and 4.8, and it can be seen that the area angle usually increases substantially for most of the severe line outages. Moreover, in the middle portion of the figures with small changes in severity from the base case (the flat portion of the maximum power into the area), the change in area angle from the base case is usually also small. This suggests, for our chosen quantification

of outage severity, that large increases in area angle usually indicate the severe line outages. In our experience, this good result relies on our use of realistic line limits. This tracking of the severity of the outages with the area angle is imperfect, but this is to be expected when trying to monitor over 700 lines in WECC area 1 and 500 lines in WECC area 2 with one scalar area angle as a single bulk area index. (Also note that we are only using a dozen or fewer synchrophasor measurements to compute the area angle.) There are several reasons for the exceptional line outages in which the changes in area angle do not track the outage severity. Large generation or load inside the area can influence the maximum power entering the area under single line outage conditions, and the discrepancy can arise from inaccurate assessment of the outage severity with the maximum power entering the area. The line limits that determine the maximum power entering the area and the outage severity may not follow the susceptance of the lines and so the susceptance of the area and hence in these cases the area angle cannot track the outage severity. These effects are also the likely cause of the outages at the right of Figure 4.8 having a maximum power into the area larger than the base case.

To numerically check the assertion that  $\theta_{\text{area}}^{(i)}$  and  $\theta_{\text{area}}^{[i]}$  are close, we compute the ratio  $\theta_{\text{area}}^{(i)}/\theta_{\text{area}}^{[i]}$  for each line outage. For WECC area 1,  $\theta_{\text{area1}}^{(i)}/\theta_{\text{area1}}^{[i]}$  has mean 0.9999, standard deviation 0.002501, and it ranges from 0.9846 to 1.014. For WECC area 2,  $\theta_{\text{area2}}^{(i)}/\theta_{\text{area2}}^{[i]}$  has mean 0.9993, standard deviation 0.006082, and it ranges from 0.9236 to 1.056.

## 4.5 Conclusion

It is useful to monitor area angle by combining together synchrophasor measurements at the borders of a suitably chosen area. The area angle and the area susceptance change when single, non-islanding line outages occur and we show that area angle and susceptance tend to change inversely using both simple examples and two examples of areas with hundreds of lines in a real power system. This approximate relation between area angle and area susceptance gives intuition about how the area angle works to detect line outages in the area.

The area angle results in a real power system also show that the amount of change in the area angle usually indicates the severity of the line outage (the exceptions generally relate to outages of lines that are connected to generation or load inside the area). This suggests that

a threshold for changes in the area angle to distinguish severe single line outages could be set, and this is established in the following chapters.

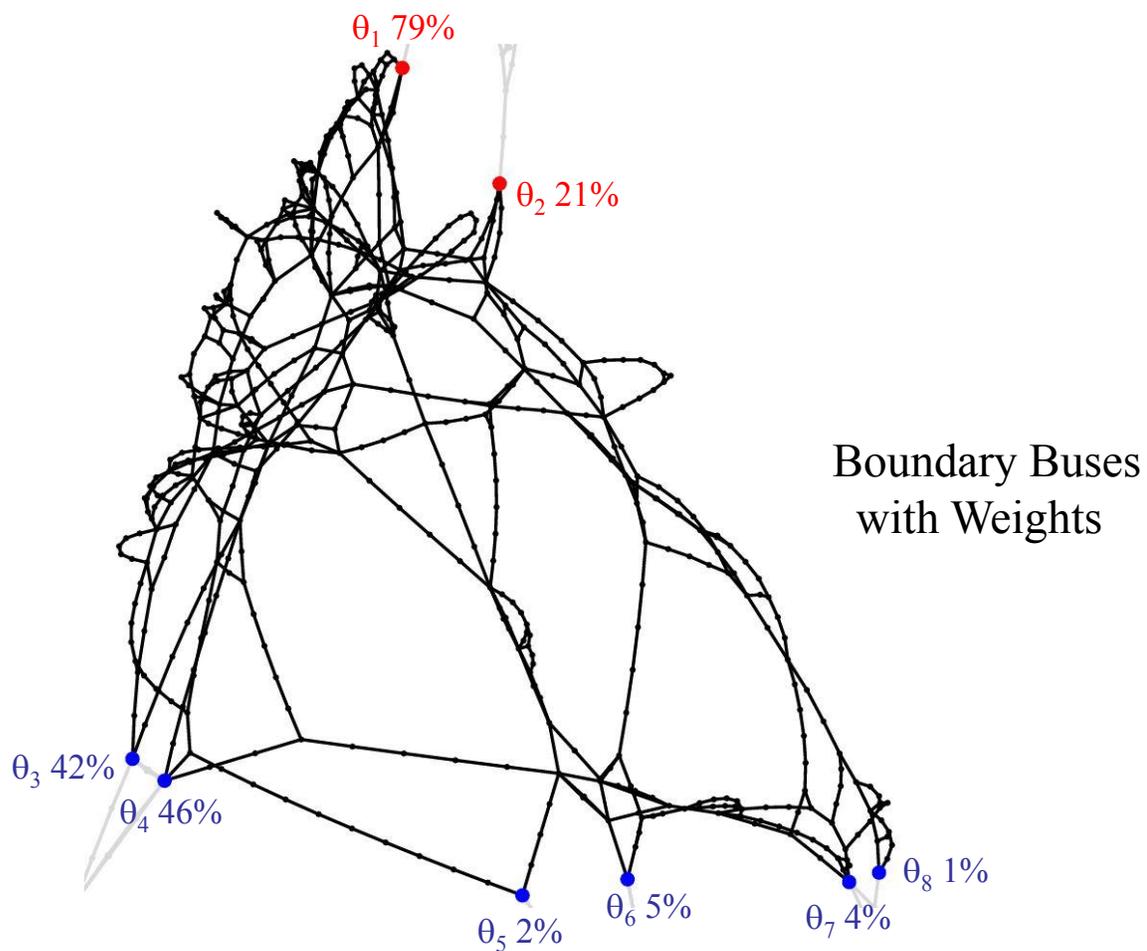


Figure 4.5 Area 1 of WECC system with area lines in black, north border buses in red, and south border buses in blue. Layout detail is not geographic.

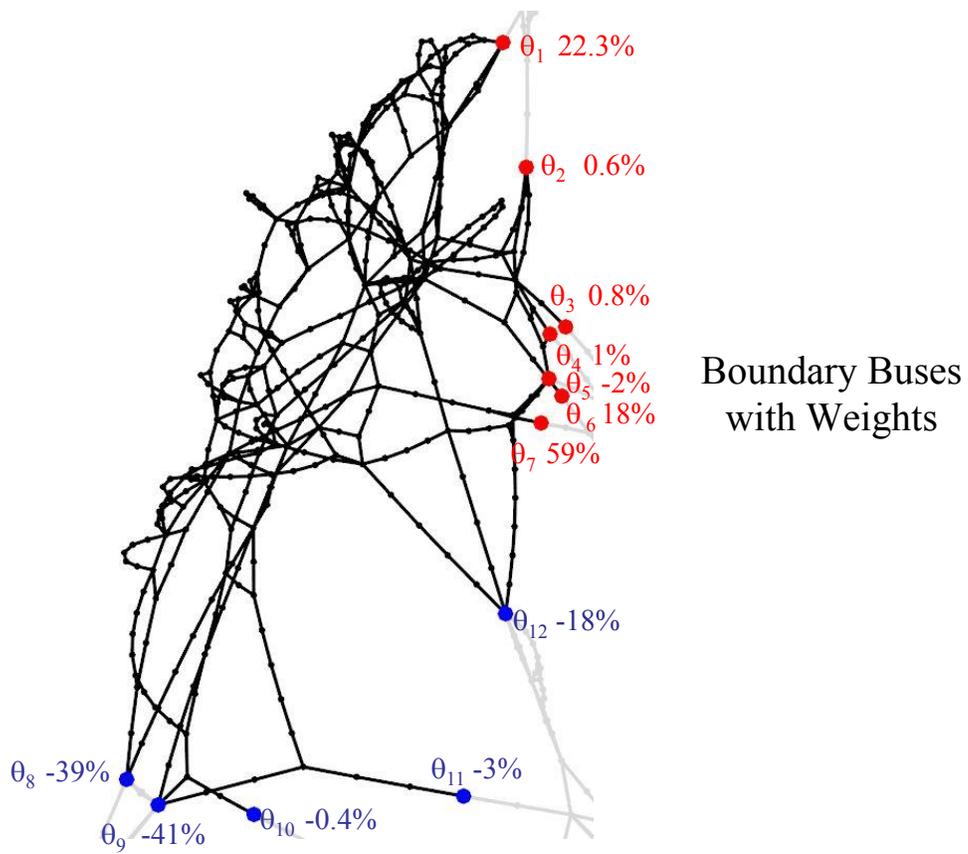


Figure 4.6 Area 2 of WECC system with area lines in black, north border buses in red and south border buses in blue. Layout detail is not geographic.

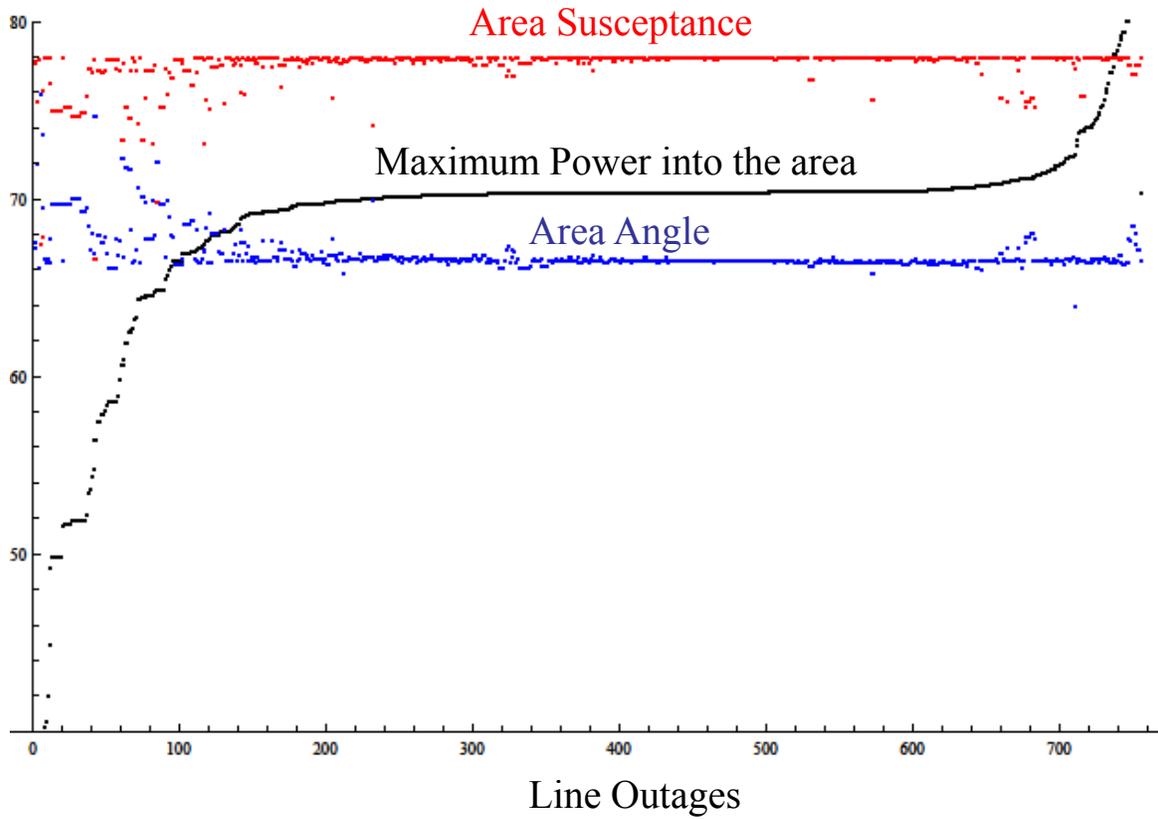
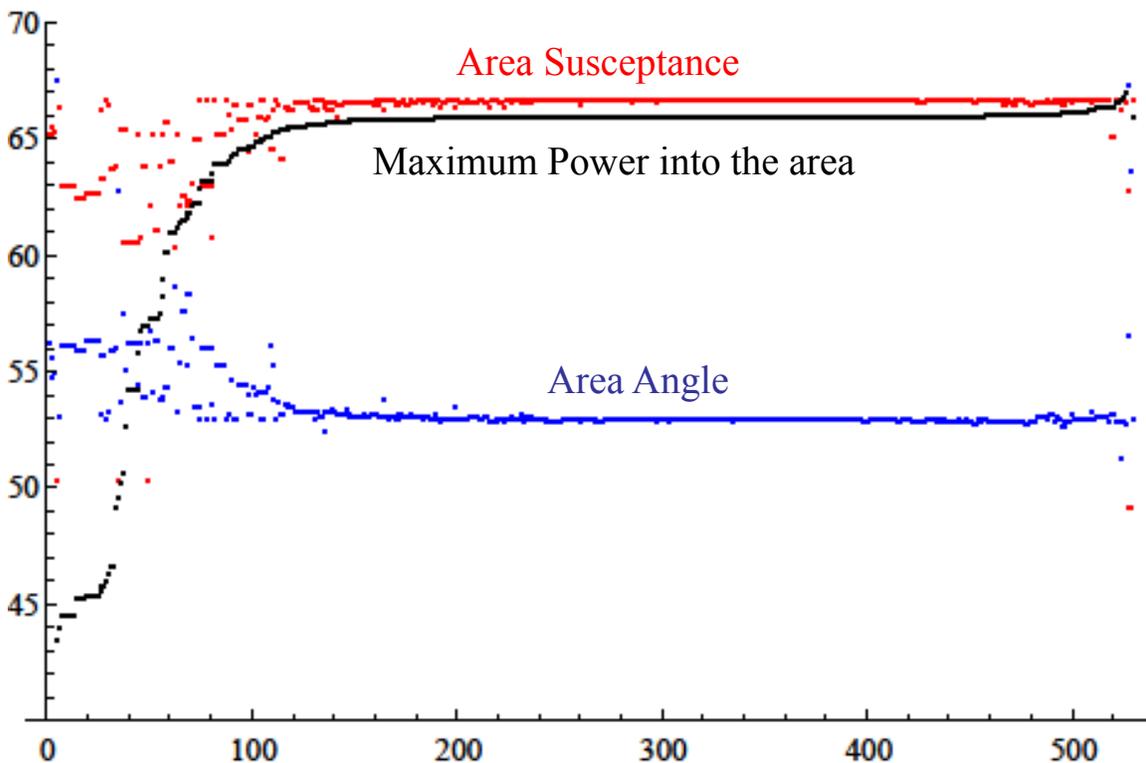


Figure 4.7 Area angle  $\theta_{\text{area1}}^{(i)}$  in degrees, area susceptance  $b_{\text{area1}}^{(i)}$ , and maximum power into the area in pu for each line outage in WECC area 1. Base case (the point at extreme right) is  $\theta_{\text{area1}} = 66.5^\circ$ ,  $b_{\text{area1}} = 39.0$  pu, max power = 46.9. For clarity, graph shows  $b_{\text{area}}$  multiplied by 2, and max power multiplied by 1.5.



## CHAPTER 5. MONITORING OF MULTIPLE OUTAGES WITH AREA ANGLE

### 5.1 Introduction

With increasing and variable demands placed on the power transmission system, areas of the power system are often stressed as bulk power is transferred through the area. Each line in the area has a power flow limit that is a thermal limit or arises as a proxy for other kinds of limits. Under contingencies such as line outages, these individual line limits become more binding on the bulk transfer of power through the area, and in severe cases, the bulk power flow through the area will have to be restricted. It is important to be able to quickly determine the severity of the outages so that the appropriate remedial actions can be taken. Especially in the case of multiple outages, a quick response could prevent further cascading and a blackout. Many observed cascading blackouts start with a few outages occurring more slowly, which gives a possibility of quick action to forestall the subsequent, faster cascading processes that lead to a widespread blackout.

This thesis demonstrates how to combine synchrophasor measurements around the border of an area to quickly monitor the severity of multiple outages inside the area. Alternatively, after some delay for state estimation calculations, one can also monitor outages via SCADA and state estimation. However, state estimation is less reliable for multiple outages. If the state estimation fails, our approach using synchrophasors changes from a faster alternative to the only indicator.

More generally, synchrophasor measurements provide fast monitoring of bus voltages over a wide area. As more synchrophasors are deployed, one of the challenges is summarizing and understanding the new data. One advantageous approach is to use physical principles to

combine together synchrophasor measurements into quantities that are more meaningful and actionable. In this thesis we combine voltage angles around the border of an area of the power system into a bulk angle across the area. The concept of the voltage angle across a power system area is new and is described in detail in (9; 10), including how it derives from circuit theory principles. The area angle concept is a generalization of the angle across a cutset area concept developed and proposed for stress monitoring in (12; 11; 13). Throughout this thesis we use a DC load flow model with voltage phasor angles and real power flows.

We note that synchrophasor measurements around the border of an area can be advantageous for other applications such as combining AC voltage measurements in a transmission corridor to monitor voltage collapse (23) or locating line outages in the area (24). More generally, the border measurements can be used to effectively decouple the area from the rest of the interconnection (25). These methods that apply to power areas will be particularly useful when utilities or ISOs in large interconnections restrict their attention to network models and phasor measurements for only their own area.

## 5.2 Simple Example of Stress Monitoring Using Area Angle

Fig. 5.1 illustrates a simple example of three equal, parallel lines connecting two buses  $a$  and  $b$ . We compare monitoring the angle difference between the buses with monitoring the power transferring between them in the case of a double outage.

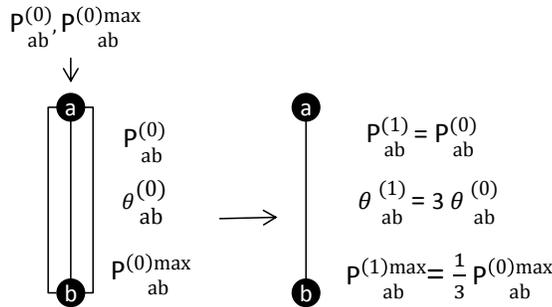


Figure 5.1 Simple example of three parallel lines with double outage

We assume lossless lines and we observe how the angle difference between buses and the

power entering into bus  $a$  varies from the base case to the double outage case. The superscripts 0 and 1 stand for the base case and double outage case respectively. In both the base case and the double outage case, the power  $P_{ab}$  entering into bus  $a$  remains the same, while the angle difference  $\theta_{ab}$  between the buses increases and triples in the case of a double outage. This increment in the angle as the double outages occur is a good indicator of increased stress in the system.

To quantify stress caused by line outages, we consider the maximum power that could enter the system. As the outages become more severe, they cause the other lines to reach their thermal limits and so the capacity of the system to transfer more power reduces. For instance, as the double line outages occur in Fig. 5.1, the maximum power that could enter bus  $a$  decreases. We can see that as the stress increases, or in other words, the maximum power that could enter bus  $a$  decreases, the power entering the area remains constant, but the area angle gets larger and indicates the increased system stress.

