On Voltage Collapse in Electric Power Systems

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Abstract

Several voltage collapses have had a period of slowly decreasing voltage followed by an accelerating collapse in voltage. In this paper we analyze this type of voltage collapse based on a center manifold voltage collapse model. The essence of this model is that the system dynamics after bifurcation are captured by the center manifold trajectory and it is a computable model that allows prediction of voltage collapse. Both physical explanations and computational considerations of this model are presented. We clarify the use of static and dynamic models to explain voltage collapse. Voltage collapse dynamics are demonstrated on a simple power system model.

1. Introduction

The continuing interconnections of bulk power systems, brought about by economic and environmental pressures, has led to an increasingly complex system that must operate ever closer to the limits of stability. This operating environment has contributed to the growing importance of the problems associated with the dynamic stability assessment of power systems. To a large extent, this is also due to the fact that most of the major power system breakdowns are caused by problems relating to the system dynamic responses. It is believed that new types of instability emerge as the system approaches the limits of stability.

One type of system instability which occurs when the system is heavily loaded is voltage collapse. This event is characterized by a slow variation in the system operating point, due to increase in loads, in such a way that voltage magnitudes gradually decrease until a sharp, accelerated change occurs.

It is interesting to note that prior to the sharp change in voltage magnitudes, bus angle and frequency remain fairly constant, a condition observed in several collapses. During a collapse, voltage control devices, such as tap-changing transformers, may not be activated if the voltage magnitudes prior to undergoing the sharp change lie in a "permissible range" and, after the change occurs, the fast rate of the change trips under-voltage relays before the transformers can respond to it. Furthermore, control center operators observe none of the classical advance warning signals since the bus angle, frequency and voltage magnitudes may remain normal until large changes in system state cause protective equipment to begin to dismantle the network.

In the past, there has been significant debate over whether the voltage collapse problem is static in nature and can therefore be studied as a parametric load flow problem or whether it is dynamic and must be studied as the trajectory of a set of differential equations. A majority of the work on the problem to date has been focused on the static problem such as load flow feasibility [1], optimal power flow [2], steady-state stability [3]. Kwatny et. al. [4] studied the static problem as a static bifurcation characterized by the disappearance of an equilibrium point and showed how bifurcation could describe instability both in voltage and angle. In [6-9] Thomas et al. proposed the minimum singular value of the Jacobian of the descriptor load flow equations as a security index and derived static control strategies based on the index. In [10] Glavistch et al. developed a voltage stability index based on the feasibility of solutions to the power-flow equations for each node. A comparison of several proposed methods was given in [11,12]. Schleuter et al. [13,14] proposed definitions of voltage stability and voltage controllability that are "based on the natural cause/effect relationships that exist at PQ buses in the power system under normal conditions." Control criteria are derived based on a linearized set of equations and the definitions.

Research is only now beginning to emerge on the dynamics associated with voltage collapse. It is clear that the collapse dynamics cannot be described solely by the generator dynamics which are traditionally believed to be responsible for transient instabilities. In [15], voltage instability is associated with tap-changing transformer dynamics by defining the voltage stability region in terms of allowable transformer settings. In [16], the voltage collapse was related to the stability of discrete models of multiple tap-changers in a power network. Transformer tap-changers were identified in [17] as a device which aggravated rapid voltage decay. In [18] the effect of incorporating induction motor characteristics on the voltage stability region is examined and a region of attraction is explored. In [19] the voltage collapse was related to small noise in load demand.

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In this paper we will analyze the dynamics of voltage collapse based on a center manifold voltage collapse model. The essence of this model is that the system dynamics after bifurcation are captured by the center manifold trajectory and it is a computable model that allows prediction of voltage collapse. Both physical explanations and computational considerations of this model will be presented. The use of static and dynamic models to explain this type of voltage collapse will be clarified. Voltage collapse dynamics will be demonstrated on a simple power system model.

2. System Model

The Generator Model

The generic generator model can be expressed as

$$\dot{y}(t) = g(y(t),z(t)) \qquad (2\text{-}1)$$

where y(t) is a vector of generator state variables such as δ_G, ω_G, E_q', etc. z(t) is a vector of system state variables such as voltage magnitude and angle at load buses. It is not clear at this moment which generator state variables should be modelled in (2-1) for the analysis of voltage collapse. Nevertheless, the center manifold voltage collapse model applies to a system with any generator model of the form (2-1), irrespective of the dimension of state space in (2-1).

The Load Model

Load characteristics are known to have a significant effect on system dynamics. It is our viewpoint that the load model is the single greatest impediment to high quality stability assessment. Considerable effort has been expended in an attempt to derive improved load models. Classical load models such as constant P-Q, constant impedance, and constant current models are not appropriate when attempting to capture severe collapse dynamics. A load model that is an affine function of frequency for real power demand and a polynomial function of voltage magnitude for reactive power demand has been adopted by several researchers [19,20]. Weedy and Cox confirmed that from the voltage stability point of view, the induction motor is a critical constituent of system loads [21]. Thomas and Tiranuchit [18,22] were the first to incorporate induction motor characteristics into their dynamic load model for the analysis of voltage instability. Dobson et. al. [23] suggested the following load model for analyzing voltage collapse. The load model comprises a dynamic induction motor model (representing an industrial load) in parallel with a constant P-Q load and a constant impedance load (representing residential plus commercial loads). The load model at bus i considered in this paper is based on load voltage dynamics due to Walve [25].

$$P_{d,i} = P_{0,i} + P_{1,i} + K_{p\omega,i}\dot{\delta}_i + K_{pv,i}(V_i + T_i\dot{V}_i) \qquad (2\text{-}2a)$$

$$Q_{d,i} = Q_{0,i} + Q_{1,i} + K_{q\omega,i}\dot{\delta}_i + K_{qv,i}V_i + K_{qv2,i}V_i^2 \qquad (2\text{-}2b)$$

where $P_{0,i}$, $Q_{0,i}$ represent the constant real and reactive powers of the motor and $P_{1,i}$, $Q_{1,i}$ are the P-Q load. We have added the term $K_{qv2,i}V^2$ to Walve's linearized model in order to better represent the nonlinear static Q-V relationship. Also, we have embedded the constant impedance part of the load in the system admittance matrix.

The real and reactive power balance equations at load bus i are expressed by

$$-P_{d,i} = \sum_{j\in J_G} V_iV_jB_{ij}\sin(\delta_i - \delta_j) + \sum_{j\in J_L} V_iV_jB_{ij}\sin(\delta_i - \delta_j) \qquad (2\text{-}3a)$$

$$-Q_{d,i} = -\sum_{j\in J_G} V_iV_jB_{ij}\cos(\delta_i - \delta_j) - \sum_{j\in J_L} V_iV_jB_{ij}\cos(\delta_i - \delta_j) \qquad (2\text{-}3b)$$

The significance of load model (2