To generalize this simple example to the real system, we consider a connected area of the power system with border buses  $M$ . The border buses  $M$  comprise the  $a$  border buses near the generation and the  $b$  border buses near the loads. The area is mostly transferring power between the generation and load. The power entering into the area (similar to the power entering bus  $a$  in the simple example) is the sum of the powers entering into the area along the tie lines connected to the border buses  $a$ . We apply the new concept of area angle to get the angle difference across the area from the  $a$  buses to the  $b$  buses (similar to angle difference of the buses  $a$  and  $b$  in the simple example). As described in detail in (9), the area angle  $\theta_{\text{area}}$  is a weighted combination of the angles around the area:

$$\theta_{\text{area}} = \frac{\sigma_a B_{eq} \theta_m}{b_{\text{area}}} = w \theta_m. \quad (5.1)$$

Here  $\sigma_a$  is the vector of the size of the number of border buses  $M$  which has the entry 1 corresponding to each  $a$  border bus and entry 0 in the rest.  $B_{eq}$  is the susceptance matrix of the Kron reduction of the area to the border buses (this Kron reduction is electrically equivalent to the original area from the perspective of the border buses).  $\theta_m$  is the vector of the angles of

the border buses  $M$ .  $b_{\text{area}}$  is the susceptance of the area which can be calculated as

$$b_{\text{area}} = \sigma_a B_{eq} \sigma_a^T. \quad (5.2)$$

As we can see, the area susceptance and area angle are inversely related. We discuss this relationship in detail in (33).

$w$  in (5.1) is a row vector of weights that depend only on the area topology and the susceptances of lines in the area. The  $a$  buses have positive weights and the  $b$  buses have negative weights. Thus in (5.1) to get the area angle across the area from the  $a$  buses to the  $b$  buses, the weighted combination of the angles in the  $b$  border buses is subtracted from the weighted combination of the angles in the  $a$  border buses.

It is important to choose the area so that the area angle is meaningful and useful for power systems operation. In this thesis, we choose an area of the transmission system between major generation and major load to try to describe with the area angle the stress resulting from the transfer of power through the area and how the stress varies with line outages inside the area.

### 5.3 Stress Monitoring Using Area Angle

#### 5.3.1 Problem set up

Our goal is to monitor a single quantity for the area that captures the severity of multiple outages inside the area. Ideally, the monitored quantity changes from its base case value if a line outages, and the amount of change should indicate the severity of the outage. Our results will show that while the real power entering the area  $P_a^{\text{into}}$  remains constant after the outages, the area angle  $\theta_{\text{area}}$  increases as the outages becomes more severe and tracks the severity of the outages inside the area. Thus the area angle  $\theta_{\text{area}}$  is a better indicator of area stress than the real power entering the area  $P_a^{\text{into}}$ .

We evaluate the severity of the outage inside the area with the maximum power that could enter the area  $P_a^{\text{intomax}}$ . We increase the power entering and leaving the area by assuming a particular pattern of load and generation injection at the border buses that increases the power transferred through the area. This power transferred through the area is increased until the first line in the area reaches its power flow limit. Each line in the area has a limit on its real

power flow that corresponds to the line thermal limit or is a proxy for other system limits. As the generation and load increase, there is increased stress on the transmission system, and lines may approach or reach their limits, especially under contingency conditions in which another line outages. We calculate the area angle and the maximum power that could enter the area that satisfy all line limits after each outage. It is of interest to find out how much monitoring  $\theta_{\text{area}}$  gives an indication of the outage severity as evaluated by  $P_a^{\text{intomax}}$ . Note that since  $P_a^{\text{intomax}}$  involves a hypothetical increase of the power entering the area from the current situation, it cannot be monitored directly.

The objective is to show how area angle can track the severity of the outages inside the area. However, there are some outages inside the area which for the area angle can not track the severity well. Plotting and ordering the relationship between the maximum power that could enter the area and the corresponding area angle can reveal and identify these outages that are outliers. In particular, we plot area angle and the maximum power transfer for all single outages, order them based on the maximum power transfer, find these outliers, and handle these exceptional cases separately. Note that this screening for the outliers can be done using the single outages only. After removing these outlier lines from the list of all lines inside the area, we can track the outages of remaining lines, plot the result and observe the relationship between the area angle and the corresponding maximum power transfer.

### 5.3.2 Formulation

We need to calculate area angle, area susceptance and the maximum power entering the area after outages. We use formulas (5.1) and (5.2), after outages to determine area angle and area susceptance. Furthermore, after finding the extra power injection in border buses after outages that stresses the area until the first limit is reached, we can calculate the maximum power could enter the area without violating any line limits. This section explains this calculation in detail.

For a general area that has paths around the area that are parallel to the power flow through the area, an outage inside the area will cause some change in the power into the area tie lines. But if there are no such parallel paths around the area, the power in the tie lines does not

change, and the power entering into the area will remain constant. In our results we use an area that has high impedance parallel paths so that the the power entering into the area will remain approximately constant.

To find the maximum power that could enter the area after outages, we need to calculate at first how much more power can be injected in the border buses and then add this injection to the base case power entering to the area. We use the power transfer distribution factor of the lines with respect to injections in border buses and the real power limits of lines to calculate the extra injection in the border buses. We increase the power entering the area with a specific pattern of injection in the border buses until the first line violates its maximum power flow limit. The pattern of injection is proportional to the base case power entering each border buses along the tie lines connected to that bus. This has the effect of increasing the area stress in the same pattern as the base case stress.

We use the following notation:

$P_{\text{line}k}$	power flow through line $k$
$P_{\text{line}}$	vector of power flows through lines
$\Delta P_{\text{inj}}^{rs}$	amount of extra power injected positive in bus $r$ and negative in bus $s$ to stress the system
$\theta$	vector of voltage angles at buses
$\theta_{\text{line}}$	voltage angle in each line
$B$	susceptance matrix
$\Lambda$	diagonal matrix of line susceptances
$A$	bus line incidence matrix
$\rho_k^{rs}$	power transfer distribution factor for line $k$ with respect to injections in buses $r$ and $s$

We describe the variables above in different conditions using the following notation:

$X$	generic variable
$X^{(i)}$	$X$ evaluated for contingency number $i$ . The base case is contingency number 0.
$X^{k\max}$	$X$ evaluated at the maximum stressed case obtained by applying stress until line $k$ reaches its maximum power flow rating.
$X^{(i)\max}$	$X$ evaluated for the maximum stressed case obtained under contingency number $i$
$X^{\text{limit}}$	operating limit established for $X$

Contingency  $i$  can be single or cascading outages.

To calculate area angle after contingency  $i$  we use (5.1) as follows:

$$\theta_{\text{area}}^{(i)} = \frac{\sigma_a B_{eq} \theta_m^{(i)}}{b_{\text{area}}} = w \theta_m^{(i)}. \quad (5.3)$$

Note that, as discussed further in (33), (5.3) uses the weights  $w$  calculated from the susceptance matrix and area susceptance evaluated *before* the outage of line  $i$ , but it uses the border buses angles  $\theta_m^{(i)}$  measured *after* contingency  $i$ . The susceptance matrix and an updated topology of the area before the outage are generally available to a control center (22).

As mentioned, we need to calculate the maximum power that could enter the area and for that we need to calculate the extra injection in border buses. It is convenient to first consider just border buses  $r$  and  $s$  and calculate the extra injection in buses  $r$  and  $s$  after contingency  $i$ . Injection in buses  $r$  and  $s$  means we add this injection in border bus  $r$  on the generation side and subtract the injection from border bus  $s$  on the load side. To find out this extra injection in border buses  $r$  and  $s$ , we need to find the margin of power flow and the generation shift factor of each line  $k$  in the area with respect to the injection in border buses  $r$  and  $s$ . To find out the margin of power flow in line  $k$  we do the following steps.

The voltage angles across the lines are

$$\theta_{\text{line}}^{(i)} = A^T \theta^{(i)}, \quad (5.4)$$

and the power flows in lines are

$$P_{\text{line}}^{(i)} = \Lambda^{(i)} \theta_{\text{line}}^{(i)}, \quad (5.5)$$

where  $\Lambda^{(i)}$  is the diagonal matrix of the susceptances after contingency  $i$ . The margin of power flow of line  $k$  until its limit is reached is:

$$\Delta P_{\text{line}k}^{(i)} = P_{\text{line}k}^{\text{limit}} - P_{\text{line}k}^{(i)}, \quad (5.6)$$

where  $P_{\text{line}k}^{\text{limit}}$  is the power flow limit of line  $k$ .

To find the generation shift factor, suppose that contingency  $i$  happens and that line  $k$  joins bus  $u$  to bus  $v$ . Then the power transfer distribution factor for line  $k$  is the amount of the increase in the power flow in line  $k$  due to a unit injection of power in bus  $r$  and a unit decrement of power in bus  $s$ :

$$\rho_k^{rs(i)} = b_k (e_u^T - e_v^T) (B^{(i)})^{-1} (e_r - e_s) \quad (5.7)$$

Here  $e_r$  denotes a vector with 1 at entry  $r$  and all other entries zero. Now, the maximum amount of injection in bus  $r$  and decrement from  $s$  until line  $k$  reaches its line limit is

$$\Delta P_{\text{line}k}^{rs(i)\text{max}} = \frac{\Delta P_{\text{line}k}^{(i)}}{\rho_k^{rs(i)}}. \quad (5.8)$$

Then the maximum possible or extra injection at the border buses  $r$  and  $s$  which satisfies all the line limits is the minimum amount of the maximum extra injections for all the lines:

$$\Delta P_{\text{inj}}^{rs(i)} = \text{Min}\{\Delta P_{\text{rs(i)1max}}, \Delta P_{\text{rs(i)2max}}, \dots, \Delta P_{\text{rs(i)nmax}}\},$$

where  $n$  is the total number of lines inside the area.

Then the maximum power  $P_r^{\text{into}(i)\text{max}}$  entering bus  $r$  corresponding to the maximum extra injection after contingency  $i$ , can be calculated as well:

$$P_r^{\text{into}(i)\text{max}} = P_r^{\text{into}} + \Delta P_{\text{inj}}^{rs(i)} \quad (5.9)$$

Now the calculation given above for the extra injection at the buses  $r$  and  $s$  can be extended to the specific pattern of extra injections assumed at the border buses  $a$  and  $b$  by appropriately weighting the generation shift factors. We multiply the pattern ratios related to each pair of

border buses to the value of generation shift factor for that pair and add them for all pairs of buses selected from  $a$  and  $b$  to get the final generation shift factor that relates increases in the injections in the given pattern to the change in power flow at each line  $k$ .

## 5.4 Results

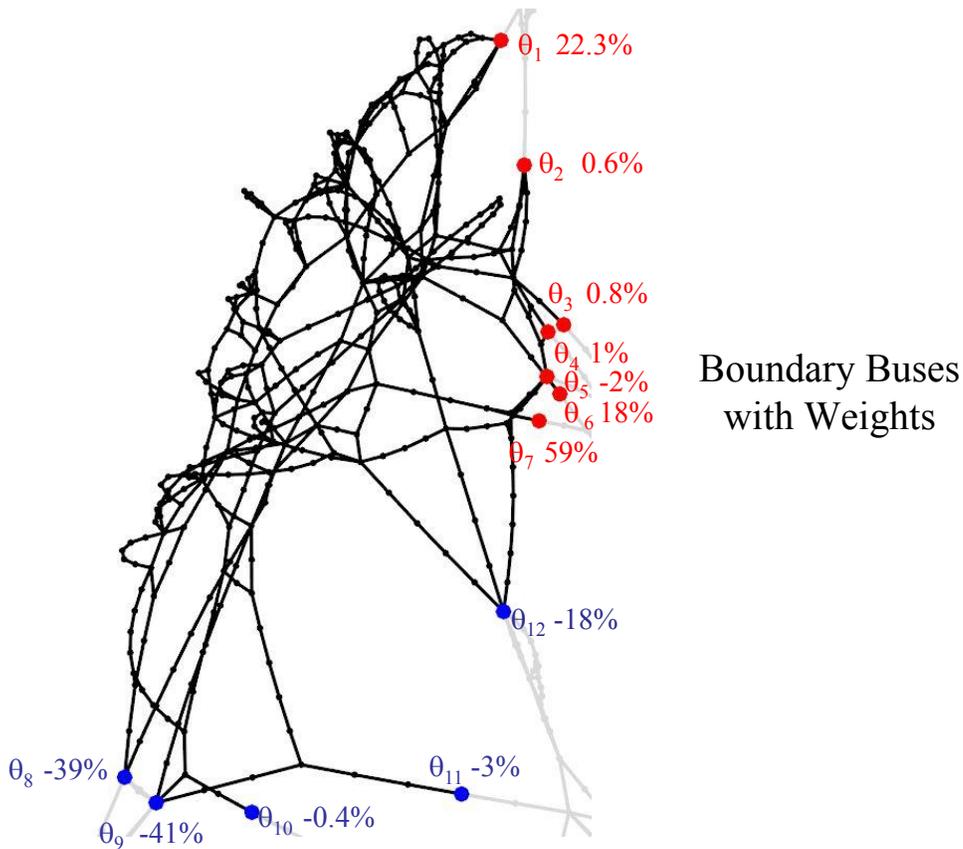
We use a 1553 bus model of WECC that was reduced from a larger model for cascading analysis (34). We select the area shown in Figure 5.2 which covers roughly Washington and Oregon states. The 7 northern (and western) border buses are near the borders of Canada-Washington, Washington-Montana, Oregon-Idaho, and the 5 south border buses are near the Oregon-California border. There are approximately 400 buses and 515 lines inside this area. The transfer through the area of interest is north to south; that is, from the north border buses to the south border buses.

The area angle is

$$\begin{aligned} \theta_{\text{area}} = & 0.223 \theta_1 + 0.006 \theta_2 \\ & + 0.008 \theta_3 + 0.01 \theta_4 + 0.02 \theta_5 + 0.18 \theta_6 + 0.59 \theta_7 \\ & - 0.39 \theta_8 - 0.41 \theta_9 - 0.004 \theta_{10} - 0.03 \theta_{11} - 0.18 \theta_{12} \end{aligned}$$

In practice the measurements with very small weights could be omitted.

We first compute the area angle and the maximum power that could enter the area for all single line outages inside the area, and order the outages based on the decreasing value of the maximum power transfer, or, equivalently, in order of increasing stress. We observed that area angle increases as the maximum power transfer decreases in almost all cases, but there are some outliers to the general trend. We find these outlier lines, take care of them separately, and remove them from the list of lines considered. There are 53 outliers from 515 lines inside the area of which only 30 of them are really of concern, since we only need to detect alarm and emergency situations and do not need to perfectly track the severity by area angle. For all the remaining lines, we sample random combinations of double and triple outages. After ordering the results by severity in the same way as before, we observe in all of them that the



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Figure 5.2 Area of WECC system with area lines in black, north border buses in red and south border buses in blue. The border bus weights are shown as percentages. Layout detail is not geographic.

area angle tracks the maximum power transfer well and can detect the severity of the multiple, and potentially cascading, outages inside the area. We will show some of these results later.

The main reason for the abnormal behavior of the outliers is that they change the local transfer of power, but not the bulk transfer of power through the area. These lines are typically near big generation and load inside the area so that their outage changes the local transfer of power. The area angle is approximately related to the susceptance of the area (33) and to the bulk transfer of power through the area, not the local power transfer. The other reason for these outliers is lack of coordination between the line limits and the susceptance of the area. Since lines limits affect the maximum power that could enter the area and the susceptance

affects the area angle, the coordination between them affects the results. In the case of our test system, some of the lines have artificial line limits because the model is reduced from a larger grid model, and this could be one factor that reduces the coordination between the line limits. We take care of all the outlier line outages separately. Synchrophasor or SCADA signals would monitor the outages of these lines and their outages need to be mitigated separately with individual actions.

We select a random samples of double and triple outages from the remaining list of lines, compute the area angle and area susceptance after their outage and then plot them in order of increasing severity as shown in Figures 5.3 and 5.4.

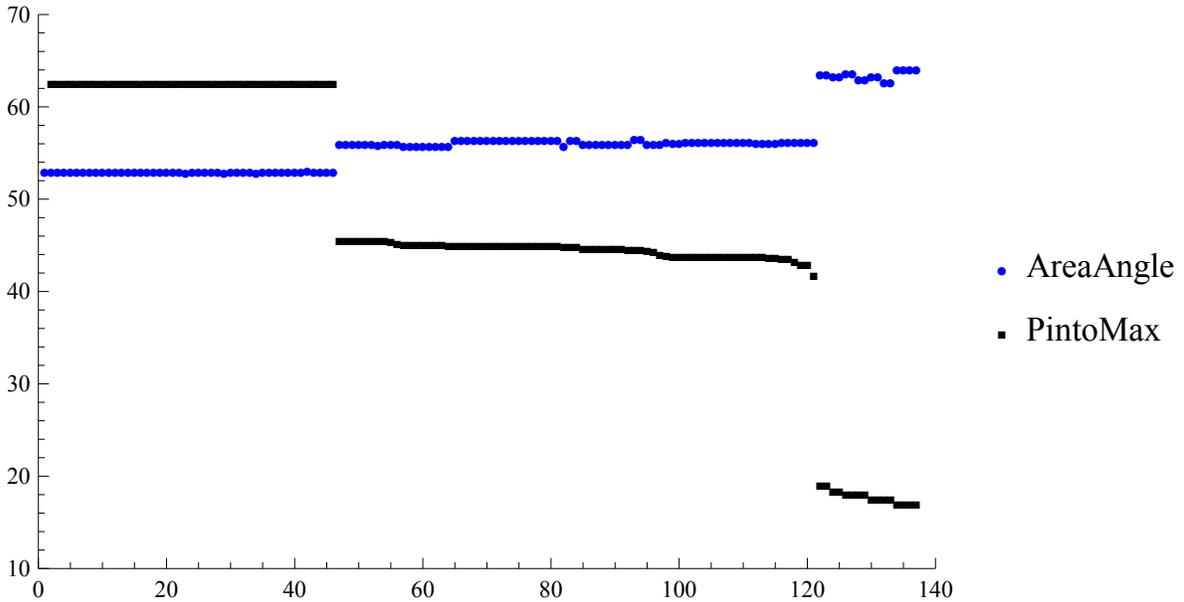


Figure 5.3 Area angle  $\theta_{\text{area}}^{(i)}$  in degrees, and maximum power into the area in per unit for a random sample of double outages in the area. Horizontal axis is outage number.

As can be easily seen in the figures, the area angle tracks the severity of the outages. From left to right, as the maximum power that could enter the area decreases, the area angle increases and detects the severity of the double and triple outages. The plots also show that area angle can separate non-severe outages from the moderate or severe ones. There are three different levels of maximum power transfer that correspond to the safe, moderate and severe outages. There are also three levels of area angle corresponding to the three levels of severity. This

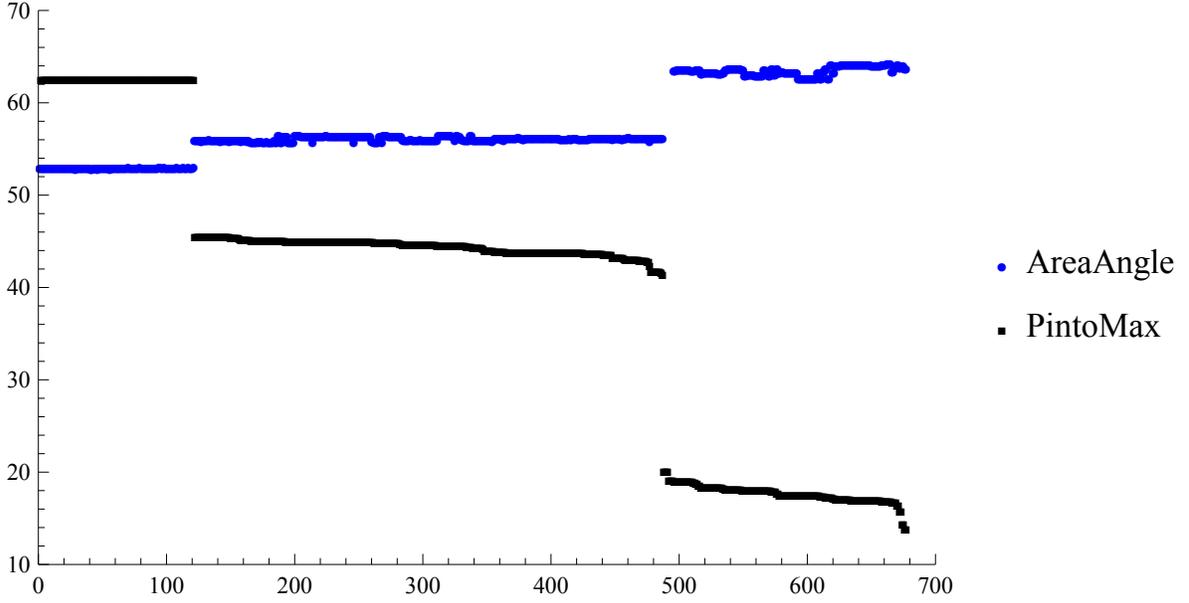


Figure 5.4 Area angle  $\theta_{\text{area}}^{(i)}$  in degrees, and maximum power into the area in per unit for a random sample of triple outages in the area. Horizontal axis is outage number.

suggests that thresholds can be set so that the area angle can distinguish safe outages from the moderate or severe ones and hence improve situational awareness. In real time, area angle can be calculated quickly from the weights and the angle data coming from synchrophasors and then it can be compared to its threshold to give an alarm in emergency situations. Moreover, the way we have formulated the outage severity indicates that the emergency action should reduce the bulk power transfer through the area.

To also show that area angle is related to the area susceptance, we did the same calculation for another random sample of triple outages and this time, we also calculate the area susceptance. The results in Figure 5.5 show that  $\theta_{\text{area}}^{(i)}$  is inversely related to the area susceptance  $b_{\text{area}}^{(i)}$ , and as the outages becomes more severe from left to right, the susceptance of the area decreases and the area angle increases. Here also separation into different stress levels for all quantities can be easily seen.

All the results above are from the list of lines from which the outlier lines associated with local problems were all removed, but if one is only interested to only classify the outage severity into the three levels, this is possible by just removing 30 lines from the list of all lines. The

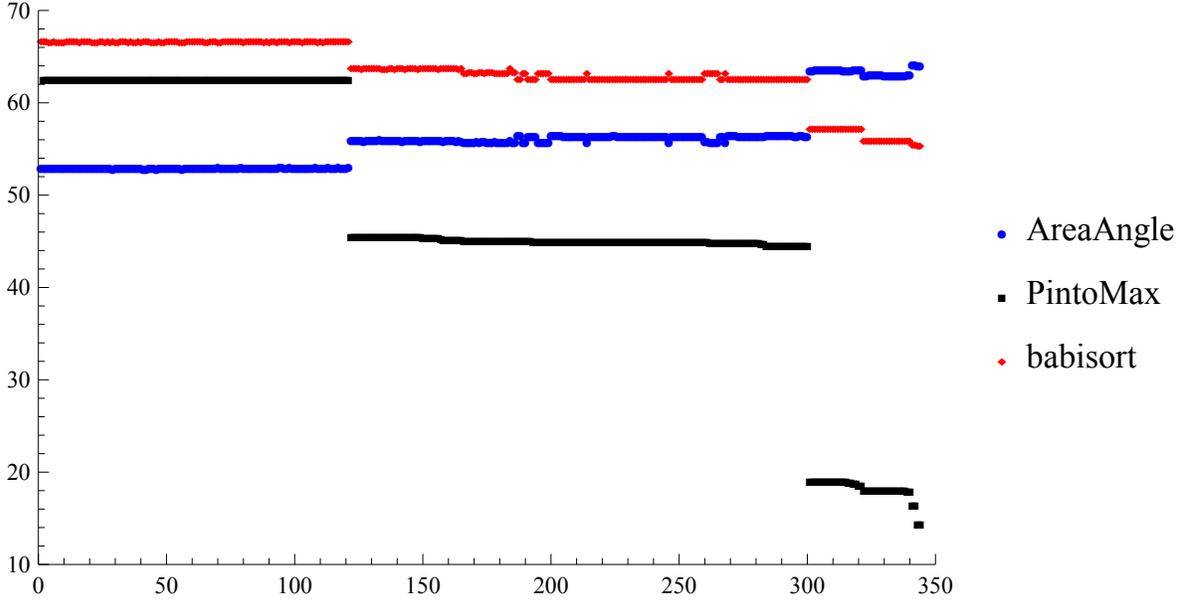


Figure 5.5 Area angle  $\theta_{\text{area}}^{(i)}$  in degrees, area susceptance  $b_{\text{area}}^{(i)}$ , and maximum power into the area in per unit for triple outages in the area. Horizontal axis is outage number.

advantage of this is that we need to take special account of fewer outlier lines compared to the other cases above, but we relax the exact tracking of the severity by the area angle. Figure 5.6 shows the area angle and the maximum power that could enter the area after triple combinations of outages chosen from such a list. It is evident that the outage severity classifications are preserved.

## 5.5 Conclusion

An area angle formed by combining together synchrophasor measurements around the border of the area can quickly track the severity of line outages inside the area. In particular, once outlier cases due to local effects inside the area are detected by analyzing the single outage cases and handled separately, we can quickly track the severity of multiple line outages with respect to limitations on the bulk power transfers through the area caused by individual lines reaching their power flow limits. This quick indication of outage severity could help forestall slowly developing cascading failures in the multiple outage case that is the most challenging case for complementary approaches based on state estimation. The separation of non-severe and severe

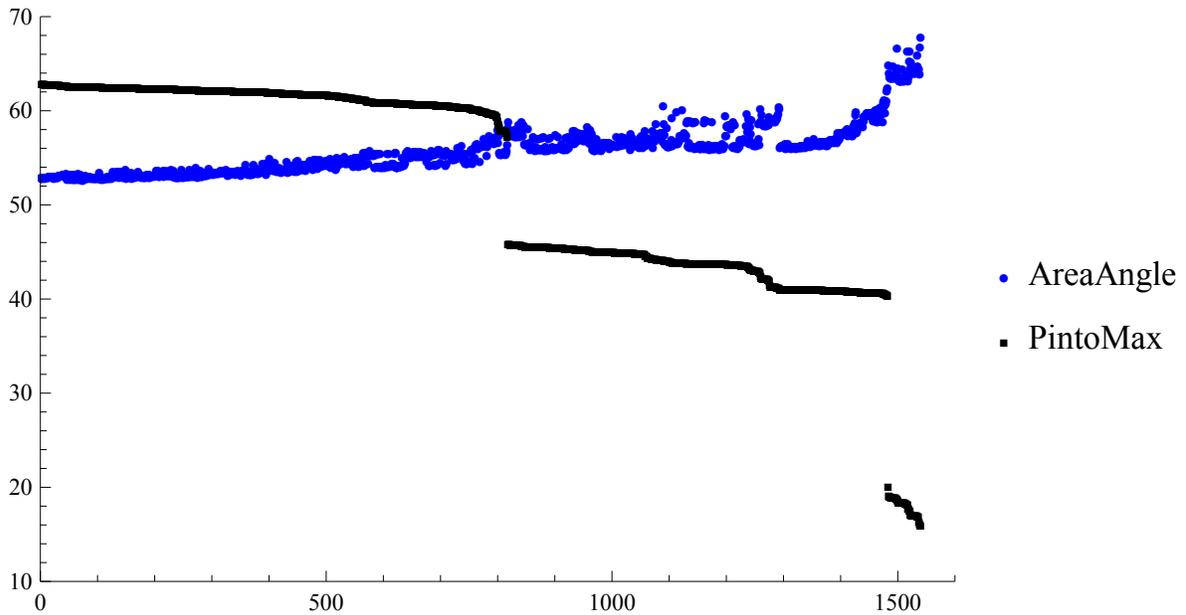


Figure 5.6 Area angle  $\theta_{\text{area}}^{(i)}$  in degrees, and maximum power into the area in pu for a random sample of triple outages in the area with fewer special cases handled separately. Horizontal axis is outage number.

outages also suggests setting thresholds of area angle corresponding to these severity levels to provide to operators improved situational awareness and recommended actions curtailing bulk power transfers when necessary. Detailed procedures for establishing these thresholds are given in the next chapter.

## CHAPTER 6. THRESHOLD-BASED MONITORING OF CASCADING OUTAGES

### 6.1 Introduction

With increasing and variable demands placed on the power transmission system, areas of the power grid are often stressed by bulk transfers through the area. It is important to be able to quickly monitor the additional stress caused by single and multiple line outages so that appropriate remedial actions can be taken. Especially in the case of multiple outages, a quick response could prevent further cascading and a blackout. It is well appreciated that major blackouts have occurred partly due to a lack of situational awareness (1).

In general, synchrophasor technology makes possible fast and accurate monitoring and control of power grids (3). Synchrophasors are becoming widespread and operation tools using synchrophasors for wide area monitoring can monitor and manage system stresses to maintain reliability (7; 4; 6; 18).

Our method focusses on measuring stress across a particular area of the power system using synchrophasor measurements around the border of the area; that is, synchrophasor measurements at all the tie lines of the area. These synchrophasor measurements around the border of the area are combined into a single angle across the area called the area angle. The area angle obeys circuit laws and is derived from circuit theory in (9; 11). In this work, we will show that the area angle tracks bulk stress caused by line outages inside the area. We consider the bulk stress to be determined by the maximum bulk transfer through the area that satisfies the line limits inside the area.

Some previous works on monitoring power system stress with phasor measurements have focused on the angle difference between two buses. Simulations of the grid conditions before

the August 2003 USA/Canada blackout show that increasing large angle differences could be a blackout precursor (19). Simulations of the New England grid show that angle differences can discriminate alert and emergency states (21). A large angle difference between two buses does indicate, in a general sense, a stressed power system, but it is difficult to interpret changes in the angle difference or set thresholds.

The advantage of combining the synchrophasor measurements around the border of an area into an area angle is that one is then monitoring stress in that particular area. Then the additional stress due to line outages inside the area can be quickly monitored in real time just after the outages occur. Furthermore, we will show that our formulation in terms of area angle allows an emergency area angle threshold to be determined based on the maximum power transfers through the area. If the monitored area angle exceeds the emergency angle threshold, the area bulk power transfer should be reduced.

Given suitable synchrophasor measurements available at a control center (22), the calculation of area angle is quick and easy so that the computations can be practical for large real systems. We note that synchrophasor measurements around the border of an area can be also advantageous for other applications such as combining AC voltage measurements in a transmission corridor to monitor voltage collapse (23) or locating line outages in the area (24) or stress between areas (13). Also we used our method for monitoring single outages (31). In this work we seek to monitor the bulk stress for general line outages in the area that include multiple outages.

In somewhat related work by other authors, static feasibility boundaries such as those associated with transmission line limits can be determined from grid models with power flows based on SCADA and state estimation. For example, (26; 27; 28) compute minimum security margins under operational uncertainty with respect to thermal overloads. Also (29) provides a tool for computation of transfer capability margins. Our work is different since we use synchrophasor measurements to monitor in real time the stress with regard to bulk power transfer through areas due to multiple outages inside the area. Methods based on the state estimator produce a much more detailed view of a representative power system condition over the SCADA sampling period, and require some computation time for actionable information.

Our method based on synchrophasors is approximate but faster, and will work under multiple outage conditions in which the state estimator may not readily converge.

## 6.2 Monitoring and Thresholds Based on Angles

### 6.2.1 Simple example

To motivate monitoring with angles, Fig. 6.1 shows a simple example of three parallel, lossless lines connecting two buses a and b. DC power flow is assumed and each line has the same power flow limit. We consider three quantities:  $P_{ab}$  is the real power entering bus a and transmitted to bus b,  $P_{ab}^{\max}$  is maximum real power that could enter bus a as determined by the line power flow limits, and  $\theta_{ab}$  is the voltage angle between a and b. The superscripts (0), (1), and (2) stand for the base, single line outage, and double line outage cases respectively and we consider  $P_{ab}$ ,  $P_{ab}^{\max}$ , and  $\theta_{ab}$  in each of these cases.

As shown in Fig. 6.1, the power  $P_{ab}$  entering bus a remains the same in all cases, while the angle  $\theta_{ab}$  between the buses increases for the single outage and triples in the more severe case of the double outage. The increment in the angle as the outages become more severe is a good indicator of increased stress in the system. That is, the increased stress due to the line outages can be monitored with the angle  $\theta_{ab}$ , but not with the power  $P_{ab}$ .

The fact that  $\theta_{ab}$  is a good indicator of stress provides the motivation to set a threshold on this quantity. To set a threshold on  $\theta_{ab}$  that distinguishes outage severity, we first consider the maximum power  $P_{ab}^{\max}$  entering bus a.  $P_{ab}^{(0)\max}$  in the base case is three times the line limit and, as shown in Fig. 6.1,  $P_{ab}^{\max}$  decreases as the outages become more severe and the stress increases. For example, for a single outage,  $P_{ab}^{(1)\max}$  is twice the line limit. Following the N-1 criteria, we may consider that a single outage is the maximum stress level the system can safely tolerate before a line limit is exceeded. This stress level corresponds to  $P_{ab}^{(1)\max}$  in the single contingency case and to the value of  $\theta_{ab}$  in the single contingency case when  $P_{ab}^{(1)} = P_{ab}^{(1)\max}$ . That is, a threshold  $\theta_{ab}^{\text{threshold}}$  for  $\theta_{ab}$  is obtained as

$$\theta_{ab}^{\text{threshold}} = \frac{x}{2} P_{ab}^{(1)\max} \quad (6.1)$$

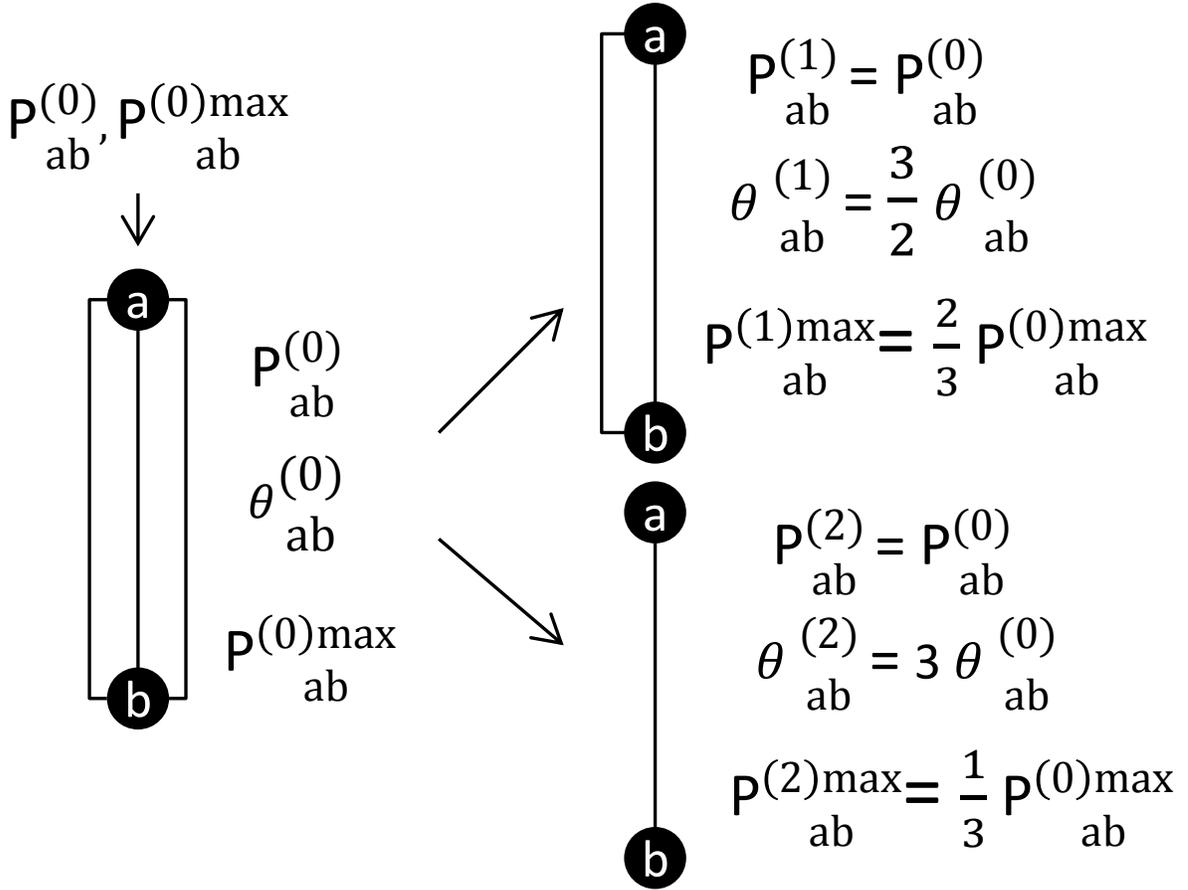


Figure 6.1 Comparing power and angle in a simple example of 3 parallel lines.

where  $x$  is the reactance of one of the lines.

We are interested in how  $P_{ab}^{\max}$  changes as line outages occur.  $P_{ab}^{\max}$  cannot be measured directly (it is calculated by increasing the power  $P_{ab}$  from its current value until a line limit is encountered). However, we can see from Fig. 6.1 that  $P_{ab}^{\max}$  is inversely proportional to  $\theta_{ab}$ , which can be monitored and compared to its threshold  $\theta_{ab}^{\text{threshold}}$ .

If an outage or outages occur,  $\theta_{ab}$  increases from its base case value and can be compared to the threshold  $\theta_{ab}^{\text{threshold}}$ .  $\theta_{ab} \leq \theta_{ab}^{\text{threshold}}$  indicates that line limits are satisfied after the outage(s). That is, the outage(s) is less severe than the highest-loaded case of a single outage satisfying the N-1 criterion, but may well require corrective action to restore operating margin. On the other hand,  $\theta_{ab} > \theta_{ab}^{\text{threshold}}$  indicates that line limits are violated after the outage(s).

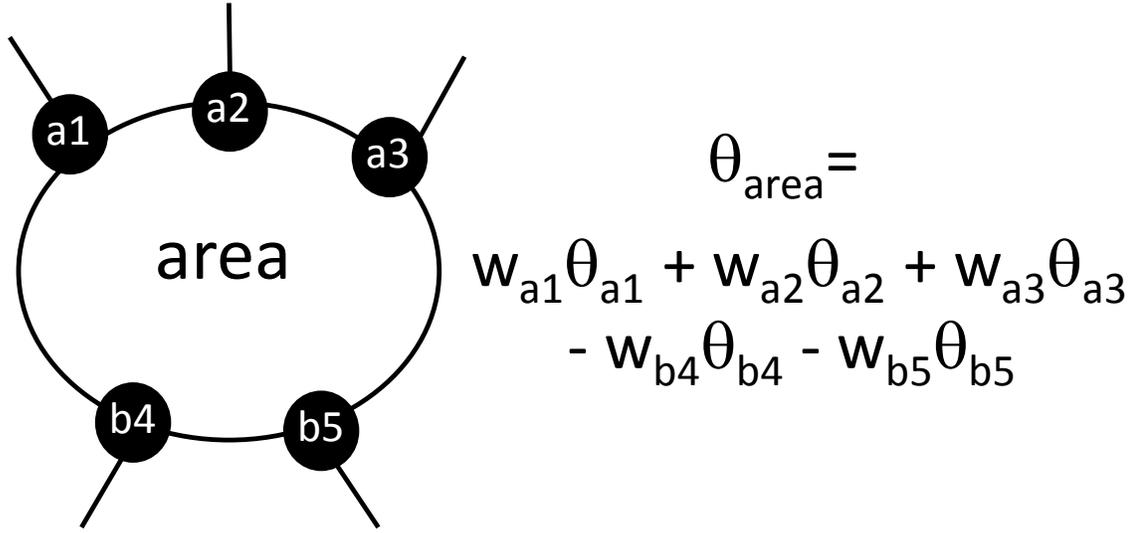


Figure 6.2 General area of power system and the area angle.

These outage(s) are more severe than the highest-loaded case of a single outage satisfying the N-1 criterion, and require emergency action reducing  $P_{ab}$  to resolve the violated line limits. The overall strategy is to set thresholds based on the line limits in terms of the economically significant maximum power transfer through the area, and then convert the threshold on the maximum power transfer to an equivalent threshold on the angle between the buses. Then monitoring the angle and comparing it to the angle threshold can detect what urgency of action is needed to reduce the power transfer in order to maintain security.

### 6.2.2 Generalization to an area of a power system

We generalize the simple example of Fig. 6.1 to the connected area of a power system of Fig. 6.2. The area is primarily transferring power from the buses in the north (marked a) to the buses in the south (marked b). The a and b buses together form a complete border of the area, so that removing the a and b buses would entirely disconnect the area from the rest of the power system. There is major generation north of the area and major load south of the area and we are interested in monitoring the area stress due to line outages inside the area.

The real power entering the area from the north is the sum of the powers entering the a buses along the tie lines connected to the a border buses. The angle difference  $\theta_{ab}$  between bus a and bus b in the simple example of Fig. 6.1 generalizes to an angle  $\theta_{\text{area}}$  across the area from the a buses to the b buses. The area angle is a new concept derived from circuit theory principles in (9). Number the area border buses  $1, 2, \dots, m$  and let the angles at these buses be  $\theta_1, \theta_2, \dots, \theta_m$ . Then the area angle  $\theta_{\text{area}}$  is computed as a weighted combination of the angles at border buses:

$$\theta_{\text{area}} = \sum_{j=1}^m w_j \theta_j \quad (6.2)$$

As might be expected for an angle across an area, the weights on the a buses are generally positive and the weights on the b buses are generally negative. The angles at all the a and b buses are obtained from the filtered quasi-steady synchrophasor measurements described in (22) that indicate the settled steady state measurement after the outage.

According to (9), the weights  $w$  are computed from the formula

$$w = (w_1, w_2, \dots, w_m) = \frac{\sigma_a B_{\text{eq}}}{b_{\text{area}}} \quad (6.3)$$

Here  $\sigma_a$  is the row vector of length  $m$  with ones at the positions of the a buses and zeros at the positions of the b buses.  $B_{\text{eq}}$  is the equivalent susceptance matrix of the border buses, which is calculated as the Kron reduction of the area susceptance matrix to the border buses.  $b_{\text{area}} = \sigma_a B_{\text{eq}} \sigma_a^T$  is the bulk susceptance of the area. Overall, it can be seen that the weights  $w$  can be obtained from the base case area topology and a DC load flow model of the area. A recent base case of the DC load flow model is generally available (22). An important detail is that we use the base case DC load flow to compute the weights  $w$ , and do not attempt to immediately update the DC load flow model based on the outage we are trying to monitor (10).

We are interested to obtain area angle thresholds corresponding to specific stress limits in the area and then observe the changes in the area angle caused by different outages inside the area in real time to be notified of different stress condition in the area. To discover the alarm or emergency thresholds based on the area angle, since we quantify stress in terms of the maximum power that could enter the area, we first determine thresholds of the maximum

power that could enter the area after the outages and then find out the corresponding area angle thresholds. Then in real time, comparing the area angle after outages with its thresholds notifies us of the different stress severities of outages inside the area.

## 6.3 Problem Setup

### 6.3.1 Overview

The first step is to define the area of interest as explained in section 6.2.2, including the particular power transfer through the area, and the border buses of the area at which synchrophasor measurements should be made. The area angle is a weighted combination of these synchrophasor measurements.

There is an offline calculation of actionable thresholds for the area angle and offline identification of any local power redistribution problems that are poorly detected by the bulk area angle. There are a limited number of these local power redistribution problems and they can be separately detected and resolved as explained below.

To apply the area angle online, we monitor the area angle computed from the measurements, and also monitor the local power redistribution problems. If there is a change in the area angle and a local power redistribution problem has not occurred, this indicates a change in bulk stress with respect to the transfer through the area and the line limits. The area angle after the outage is compared to its precomputed thresholds so that the appropriate action to reduce the power transfer (emergency action, some action needed to restore full security, no action required) can be chosen. The emergency threshold distinguishes outages that require emergency action from outages that require some action and the alarm threshold distinguishes outages that require some action from outages that require no action.

The procedures are summarized in the following steps:

#### 1) Offline calculations to set thresholds and identify local problems

1. For each single outage inside the area, after the outage, calculate the maximum power that can enter the area before the first line limit is encountered. The maximum power that can enter the area for the worst case single outage is the emergency threshold for

the maximum power entering the area. Also define the alarm threshold on the maximum power entering the area.

2. Set the base case power entering the area to the emergency threshold of the maximum power. Then for all single outages inside the area, calculate the area angle after the outage.
3. By finding outliers to the bulk relationship between the area angle and the maximum power that can enter the area, outages that cause local power redistribution problems can be identified, and these cases that are poorly detected by the area angle are dealt with separately.
4. Convert the emergency and alarm thresholds of the maximum power entering the area to the area angle emergency and alarm thresholds using the bulk relationship between the maximum power that can enter the area and the area angle.

## 2) Online implementation

1. In the control center, compute the area angle from the synchrophasor angles at the border of the area and monitor the occurrence of any of the outages causing local power redistribution problems.
2. If outages which are causing local power redistribution problems have not occurred, then compare the area angle to its thresholds to take no action or to take proper action with the appropriate urgency.
3. If outages that cause local power redistribution problems occur, then take the appropriate local action.

We now discuss some of these steps in more detail.

### 6.3.2 Setting thresholds on maximum power and angle

Since the system is operated with respect to the N-1 criterion for line limits, no single line outage will violate a line limit in the base case. We want to quickly detect from the

measurements how the severity of the outages compares to the worst case single outage. To do this, we set the threshold on the maximum power entering the area to be the maximum power entering the area satisfying the line limits when the most severe single outage occurs. Equivalently, this threshold is the minimum of the maximum power entering the area over all single outages, since the most severe single outage restricts the maximum power entering the area the most.

Now we convert the maximum power emergency threshold into an area angle emergency threshold because we can measure and monitor the area angle. The area angle emergency threshold is the area angle under the worst case single contingency when the power entering the area is equal to the maximum power that could enter the area. This area angle threshold is effective because, after the exceptional cases related to local outages are excluded, the area angle approximately increases as the maximum power that could enter the area decreases. That is, if multiple outages occur and the area angle after the outages is below its emergency threshold, then the corresponding maximum power entering the area is above its emergency threshold and the outages are comparable in severity to a single outage that does not violate line limits. After such outages, action may be needed to restore the N-1 security, but no emergency action is required. On the other hand, if the area angle after the outages exceeds the emergency threshold, then the corresponding maximum power entering the area is below its emergency threshold and the outages are comparable in severity to a single outage that violates line limits. After such outages, emergency action to resolve the problem is appropriate.

It is also useful to set an area angle alarm threshold below which no action is needed. This alarm threshold corresponds to a suitably small decrease in the maximum power entering the area from the base case maximum power entering the area. There are many multiple outages that have little effect on the system performance and if the area angle after these outages is below the alarm threshold, then no action needs to be taken.

To summarize, if the area angle after the outage is less than the alarm threshold, the area is safe and we do not need to take any action. If it is between the alarm and the emergency threshold, we need to take some moderate action. If it is more than the emergency threshold, we need to take emergency action to immediately reduce the bulk power transfer through the area.

### 6.3.3 Finding outages that cause local power redistribution problems

It is approximately the case that the area angle gets larger as the maximum power that could enter the area decreases. This relationship describes a bulk property of the area. Plotting this relationship between the maximum power that could enter the area and the area angle can reveal and identify those exceptional outages that are outliers that do not follow the bulk relationship.<sup>1</sup> The most common reason for these exceptional line outages causing a local power redistribution problem is proximity to large generation or load inside the area<sup>2</sup> (31).

The exceptional outages and consequent potential overloads are handled separately. For example, if the outage is near a larger load inside the area, then we may redispatch the local generation to serve that load. The mitigation or correction of these exceptional outages can be local or by a more wide area scheme, and can use SCADA or synchrophasor data, but in any case, a signal is sent to the control center when one of the exceptional outages occurs. Our experience so far is that there are a limited number of these exceptional outages to resolve.

We illustrate the effect of large load or generation inside the area in a simple example. Fig. 6.3 first shows the base case of a three bus system with buses a and b as border buses and load bus c inside the area, and then shows the effects of the outage of line a-c and the outage of line a-b. The line limits are chosen to satisfy the N-1 criterion in the base case and are specified in Fig. 6.3. We calculate the area angle  $\theta_{\text{area}} = \theta_a - \theta_b$  and the maximum power  $P_{\text{into}}^{\text{max}}$  that could enter bus a after each outage based on the line limits. Fig. 6.3 shows that after the outages of line a-c and of line a-b, while the maximum power  $P_{\text{into}}^{\text{max}}$  that could enter bus a is the same, the area angle  $\theta_{\text{area}}$  after these outages varies considerably. The outage of line a-c is an exceptional outage in which load in bus c makes  $\theta_{\text{area}}$  less than expected; some power will be redistributed from bus b to bus c in the opposite direction of the bulk power transfer between bus a and bus b.

There is a tradeoff between how closely the area angle tracks the outage severity and the

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<sup>1</sup> Working with all the single outages in steps 1(a) and 1(b) of section 6.3.1 identifies all of the outages causing local problems. Repeating 1(a) and 1(b) for a random selection of double outages can further help to identify these outages.

<sup>2</sup> Some grid models combine together lines and generation, especially at lower voltages, leading to lack of coordination between line limits and between line limits and generation; these reduced models can contribute to the exceptional cases. Also, the area can sometimes be adjusted to exclude large generators or loads.

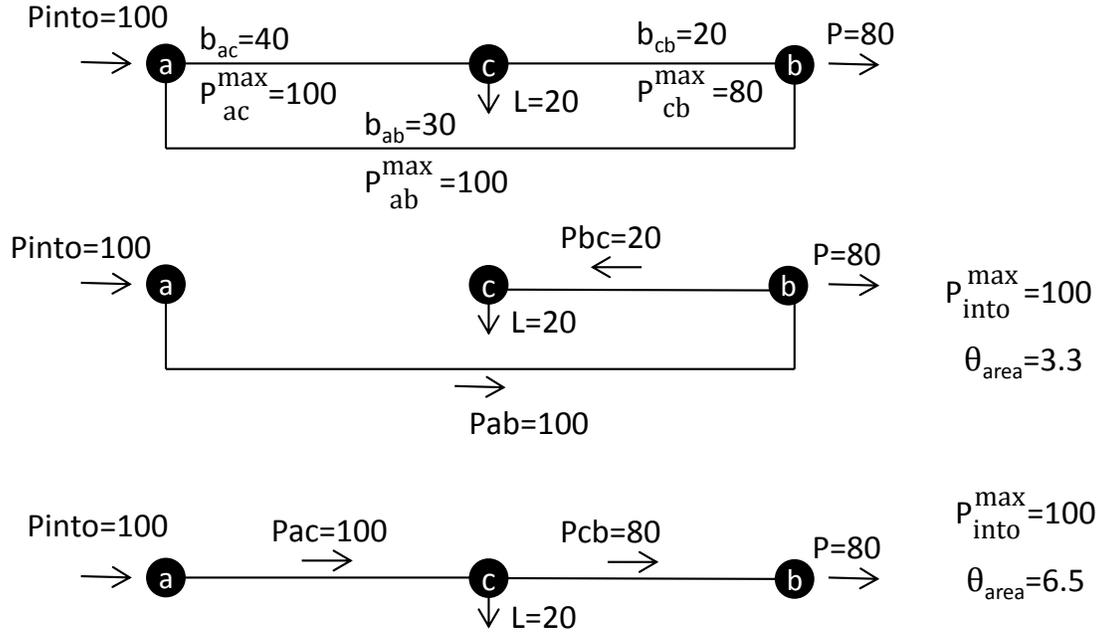


Figure 6.3 Simple example of a local power redistribution problem

number of exceptional line outages requiring special treatment. The outage severity tracking can be relaxed by only requiring the correct classification of outages by the thresholds. In this case the number of exceptional line outages will be smaller and applying the monitoring will be easier. A further reduction in the number of exceptional line outages can be achieved if one requires classification of outages only with respect to the emergency threshold.

Also it should be noted that once an outage of one of the lines which cause local problems is detected, we can have a estimate of the network situation, since we have observed that they cause a similar effect when they combine with other outages as when they occur singly, so studying the effect of the exceptional line outages in the single outage case will be very useful in the case of their combination with other outages.

### 6.3.4 Detail of formulation and calculations

This subsection gives the details of the formulation and calculations of the maximum power that could enter the area and the area angle. We use the following notation:

$X$	generic variable
$X^{(i)}$	$X$ evaluated for contingency number $i$ .
$X^{k\max}$	$X$ evaluated at the maximum power injection case obtained by applying power injection until line $k$ reaches its maximum power flow rating.
$X^{(i)\max}$	$X$ evaluated for the maximum power injection case obtained under contingency number $i$
$X^{\text{limit}}$	operating limit established for $X$

To evaluate the maximum power that can enter the area it is necessary to stress the area with additional power injections. These additional power injections are made at each border bus in proportion to the tie line flows entering or leaving that bus, as described later in this subsection. To calculate the effect of these power injections, we start with the simpler case of injecting additional power at a particular northern border bus  $r$  and removing the same amount of additional power from a southern border bus  $s$ .

To calculate the maximum possible extra power injection at the border buses that satisfies the line limit of all lines after each contingency, we first calculate  $\Delta P^{rs(i)k\max}$ , the maximum possible extra power injection at the border buses satisfying only the line limit of line  $k$ :

$$\Delta P^{rs(i)k\max} = \frac{\Delta P_k^{\text{limit}(i)}}{\rho_k^{rs(i)}}. \quad (6.4)$$

$\Delta P_k^{\text{limit}(i)}$  is the margin in line  $k$  after contingency  $i$  which is the power in line  $k$  after the contingency  $i$  subtracted from the line power limit of line  $k$ .  $\rho_k^{rs(i)}$  is the generation shift factor of line  $k$  with respect to the power injection in border buses  $r$  and  $s$ , which can be calculated as

$$\rho_k^{rs(i)} = b_k(e_u^T - e_v^T)(B^{(i)})^{-1}(e_r - e_s). \quad (6.5)$$

Here  $B^{(i)}$  is the susceptance matrix when line  $i$  is outaged,  $e_r$  is the vector with 1 at entry  $r$  and all other entries zero,  $b_k$  is the susceptance of line  $k$ , and  $u$  and  $v$  are the sending and receiving buses of line  $k$ .

Then  $\Delta P_{\text{inj}}^{rs(i)}$ , the maximum possible extra power injection at the buses  $r$  and  $s$  which satisfies all the line limits, is the minimum value of all the injections corresponding to each of

the  $n$  lines inside the area:

$$\Delta P_{\text{inj}}^{rs(i)} = \text{Min}\{\Delta P^{rs(i)1\text{max}}, \dots, \Delta P^{rs(i)n\text{max}}\}. \quad (6.6)$$

We add the extra injection in bus  $r$  from (6.6) to the base case power  $P_{\text{intor}}$  entering bus  $r$  to calculate the maximum power  $P_{\text{intor}}^{(i)\text{max}}$  that could enter bus  $r$  after contingency  $i$ :

$$P_{\text{intor}}^{(i)\text{max}} = P_{\text{intor}} + \Delta P_{\text{inj}}^{rs(i)}. \quad (6.7)$$

We expand this calculation from pair  $r$  and  $s$  to all the border buses of sets  $a$  and  $b$  and calculate the maximum power that could enter the area through the border buses  $a$ . In that case the first term on the right hand side of (6.7) will change to the power entering the area, which is the sum of the powers entering border buses  $a$ . To find the second term that is the extra injection considering buses  $a$  and  $b$  as border buses, we first calculate the generation shift factor of line  $k$  with respect to sets  $a$  and  $b$  and then update (6.4) and (6.6) accordingly.

We calculate the generation shift factor of line  $k$  with respect to sets  $a$  and  $b$  in the following way. The change in power flow of line  $k$  caused by proportional increases in injection in border buses  $a$  and  $b$  is

$$\rho_k^{ab(i)} = b_k(e_u^T - e_v^T)(B^{(i)})^{-1}(e_a - e_b). \quad (6.8)$$

Here  $e_a$  and  $e_b$  have the entry  $\alpha_j$  in positions corresponding to the sets  $a$  and  $b$  and the rest of the entries zero. The ratio  $\alpha_j = P_{\text{intoj}}/P_{\text{intoa}}$  is obtained by dividing the power entering or leaving border bus  $j$  by the total power entering or leaving all border buses in  $a$  or in  $b$ .

To calculate the area angle  $\theta_{\text{area}}^{(i)}$  corresponding to the maximum power entering the area with the worst case outage number  $i$ , the system is placed in the condition of limit of the maximum power entering the area with outage  $i$ , line  $i$  is outaged, and then the area angle is evaluated using (6.2) so that

$$\theta_{\text{area}}^{(i)} = \sum_{j=1}^m w_j \theta_j^{(i)}. \quad (6.9)$$

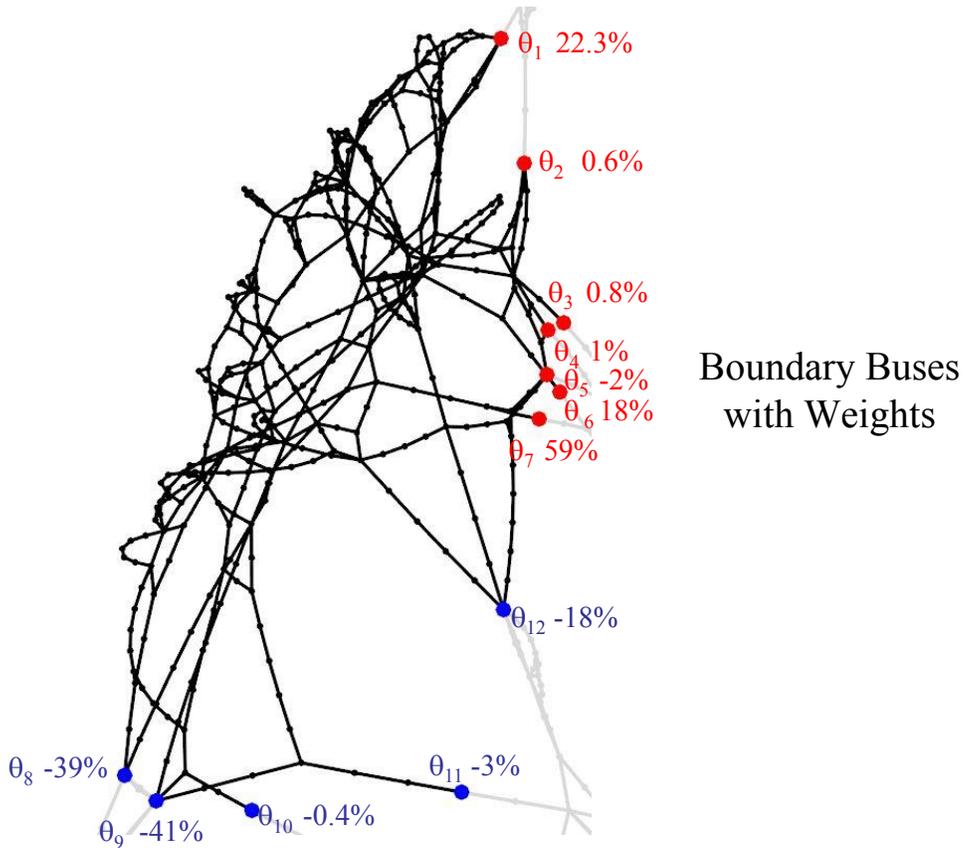
## 6.4 Case Study

We use an area of a 1553 bus reduced model of WECC shown in Fig. 7.2 that covers roughly Washington and Oregon states. The 7 north (and east) border buses are near the borders of

Canada-Washington, Washington-Montana, and Oregon-Idaho, and the 5 south border buses are near the Oregon-California border. There are 407 buses and 515 lines inside this area. The bulk power transfer of interest is north to south.

The area angle is the following weighted combination of the border bus angles:<sup>3</sup>

$$\begin{aligned} \theta_{\text{area}} = & 0.223 \theta_1 + 0.006 \theta_2 \\ & + 0.008 \theta_3 + 0.01 \theta_4 - 0.02 \theta_5 + 0.18 \theta_6 + 0.59 \theta_7 \\ & - 0.39 \theta_8 - 0.41 \theta_9 - 0.004 \theta_{10} - 0.03 \theta_{11} - 0.18 \theta_{12} \end{aligned}$$



8

Figure 6.4 Area of WECC system with area lines in black, north border buses in red and south border buses in blue. Layout is not geographic.

<sup>3</sup>The angle at border bus 5 has a negative weight due to incident lines with negative susceptances arising from grid model reduction. In practice the measurements with very small weights could be omitted.

In the following subsections, we do the offline calculation for this area to find area angle thresholds and the lines which cause local power redistribution problems. Then, to test how it would perform when monitoring multiple outages online, we use a random sample of triple outages to check that the thresholds can discriminate outage severity.

#### 6.4.1 Offline calculation to find thresholds and local power redistribution problems

**Finding emergency and alarm thresholds of maximum power transfer** We take out all single lines inside the area in turn and calculate the maximum power that could enter the area for each. It should be noted that the maximum power that could enter the area is related to the line limits and does not depend on the base case load level. We order the outages by decreasing amount of the maximum power transfer so that the outages are ordered with increasing severity and show the results in Fig. 6.5. Then it can be seen that the lowest value of the maximum power transfer (the worst case single outage) is near 35 pu and so we initially set the emergency maximum power threshold to be 35 pu. However, in the case considered, the worst case single outage turns out to be an exceptional outage in step (c) below. Thus there arises a choice, after step (c) in the detail of setting the emergency threshold of whether the worst case outage should be the worst case outage over all outages or the worst case over the non-exceptional outages. Since we are looking for the thresholds with respect to bulk power transfer, after step (c), we decide to set the emergency threshold according to the worst case non-exceptional outage and revise the emergency threshold for the maximum power entering the threshold accordingly to 40 pu. Considering that the maximum power transfer for the base case (no outage) is 62.5 pu, we set the alarm threshold on the maximum power transfer to 60 pu. (The maximum power transfer decreases more quickly below 60 pu so that those outages start to be more severe.)

**Calculate the area angle for outages** This calculation is done with the bulk power transfer set to 35 pu (maximum power transfer for the worse single outage). We find the area angle after all single outage, and then order the outages according to increasing severity. We

also do the same calculation for a random combination of double outages, since this can help us to find the local problems more easily. Figs. 6.5 and 6.6 show the results ordered by outage severity for single and double outages.

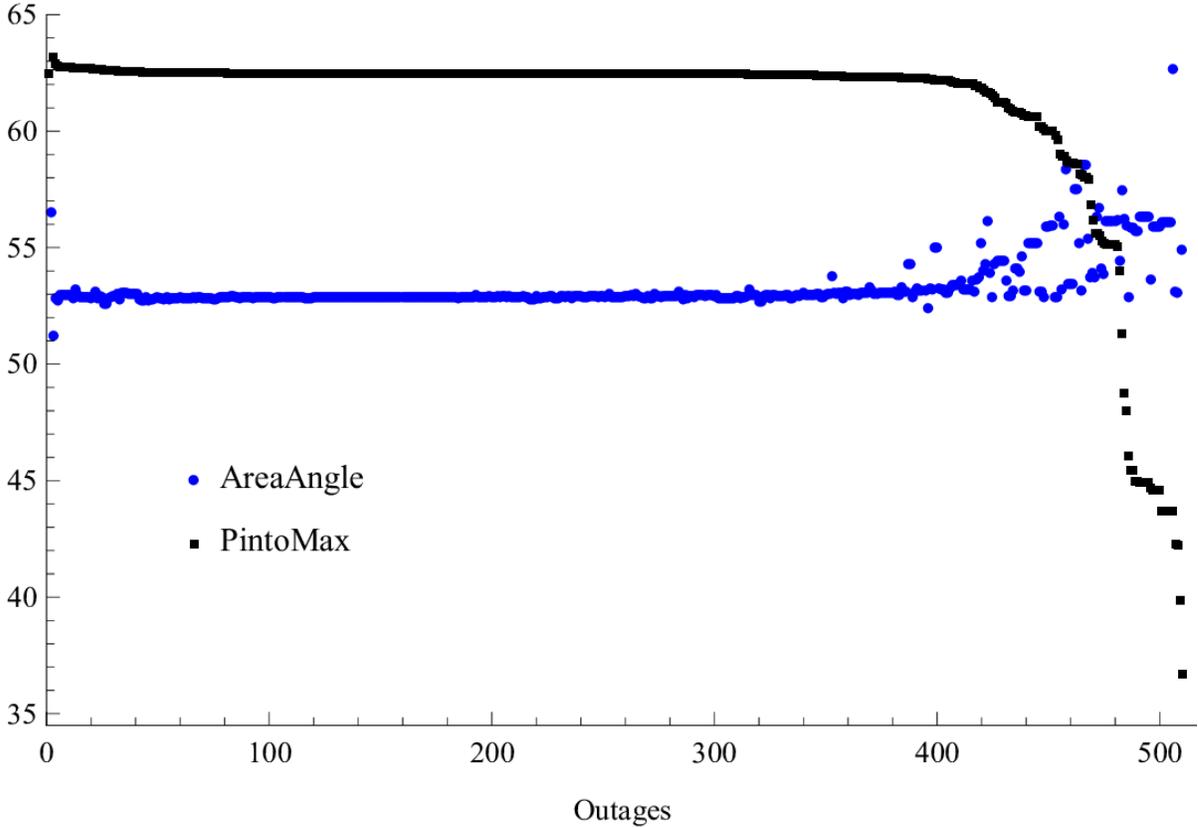


Figure 6.5 Area angle  $\theta_{\text{area}}^{(i)}$  in degrees, and maximum power into the area in per unit for all single outages inside the area. Horizontal axis is outage number.

**Finding the exceptional outages** As we can see in Figs. 6.5 and 6.6, generally the area angle increases as the maximum power that could enter the area decreases. But there are some outliers that indicate exceptional outages that do not follow this pattern whose severity is poorly indicated by the area angle. These exceptional outages are associated with local power redistribution problems and appear in the form of individual points in the single outage case and as sets of points in the double case (in the double outage case, the combination of each exceptional line outage with all the other line outages makes a set of points). In the case

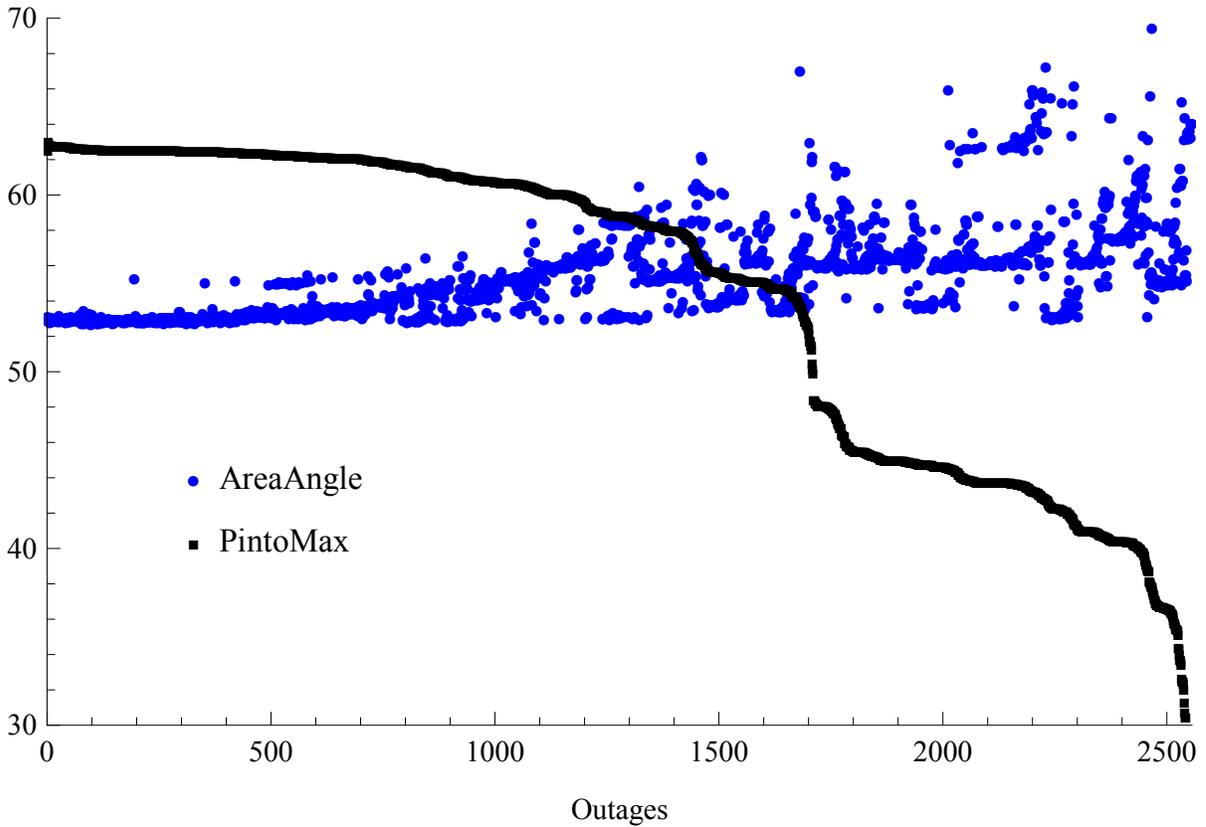


Figure 6.6 Area angle  $\theta_{\text{area}}^{(i)}$  in degrees, and maximum power into the area in per unit for a random sample of double outages inside the area.

studied, there are about 54 outlier lines out of 515 lines inside the area. Removing these lines from Fig. 6.5 yields Fig. 6.7. Only about 30 of these 54 outliers are of concern in causing a local power redistribution problem. Removing the 30 main outliers from Fig. 6.6 yields Fig. 6.8.

**Convert the thresholds of the maximum power entering the area to angle thresholds** We use Figs. 6.7 and 6.8 to convert the maximum power emergency threshold of 40 pu to the emergency area angle threshold of 63 degrees. For all the severe double outages in Fig. 6.8 that reduce the maximum power that could enter the area below 40 pu, the area angle is above 63 degrees.

We can use either Fig. 6.7 or Fig. 6.8 to convert the alarm power threshold of 60 pu to the area angle threshold of 56 degrees. For all the moderate severe outages that reduce the

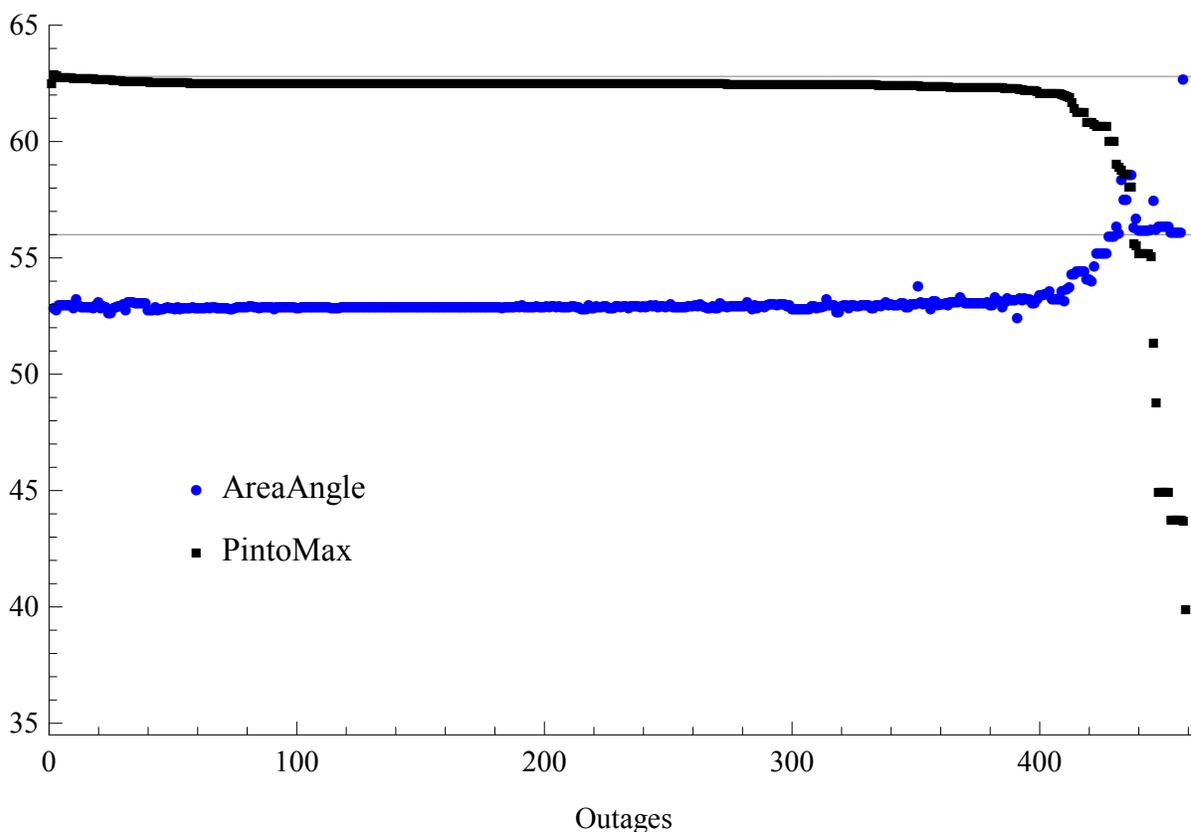


Figure 6.7 Area angle  $\theta_{\text{area}}^{(i)}$  in degrees, and maximum power into the area in per unit for all non-exceptional single outages inside the area. Horizontal line is area angle alarm threshold. Horizontal axis is outage number.

maximum power transfer to between 40 and 60 pu, the area angle is between 56 and 63 degrees.

#### 6.4.2 Test for online implementation

For online monitoring, we would first check using SCADA or synchrophasor data whether the line outage is one predetermined to cause a local problem. If the outage is an outage causing local problems, this is resolved by local actions. If the outage is not an outage causing local problems, we compute the area angle from the synchrophasor measurements of angles at the border buses after the outage and compare this area angle to the area angle thresholds to determine if the outage is safe, moderately severe, or severe.

To test the method, we randomly sampled triple outages from all lines except the 30 lines

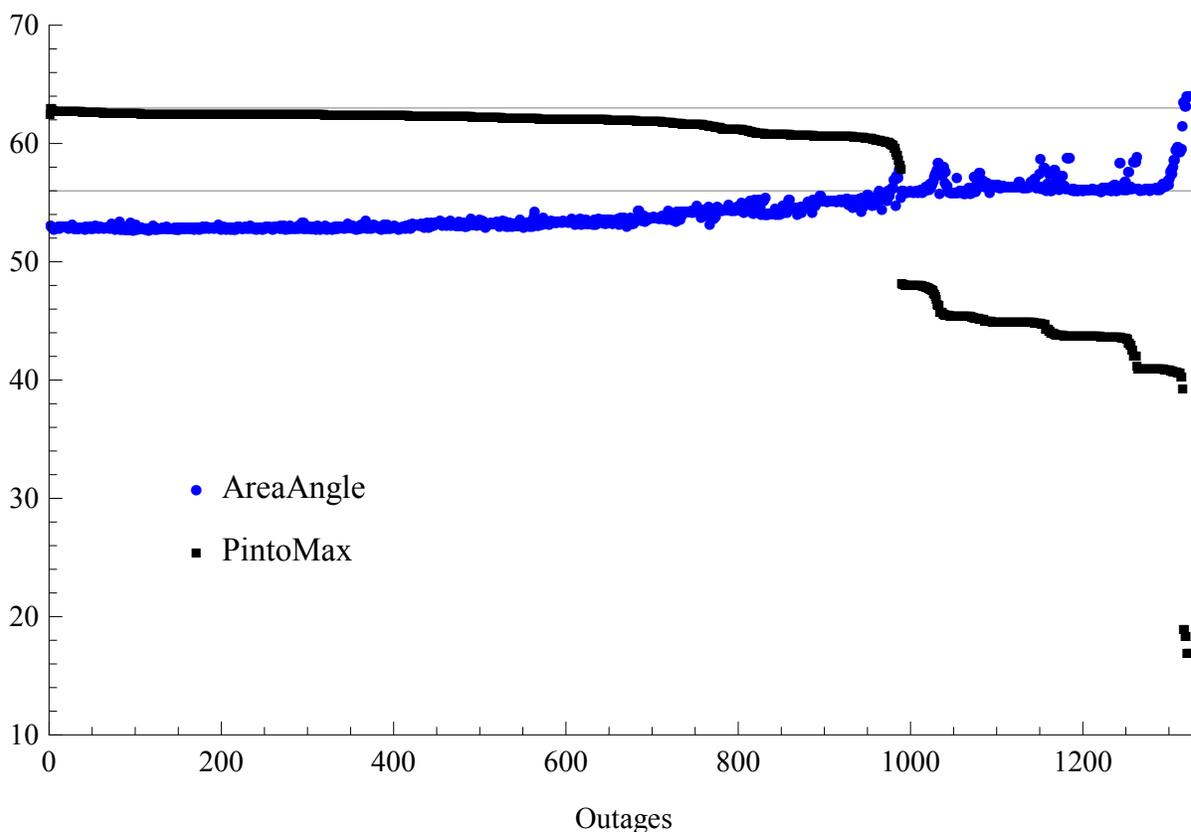


Figure 6.8 Area angle  $\theta_{\text{area}}^{(i)}$  in degrees, and maximum power into the area in per unit for a random sample of non-exceptional double outages in the area. Upper horizontal line is area angle emergency threshold and lower horizontal line is area angle alarm threshold. Horizontal axis is outage number.

that cause local problems and computed the area angle and maximum power entering the area after each of these triples outages. We ordered the results and plotted them in Fig. 6.9. As can be seen in Fig. 6.9, for the most severe triple outages (numbered from 310 to 350) that reduce the maximum power coming to the area below 40 pu, the area angle is above the emergency threshold of 63 degrees and for all triple outages numbered from 100 to 310 which decrease the maximum power transfer from 60 pu to 40 pu, the area angle is between 56 and 63 degrees, and for the rest of triple outages numbered less than 100 which decrease the maximum power transfer only to 60 pu, the area angle is below 56 degrees.

The emergency threshold is also effective for multiple outages in discriminating emergency cases in which line overloads are caused, since for all the multiple outages that are above the

emergency limit of 63 degrees the maximum power that could enter to the area is below 35 pu, the maximum power transfer for the worst case single outage over all single outages. This implies that for all multiple outages above the emergency limit some line limits are violated.

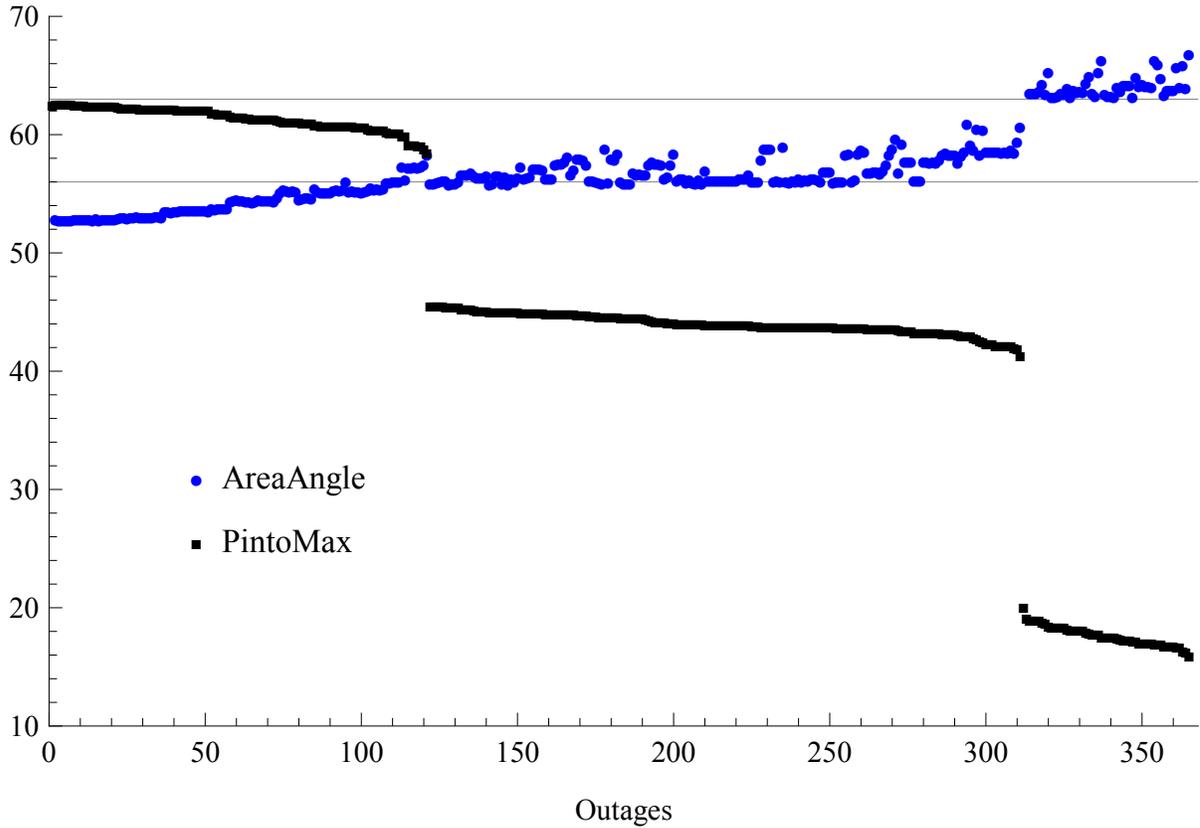


Figure 6.9 Area angle  $\theta_{\text{area}}^{(i)}$  in degrees, and maximum power into the area in per unit for a random sample of non-exceptional triple outages in the area.

#### 6.4.3 Real time change in load

when there is a change in the base case load or the base case power that enters to the area, the real time area angle changes and indicates the change in the stress level because of change in load.

Fig. 6.10 shows how real time area angle is changing after same multiple outages of Fig. 6.9 happen when the base case power that enters to the area increased by 5 pu.

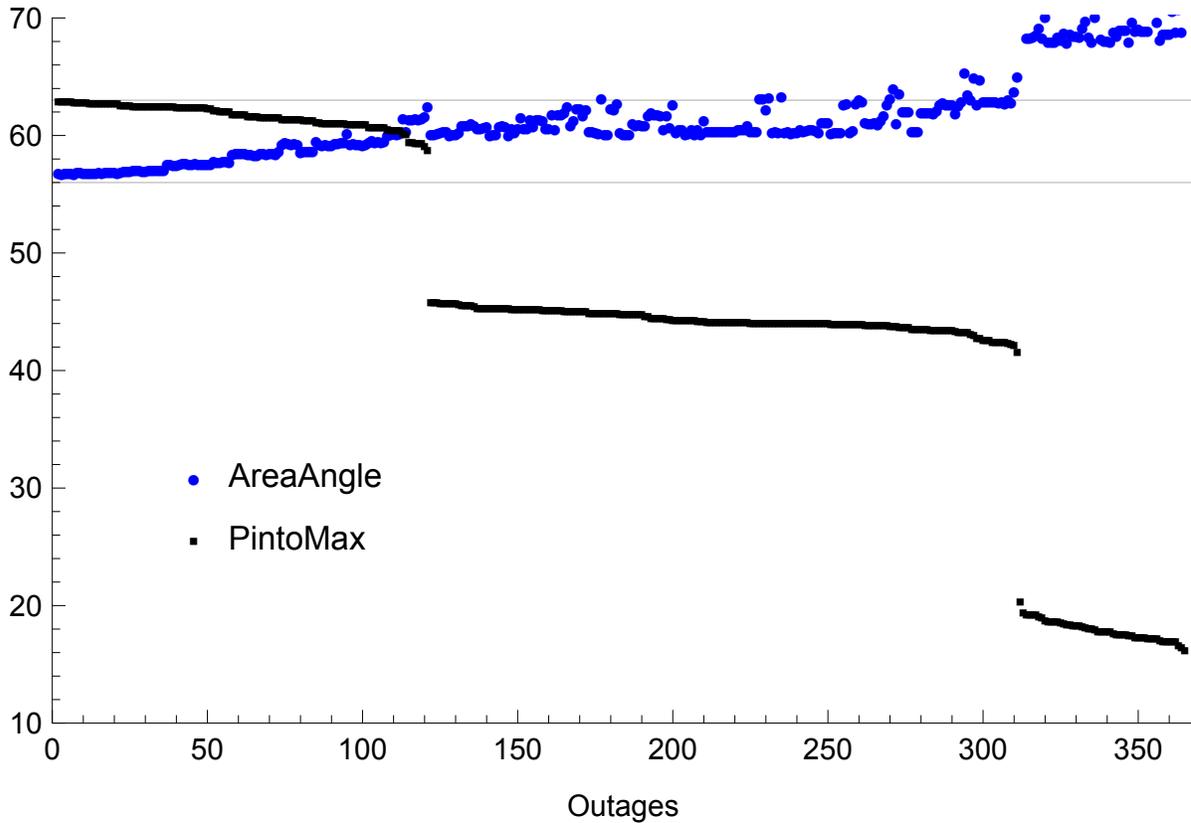


Figure 6.10 Area angle  $\theta_{\text{area}}^{(i)}$  in degrees after load increases, and maximum power into the area in per unit for a random sample of non-exceptional triple outages in the area and 5 pu increase to the power entering the area.

Fig. 6.10 shows that area angle increases compare to Fig. 6.9. Since the power transfer through the area increases after increase in base case load, the area angle also increases. This increase in area angle indicates the extra stress in transmission lines due to increase in load and then power transfer as well.

#### 6.4.4 Trade off between classification accuracy and the number of exceptional outages

Fig. 6.9 shows that area angle can track the severity and can classify the outages into alarm and emergency cases with 30 exceptional line outages. But if one is less interested in tracking severity and only interested in discriminating the emergency outages, the number of exceptional

outages can be reduced to 15.

Fig. 6.11 shows this more relaxed classification for triple outages. Fig. 6.11 preserves the emergency threshold, but loses the exact track of the severity by the area angle and the exact classification between the safe and moderate outages.

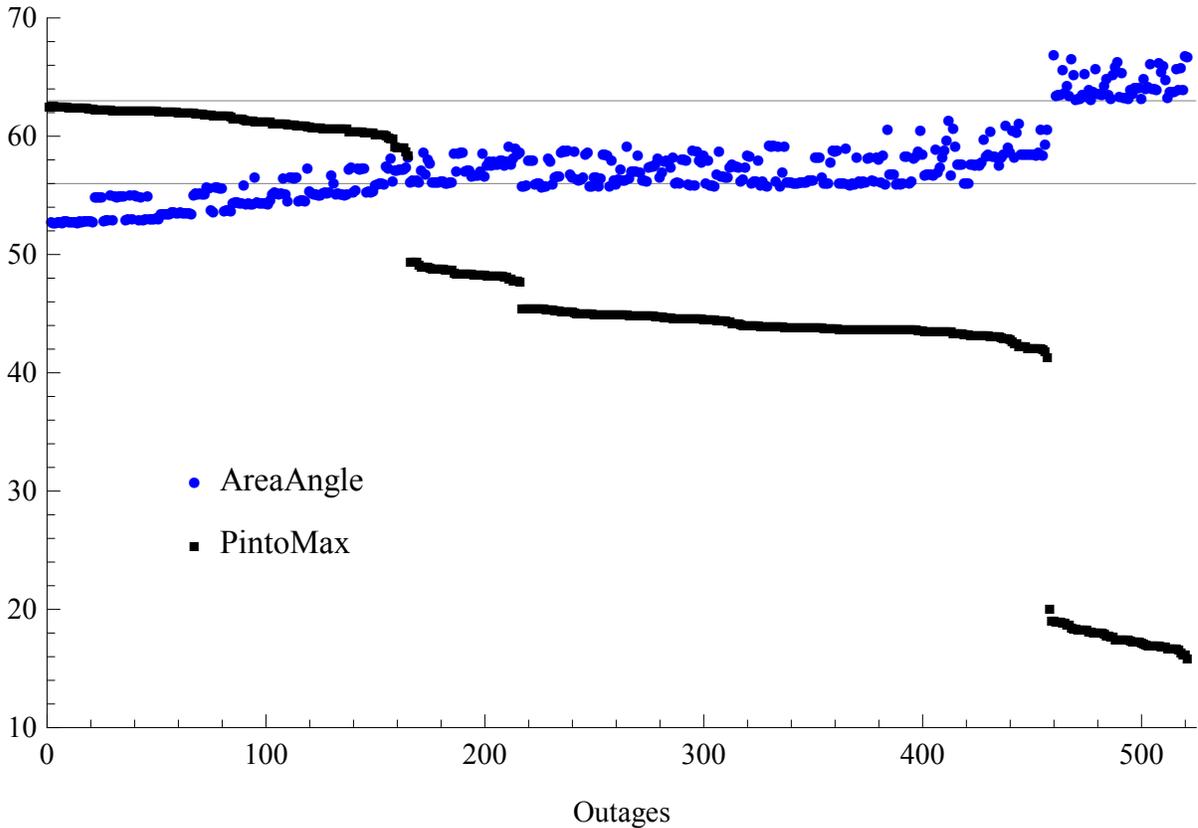


Figure 6.11 Area angle  $\theta_{\text{area}}^{(i)}$  in degrees, and maximum power into the area in per unit for a random sample of triple outages in the area with 15 exceptional outages excluded. Horizontal axis is outage number.

We need to monitor the 15 exceptional outages and resolve them separately with local actions. Our experience is that these outages cause the same discrepancy in area angle (usually underestimating, but a few overestimating severity) in both the single and double outage cases. Therefore appropriate actions can be deduced from the single action case.

### 6.4.5 Sensitivity analysis of area angle to the weights

As discussed in detail, the area angle is calculated in real time from the PMU angles with constant weights that are calculated from the base case DC load flow of the area. Suppose that a line outage occurs. Then the area angle is calculated with weights from the base DC load flow. However, we could also calculate the area angle with updated weights to evaluate the sensitivity of area angle to the change in the weights, although this is impractical to implement in real time.

Fig. 6.12 shows how border-bus weights are changing after all non-exceptional single outages inside the area of WECC system discussed in the work. The value of weights for outage 0 is the base case value of the weights. Fig. 6.12 shows that for most border buses the weights do not change much from their base case value.

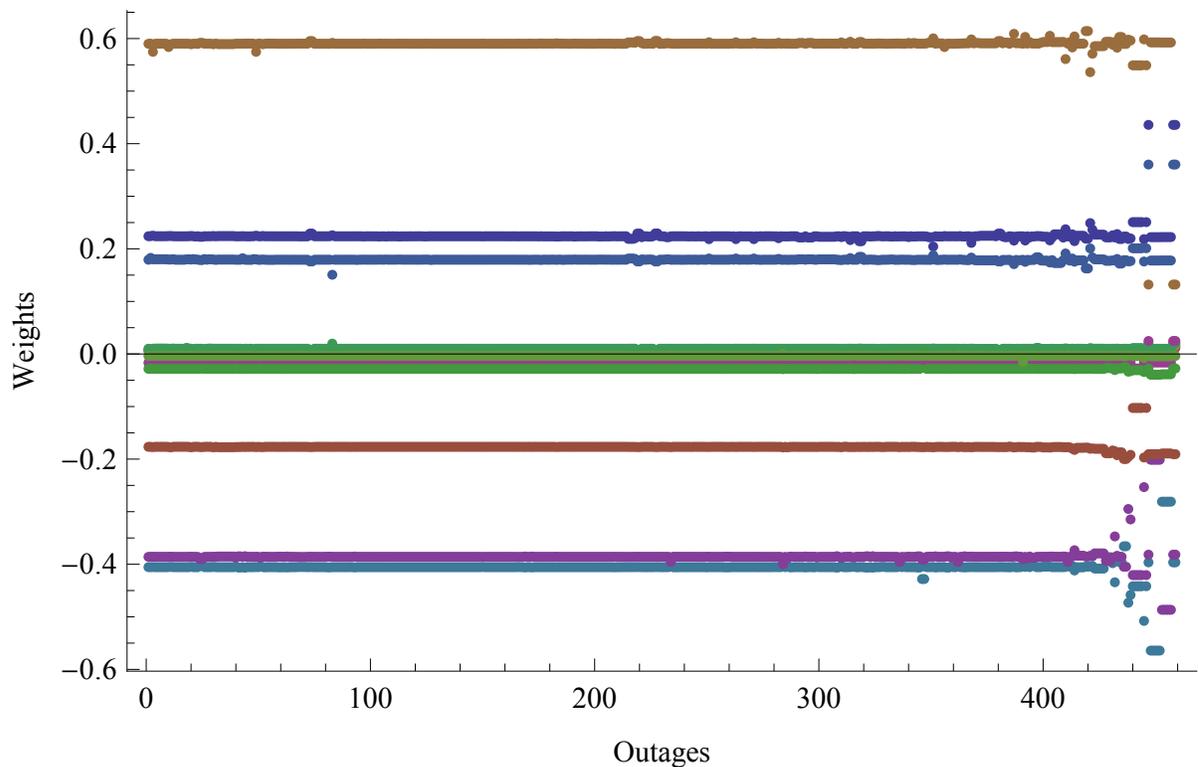


Figure 6.12 Weights for all border buses after all non-exceptional single outages in the area.

Furthermore, it is the resulting changes in the value of the area angle computed from the weights that matters the most, rather than the individual change of weights. After all, it is

the area angle that it is monitored. Fig. 6.13 shows how the area angle with updated weights changes after all non-exceptional outages and one can see in Fig. 6.13 that this value is near to the value that is computed with constant weights and the emergency threshold will not change.<sup>4</sup> We discussed this issue in detail with formulation and numerical results for different areas of WECC in (33) and also summarized in chapter 4.

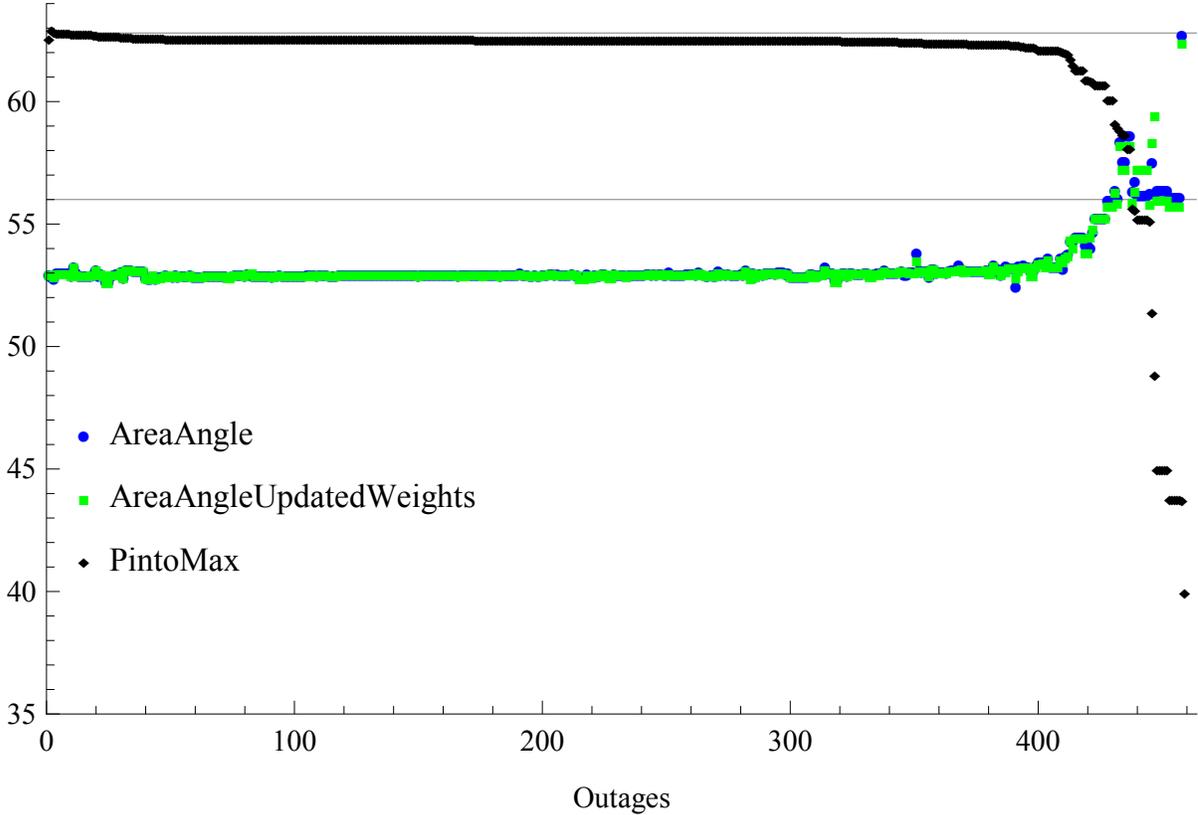


Figure 6.13 Area angle with weights calculated from the base case DC load flow in blue, and area angle with weights updated after the outage in green for all non-exceptional single outages. Angles are in degrees and maximum power into the area is in per unit.

Fig. 6.13 shows that the area angle is largely insensitive to the change in the weights and also that the thresholds do not change. We explained this insensitivity in (33).

<sup>4</sup> The mean value of the ratio between area angle with constant weights to the area angle with updated weights is 0.9993, the standard deviation is 0.006082, and it ranges from 0.9236 to 1.056.

## 6.5 Conclusion

A limited number of synchrophasor measurements at the border of a suitable chosen area of the power system are combined to obtain an area angle that can quickly monitor the severity of multiple outages inside the area. This capability could help to mitigate cascading outages in the early, slower stages of cascading. This approach relates the area angle to the maximum power that can be transferred through the area for a particular bulk power transfer direction through the area.

This chapter describes a procedure to set a meaningful emergency threshold for the area angle after multiple outages according to the worst case single outage. This worst case corresponds to the N-1 criterion. Moreover, our results show that this emergency threshold is also effective for multiple outages in discriminating emergency cases in which line overloads are caused. The procedure also identifies line outages associated with local power transfer problems that can also limit the bulk power transfer; these line outages are addressed separately by separate monitoring and control actions.

The angle severity and thresholds are obtained by considering the bulk power transfer of power throughout the area as limited by overloads of lines inside the area. This formulation limits the monitoring to the limits on this bulk transfer, but has the benefit that if the area angle exceeds the threshold, then the mitigating action of reducing the transfer is clear.

## CHAPTER 7. INTUITION FOR MONITORING POWER TRANSFER WITH AREA ANGLE

### 7.1 Introduction

As discussed fully in other chapters, area angle can monitor the severity of the outages in the area. Also by setting thresholds the emergency outages and situation can be monitored and quick actions to decrease power transfer can be applied.

This chapter investigates the relationship between area angle and maximum power transfer after multiple outages happen. It relates the maximum power transfer to the generation shift factor of the congested line after outages happen. After maximum power transfer limits can be converted to angle limits, the real time monitoring of power transfer between areas would be possible.

### 7.2 Monitoring Of Maximum Power Transfer With Area Angles

#### 7.2.1 Simple example

Fig. 7.1 shows a simple example of three parallel lines connecting two buses a and b and compares  $\theta_{ab}$ , the voltage angle between a and b, with  $P_{ab}^{\max}$ , the maximum real power that could enter bus a as determined by the line power flow limits. It also shows  $\rho_{ab}$ , the generation shift factor of any of the lines between a and b, which is defined as the increase in power flow of the line for a one unit increase in power entering bus a. The superscripts (0), (1), and (2) stand for the base, single line outage, and double line outage cases respectively.

Fig. 7.1 shows that as outages become more severe,  $P_{ab}^{\max}$  decreases while  $\rho_{ab}$  and  $\theta_{ab}$  increase. For example, for the severe case of the double outage, the remaining line carries all the power entering bus a and the generation shift factor of the remaining line increases.

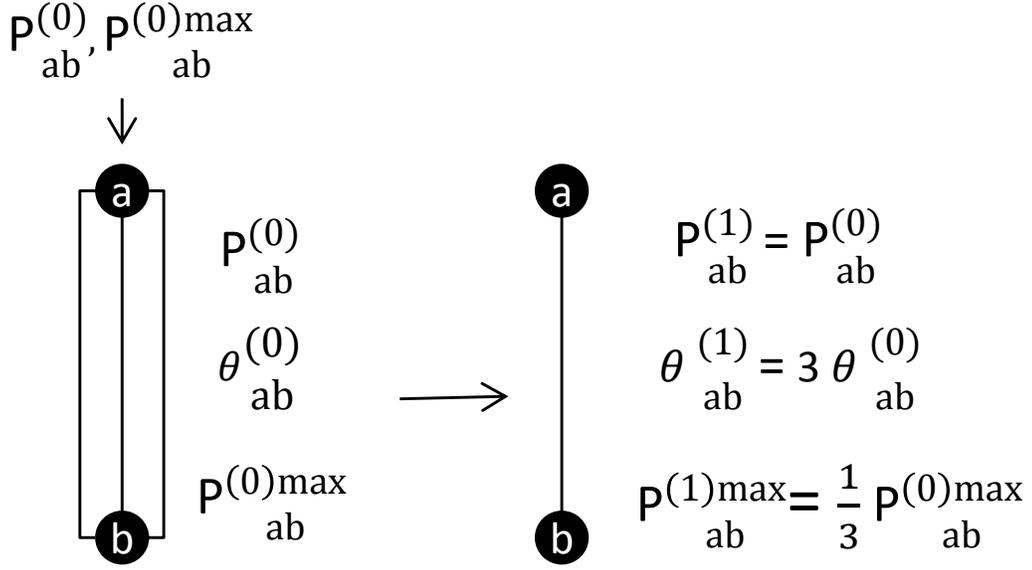


Figure 7.1 Maximum power, angle, and generation shift factor in 3 parallel lines

The extra power flowing in the remaining line between  $a$  and  $b$  causes the angle difference between  $a$  and  $b$  to increase while it limits the maximum power that could enter bus  $a$ . This inverse relation between the angle difference of  $a$  and  $b$  with the maximum power entering bus  $a$  suggests that  $\theta_{ab}$ , which can be directly measured, can be used to monitor  $P_{ab}^{\max}$ . Moreover, the fact that  $\theta_{ab}$  is a good indicator of  $P_{ab}^{\max}$  gives the motivation to convert thresholds for the maximum power transfer to angle thresholds, since we can directly monitor the angles.

Consider the maximum power  $P_{ab}^{\max}$  entering bus  $a$ .  $P_{ab}^{(0)\max}$  in the base case is three times the line limit and  $P_{ab}^{\max}$  decreases as the outages become more severe. Following the N-1 criteria, we may consider that the maximum power entering bus  $a$  after single outage  $P_{ab}^{(1)\max}$  is a suitable threshold for  $P_{ab}$ . The threshold  $\theta_{ab}^{\text{threshold}}$  for  $\theta_{ab}$  corresponding to  $P_{ab}^{(1)\max}$  entering bus  $a$  is obtained as

$$\theta_{ab}^{\text{threshold}} = \frac{x}{2} P_{ab}^{(1)\max}, \quad (7.1)$$

where  $x$  is the reactance of one of the lines.

We are interested in how  $P_{ab}^{\max}$  changes as line outages occur.  $P_{ab}^{\max}$  cannot be measured

directly (it is calculated by increasing the power  $P_{ab}$  from its current value until a line limit is encountered). However, we can see from Fig. 7.1 that  $P_{ab}^{\max}$  is inversely proportional to  $\theta_{ab}$ , which can be monitored and compared to its threshold  $\theta_{ab}^{\text{threshold}}$  after outages occur.  $\theta_{ab} \leq \theta_{ab}^{\text{threshold}}$  indicates the outages are less severe than the highest-loaded case of a single outage satisfying the N-1 criterion, but may well require corrective action to restore operating margin. On the other hand,  $\theta_{ab} > \theta_{ab}^{\text{threshold}}$  indicates that line limits are violated and emergency action is required to reduce  $P_{ab}$ .

The overall strategy is to set thresholds based on the line limits in terms of the economically significant maximum power transfer through the area, and then convert the threshold on the maximum power transfer to an equivalent threshold on the angle between the buses. Then monitoring the angle and comparing it to the angle threshold can detect what urgency of action is needed to reduce the power transfer in order to maintain security.

We expand this simple example to the real system by considering a connected area of the power system with border buses  $M$ . Border buses  $M$  are comprised of two sets of buses,  $a$  border buses near the generation and  $b$  border buses near the loads. The area transfers bulk power from the generation to the load. As described in detail in (9), the area angle  $\theta_{\text{area}}$  is a weighted combination of the angles around the area:

$$\theta_{\text{area}} = \frac{\sigma_a B_{mm}^{\text{red}} \theta_m}{b_{\text{area}}} = w \theta_m \quad (7.2)$$

Here  $\sigma_a$  is the vector of the size of the border buses  $m$  which has the entry 1 in all  $a$  border buses and 0 entry in the rest.  $B_{mm}^{\text{red}}$  is the susceptance matrix of a reduced subnetwork of border buses which is electrically equivalent to the original network from the perspective of the border buses.  $\theta_m$  are the angles of the border buses  $M$ .  $w$  is a row vector of the weights that depends only on the area topology and the susceptances of lines in the area.  $b_{\text{area}} = \sigma_a B_{mm}^{\text{red}} \sigma_a^T$  is the total susceptance of the area.

It is not enough to choose an area and define a valid area angle according to these requirements; it is also important to choose an area angle that is meaningful and useful for power systems operation. In this thesis, we choose an area of the transmission system between major generation and major load to try to describe with the area angle the major power entering the

area. In that case, the power entering in to the area similar to the power entering to the bus  $a$  in the simple example case will be the sum of the powers entering in to the area along the tie lines connected to the border buses  $a$ .

Our goal is to monitor a single quantity for the area that captures the maximum power transfer capability of the area after single or multiple outages inside the area. We show that the area angle  $\theta_{\text{area}}$  can track the maximum power transfer. Furthermore we can convert thresholds of the maximum power to the thresholds of the angle.

We calculate the maximum power transfer after the outages using the margin of the power and the generation shift factor of the first line that congests after the outages in the area. We show how the maximum power transfer is related to the generation shift factor of the congesting line and then show how area angle and generation shift factor of the congesting line change when outages occur in the area. In this way the relationship between area angle and maximum power transfer will become clear as outages occur in the area.

### 7.2.2 Formulation

To evaluate the maximum power that can enter the area after the outage  $i$ , it is necessary to stress the area with additional power injections until the first line in the area violates its power flow limit. Suppose that line  $uv$  is this congesting line. (This line may vary somewhat as different lines outage.) The additional power injections are made at each border bus in proportion to the tie line flows entering or leaving that bus. Then the maximum possible extra power injection  $\Delta P_{\text{intoa}}^{(i)}$  at the border buses that satisfies the line limit of all lines after line  $i$  outages is:

$$\Delta P_{\text{intoa}}^{(i)} = \frac{\Delta P_{uv}^{\text{limit}}}{\rho_{uva}^{(i)}}. \quad (7.3)$$

$\Delta P_{uv}^{\text{limit}}$  is the margin in power flow of line  $uv$  after contingency  $i$  and  $\rho_{uva}^{(i)}$  is the generation shift factor of the congested line  $uv$  with respect to the power injection in border buses  $a$  after contingency  $i$ , which can be calculated as

$$\rho_{uva}^{(i)} = \frac{b_{uv} e_{uv}^T}{P_{\text{intoa}}} \theta^{(i)}. \quad (7.4)$$

$b_{uv}$  is the susceptance of line  $uv$  and  $e_{uv}$  is the vector of size the number of buses in the area with 1 at entry  $u$ ,  $-1$  at entry  $v$ , and all other entries zero and  $P_{\text{intoa}}$  is the power entering the area at the border buses  $a$ .  $\theta^{(i)}$  is the vector of angles at all the area buses after outage  $i$ .

We add the extra injection into the  $a$  buses (7.3) to the base case power entering buses  $a$  to calculate  $P_{\text{intoa}}^{(i)\text{max}}$ , the maximum power that could enter the border buses  $a$  after contingency  $i$ :

$$P_{\text{intoa}}^{(i)\text{max}} = P_{\text{intoa}} + \Delta P_{\text{intoa}}^{(i)} \quad (7.5)$$

Equation (7.3) shows that the maximum possible extra power injection and so the maximum power transfer is inversely related to  $\rho_{uva}^{(i)}$ , the generation shift factor of the congested line with respect to injection in border buses  $a$ .

To calculate the area angle after outage  $i$  we simply use (7.2) with the updated angles at the border buses after the outage  $i$ .

$$\theta_{\text{area}}^{(i)} = w\theta_m^{(i)} = we_{mn}\theta^{(i)}, \quad (7.6)$$

where  $e_{mn}$  is the  $m \times n$  matrix of ones and zeros that extracts the border bus angles  $\theta_m$  from the area angles  $\theta$  according to  $\theta_m = e_{mn}\theta$ .  $m$  is the number of border buses and  $n$  is the number of buses.

Equation (7.6) shows that area angle is related to the vector of weights which remains constant and the angle difference between border buses which changes after outage  $i$ . If there is a severe outage in the area which significantly increases the angle differences between the border buses  $a$  and the border buses  $b$ , it will also increase the area angle.

We explain the typical way that a line outage redistributes the bulk power in the area, and how the generation shift factor and area angle respond. When the power entering to the area remains constant, if there is a severe outage in the area, it causes the other lines, including the congesting line, to carry more power. That is, the outage redistributes the power flow through the area to the other lines. Then the generation shift factor of the congesting line with respect to the power injection into the border buses increases. Moreover, the angles across the lines increase and also the angles from border buses  $a$  near generation to the border buses  $b$  near the load increase. Hence the area angle computed by (7.6) increases since it is an overall angle difference between buses  $a$  and buses  $b$ .

We numerically find the maximum power transfer, area angle and the generation shift factor in the results presented in section 7.3. These results show that as outages become more severe, the maximum power transfer decreases, and the area angle and the generation shift factor of the congested line increase.

### 7.3 Results

We use an area of a 1553 bus reduced model of WECC shown in Fig. 7.2 that covers roughly Washington and Oregon states. The 7 north (and east) border buses are near the borders of Canada-Washington, Washington-Montana, Oregon-Idaho, and the 5 south border buses are near the Oregon-California border. There are 407 buses and 515 lines inside this area. The bulk power transfer of interest is north to south.

The area angle is the following weighted combination of the border bus angles:<sup>1</sup>

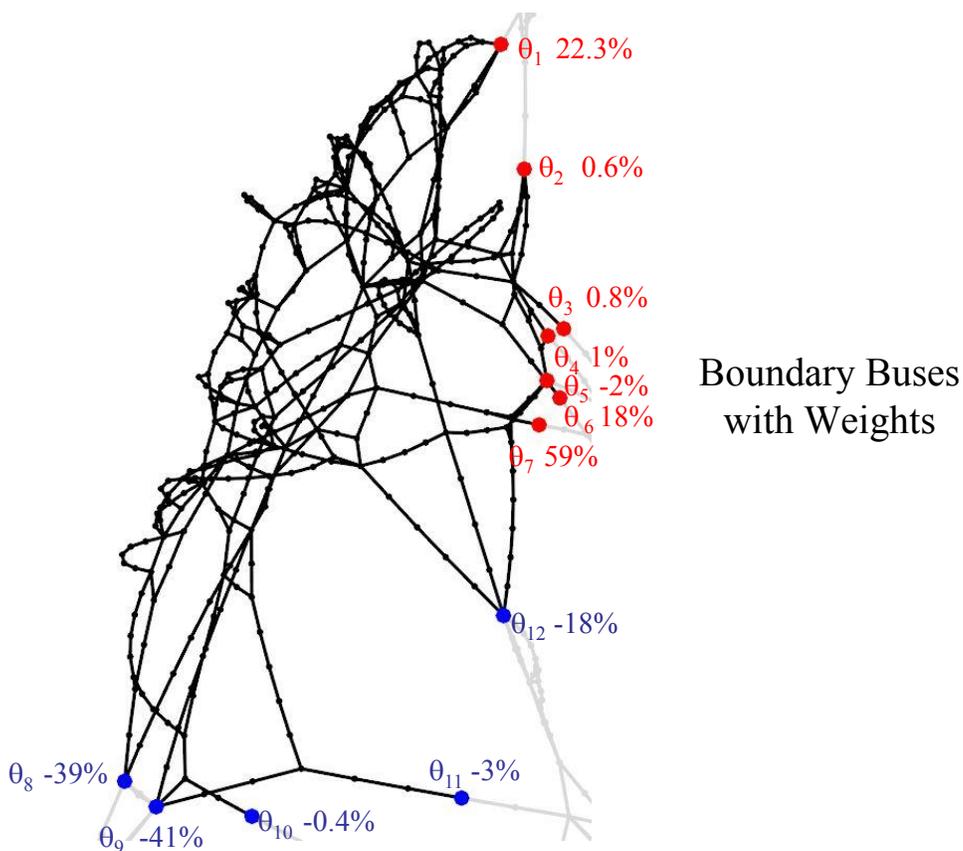
$$\begin{aligned} \theta_{\text{area}} = & 0.223 \theta_1 + 0.006 \theta_2 \\ & + 0.008 \theta_3 + 0.01 \theta_4 - 0.02 \theta_5 + 0.18 \theta_6 + 0.59 \theta_7 \\ & - 0.39 \theta_8 - 0.41 \theta_9 - 0.004 \theta_{10} - 0.03 \theta_{11} - 0.18 \theta_{12} \end{aligned}$$

We will show how area angle tracks the maximum power transfer after multiple outages in this area. This power transfer is related to bulk power transfer from set  $a$  to set  $b$ , but there are a few outages that cause local power redistribution and can not be captured by area angle which is related to bulk power transfer. We identify these exceptional outages and take care of them separately using PMU or SCADA data from these lines as explained in detail in (30).

We select the double and multiple outages from the list of non-exceptional lines to see the tracking of the maximum power transfer by area angle. We find the area angle, the maximum power transfer and the generation shift factor of the congested line after random double and triple outages inside the area. Figure 7.3 and 7.4 show the results.

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<sup>1</sup>The angle at border bus 5 has a negative weight due to incident lines with negative susceptances arising from grid model reduction. In practice the measurements with very small weights could be omitted.



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Figure 7.2 Area of WECC system with area lines in black, north border buses in red and south border buses in blue. Layout detail is not geographic.

The results show that  $\theta_{\text{area}}^{(i)}$  is approximately inversely related to the maximum power transfer and suggest that area angle can be used to monitor the maximum power transfer change in both the double and multiple outage case. The results also show that area angle is related to the generation shift factor  $\rho_{uva}^{(i)}$  of the congesting line that causes the change in the maximum power transfer. As discussed in section 7.2.2, as the outages get more severe, the congesting line exceeds its limit and the maximum power transfer decreases. Also after the outage, the congesting line has an increased power flow and also the generation shift factor  $\rho_{uva}^{(i)}$  of the congesting line with respect to injection in the border buses increases. This increase in the generation shift factor due to severe outages is captured in the area angle.

Furthermore we can also determine threshold of the maximum power transfer as discussed

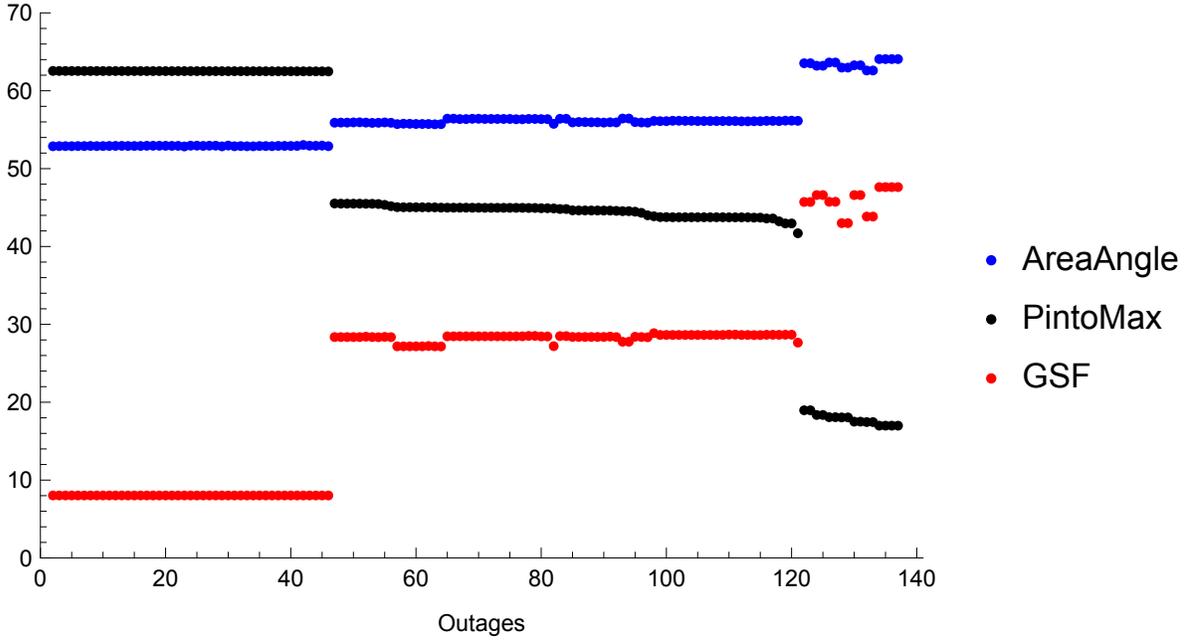


Figure 7.3 Area angle  $\theta_{\text{area}}^{(i)}$  in degrees, maximum power into the area in per unit, and generation shift factor  $\rho_{uva}^{(i)}$  of the congested line for a random sample of non-exceptional double outages in the area. Horizontal axis is outage number.

and monitor limits of maximum power transfer using these thresholds and real time value of area angle.

## 7.4 Conclusion

The maximum power transfer through a power system area is of economic and security importance. This chapter discussed monitoring of changes in the maximum power transfer after multiple outages inside the power system area. Results on a large power system area suggests that, barring some exceptional cases (30), area angle is related to the generation shift factor of the congested line after multiple outages which itself is related to the maximum power transfer. Furthermore the thresholds of the maximum power transfer can be converted to the thresholds of the area angle. Comparing the monitored area angle with the area angle thresholds can monitor change in power transfer and also violation of power transfer limits.

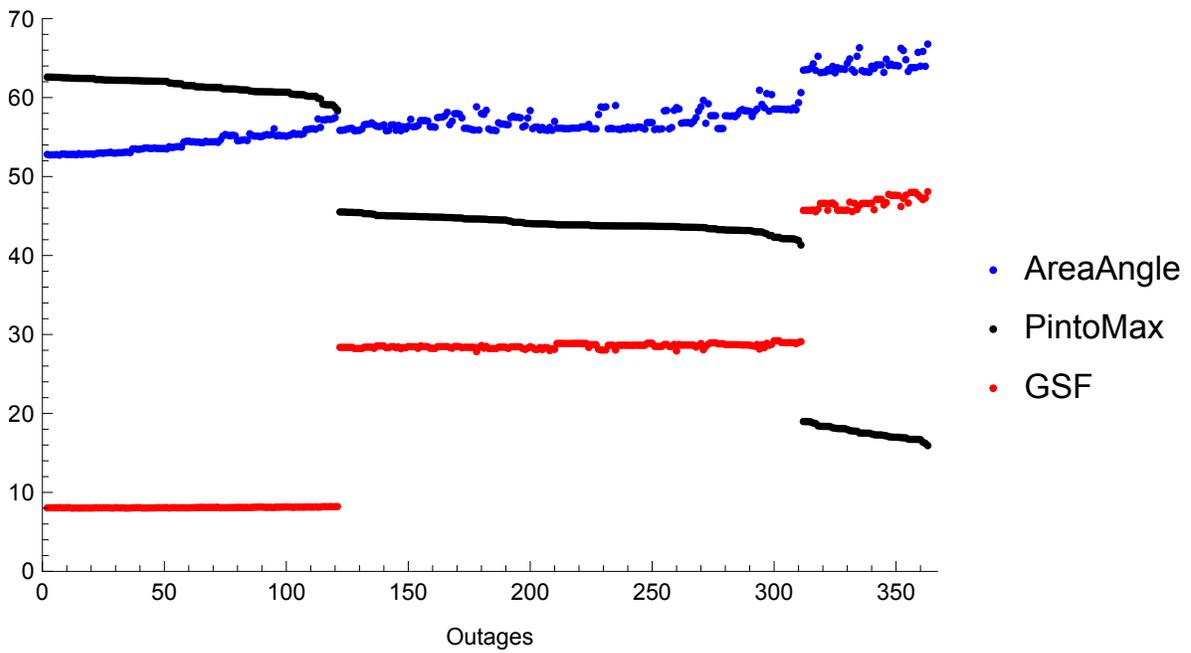


Figure 7.4 Area angle  $\theta_{\text{area}}^{(i)}$  in degrees, maximum power into the area in per unit, and generation shift factor  $\rho_{uva}^{(i)}$  of the congested line for a random sample of non-exceptional triple outages in the area. Horizontal axis is outage number.

## CHAPTER 8. Discussions And General Conclusion

### 8.1 Introduction

This chapter discusses some aspect of the work that can be further investigated in future research and then summarizes and concludes the thesis. Specifically this chapter shows how load or generation injection inside the area can affect the area angle; this could be further investigated in future work. It also shows the sensitivity of the result with respect to different pattern of load change. Finally this chapter gives a summary of all the chapters and concludes this work.

### 8.2 Discussions and Recommendations for Future Research

#### 8.2.1 Load or generation inside the area

We first discuss how area angle can be presented as the combination of the change of parameters inside and outside of the area. Based on (9), we decompose the area angle into an internal angle due to power injections inside the area and an external angle due to power flows from other areas. The internal and external area angles offer more specific stress information about the area. Consider a connected resistive network in which the buses  $M$  of the network are partitioned into two subsets  $M_a$  and  $M_b$ . Let  $c$  be the cutset of lines in the subnetwork that separates  $M_a$  and  $M_b$ . Then the power flowing from  $M_a$  to  $M_b$  decomposes as

$$P_{ab} = P_{\text{intoa}} + P_{\text{inRa}}. \quad (8.1)$$

$P_{\text{intoa}}$  is the power flowing into area R along the external tie lines attached to the border buses in  $M_a$ .  $P_{\text{inRa}} = \sigma_a P_m - \sigma_a B_{mn} B_{nn}^{-1} P_n$  is the total equivalent power injected into the buses  $M_a$

that corresponds to power injected in R. Moreover, since  $P_{ab} = -P_{ba}$ ,

$$\begin{aligned} P_{ab} &= P_{\text{intoa}} + P_{\text{inRa}} = -P_{\text{intob}} - P_{\text{inRb}} \\ &= \frac{1}{2}(P_{\text{intoa}} - P_{\text{intob}}) + \frac{1}{2}(P_{\text{inRa}} - P_{\text{inRb}}). \end{aligned} \quad (8.2)$$

Equation (8.2) decomposes  $P_{ab}$  into two parts. The first part is related to the difference of the external power flows injected at buses  $M_a$  and  $M_b$ . The second part is related to the difference of the power flows equivalent to the power injected in area R. Dividing (8.2) by the susceptance  $b_{ab}$  and using the Ohm's law decomposes the angle across the area as:

$$\theta_{\text{area}} = \theta_{\text{area}}^{\text{into}} + \theta_{\text{area}}^{\text{inR}}, \quad \text{where} \quad (8.3)$$

$$\theta_{\text{area}}^{\text{into}} = \frac{P_{\text{intoa}} - P_{\text{intob}}}{2b_{ab}} \quad \text{and} \quad \theta_{\text{area}}^{\text{inR}} = \frac{P_{\text{inRa}} - P_{\text{inRb}}}{2b_{ab}}. \quad (8.4)$$

The angle  $\theta_{\text{area}}^{\text{into}}$  is caused by differences in the powers entering into the area at  $M_a$  and  $M_b$  and measures the external stress on area R. The angle  $\theta_{\text{area}}^{\text{inR}}$  is caused by the differences at  $M_a$  and  $M_b$  of the powers equivalent to the powers generated or consumed inside area R and measures the internal stress on R. In particular,  $\theta_{\text{area}}^{\text{inR}}$  only depends on the generation and loads inside R and the lines in service inside R.

Now that we show area angle as the combination of both internal and external angle, we assume there is a generation loss inside the area. Then there can be several scenarios based on the position of the outaged generation and the location of other generation compensating after the generation outage to serve the load.

We take the simple example of Fig. 1 that includes one bus in the middle of lines. In this scenario generation outside the area near a border buses will compensate for the loss of generation and the extra power flows through the area to serve the load.

Then both  $P_{\text{intoa}}$  and  $P_{\text{inRa}}$  will change. We use (8.3) and (8.4) to show how the area angle changes after the loss of generation in bus c. Superscript 0 indicates no generation outage and superscript 1 indicates generation outage in bus c.

$$\theta_{\text{area}}^{(0)} = \theta_{\text{area}}^{\text{into}(0)} + \theta_{\text{area}}^{\text{inR}(0)}, \quad \text{where}$$

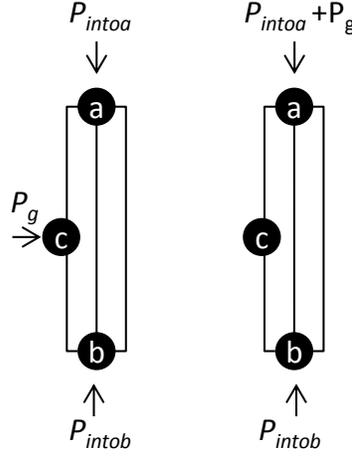


Figure 8.1 Simple example of a generation outage inside the area

$$\theta_{\text{area}}^{\text{into}(0)} = \frac{P_{\text{intoa}} - P_{\text{intob}}}{2b_{ab}} \quad \text{and} \quad \theta_{\text{area}}^{\text{inR}(0)} = \frac{P_{\text{inRa}} - P_{\text{inRb}}}{2b_{ab}}. \quad (8.5)$$

$$\theta_{\text{area}}^{(1)} = \theta_{\text{area}}^{\text{into}(1)} + \theta_{\text{area}}^{\text{inR}(1)}, \quad \text{where}$$

$$\theta_{\text{area}}^{\text{into}(1)} = \frac{P_{\text{intoa}} + P_g - P_{\text{intob}}}{2b_{ab}} \quad \text{and} \quad \theta_{\text{area}}^{\text{inR}(1)} = 0 \quad (8.6)$$

In (8.6) we can see that the external part of area angle  $\theta_{\text{area}}^{\text{into}}$  will increase, showing the extra stress caused by the increase in the power entering into bus a. The change in the internal part of the area angle is proportional to the change in the internal generation  $P_g$ .

The area angle will be affected by the changes in the internal and external angles, and it overall changes to reflect the change in the power transfer through the area. It seems likely that dividing the area angle into internal and external components as explained above will be very useful in pursuing this direction.

### 8.2.2 Pattern of load change

This work assumes a proportional increase pattern since this is one of the reasonable assumptions, but any fixed pattern of increase may be assumed for the calculations. For example,

some areas may have some specific pattern of stress based on historical data that can be assumed. This work used the maximum power transfer to measure the severity of the outages with respect to bulk power transfer and therefore considers a proportional increase pattern that is related to bulk power transfer.

It is also of interest to find out how sensitive the results are to changing the assumptions about the pattern of loading increase. We significantly changed the pattern of stress to show how it affects the area angle, maximum power transfer and emergency threshold of angle for the WECC area. In particular the weights are changed to be zero except for two border buses one in north and one in south and consider the stress in lines with respect to injections in only these two buses. Fig. 8.2 shows the change in the maximum power that could enter the area after random triple outages and the thresholds. As we see in Fig. 8.2, the change in the pattern does not affect the emergency threshold.

### 8.2.3 Discussions on DC load flow analysis

The DC load flow can become more inaccurate in extreme conditions, but we are thinking that the core issue is the accuracy for the outages near the threshold for emergency action, which are comparable to the worst N-1 contingency. That is, multiple outages significantly more extreme than this threshold can be correctly classified as emergencies even if there is inaccuracy in their precise degree of severity, and the advantage of DC calculation for this extreme situation is that it is faster and a quick decision is important in extreme cases. We know that DC load flow will be sufficiently accurate for single contingencies in the present context of fast approximate methods to discriminate whether emergency actions are required for reasons of line overloads. Now it is certainly true that outages (even single outages) can degrade performance in other ways such as lower voltage magnitudes or proximity to collapse or to oscillations, but our view is that these problems should be addressed by AC monitoring methods tailored to these problems such as in (23). The advantage of focussing more narrowly on problems in this way is that the actions to be taken can more clearly be linked to the monitoring. That is, we require our monitoring method to lead to credible actions when thresholds are crossed, and this requires

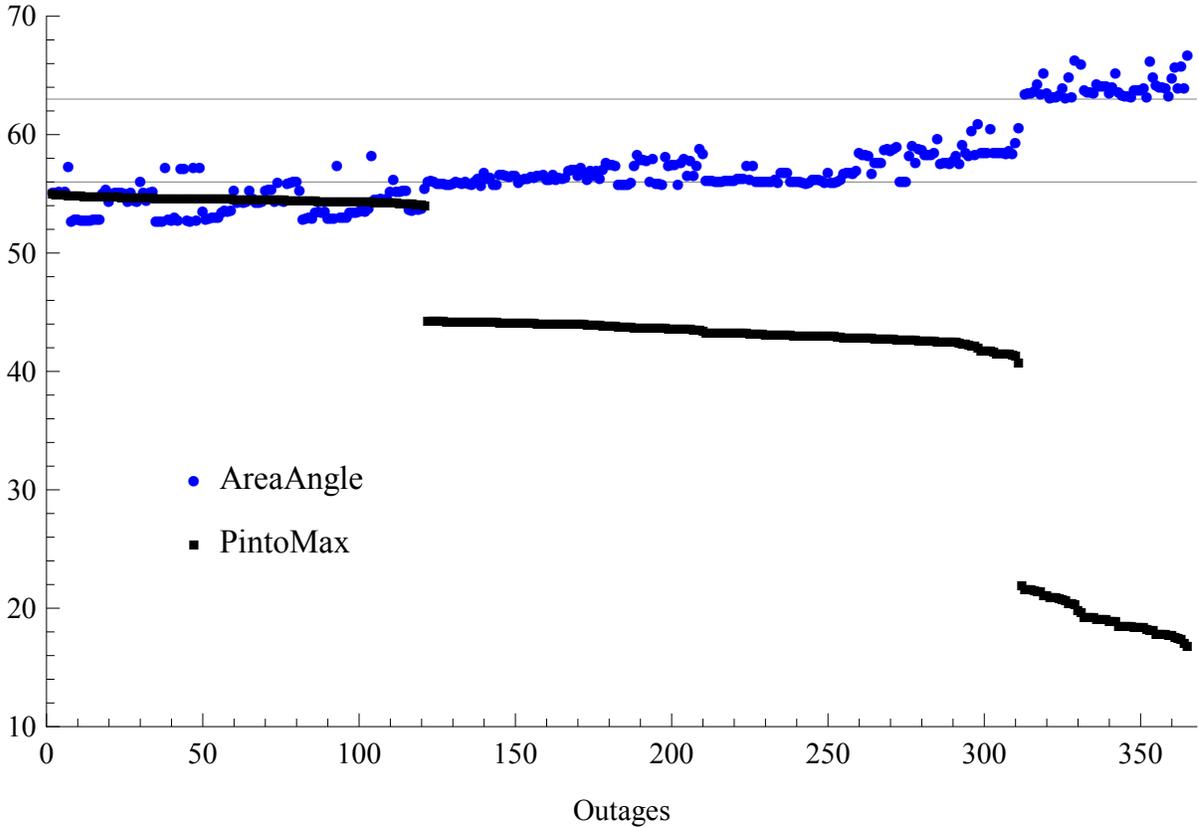


Figure 8.2 Area angle  $\theta_{\text{area}}^{(i)}$  in degrees, and maximum power into the area in per unit for a random sample of non-exceptional triple outages in the area and injection in one generation in north and one load in south.

limiting the scope of the method to a single phenomenon, in this case static overloads, and to exclude other phenomenon such as voltage problems that are detected and mitigated in other ways.

### 8.3 Summary and Conclusion

This thesis developed, analyzed and tested the real-time monitoring of single and multiple outages using area angle. We used the real time value of PMUs to monitor real time multiple outages and furthermore we set thresholds based on angles to detect emergency situations for quick real time actions.

To perform this analysis, first chapter 1 introduced an overview of the materials and ap-

proaches used in the thesis for monitoring of multiple outages. This chapter also gave a summary of the materials in each chapter.

Chapter 2 provided the literature review of the studies and papers related to monitoring with PMU data. We furthermore discussed in this chapter the remaining challenges such as fast monitoring of emergency situations after single and multiple outages.

Chapter 3 discussed the monitoring of single outages inside areas of a Japanese power system and explored different aspect of choosing the border buses and load and generation inside the area.

Chapter 4 explored the relationship between area angle and area susceptance after single outages in different areas of the WECC system and showed that area angle can monitor and track change in the susceptance of the area after outages. This chapter also explored further the approximate relation between area angle and area susceptance.

Chapter 5 expanded the monitoring of single outages to multiple outages and supported the findings with results from the WECC system.

Chapter 6 discussed thresholds of area angle and monitoring emergency situations using these thresholds. In this chapter the detailed procedure applied to find the weights and thresholds off line and then finding safe, alarm and emergency situations in real time using these thresholds have been discussed.

Chapter 7 investigated the interesting relationship between the area angle and the maximum power transfer after multiple outages using the generation shift factor of the congested and outaged line.

In this chapter, chapter 8, we discussed some future work directions and gave some discussions about the assumptions made in this work. We then summarized the chapters and concluded the dissertation.

As described in detail in the chapters, this work proposed a new formulation to find stress with respect to power transfer inside areas of power system. We used the area angle in real time that is the weighted combination of angles around the border buses to find out stress with respect to power transfer. This formulation helped later to convert the thresholds of power transfer to the thresholds of are angle offline. Then monitoring area angle in real time

and comparing the area angle with its thresholds discriminates different stress level and corresponding actions after multiple outages. The methodology proposed in this work to find out the stress with respect to power transfer is quick so that it is practical for mitigation actions to be applied in emergency situations.

#### 8.4 List of Publications

1. A. Darvishi, I. Dobson, Threshold-based monitoring of multiple outages with PMU measurements of area angle, accepted in IEEE Transactions on Power Systems and in final preprint step. (preprint can be find online at [arXiv:1411.6130 \[cs.SY\]](https://arxiv.org/abs/1411.6130)).
2. A. Darvishi, I. Dobson, Synchrophasor monitoring of single line outages via area angle and susceptance, NAPS North American Power Symposium, Pullman WA USA, September 2014.
3. A. Darvishi, I. Dobson, Area angle can monitor cascading outages with synchrophasors, IEEE ISGT Innovative Smart Grid Technologies, Washington DC USA, February 2015.
4. A. Darvishi, I. Dobson, A. Oi, C. Nakazawa, Area angles monitor area stress by responding to line outages, NAPS North American Power Symposium, Manhattan KS USA, September 2013.

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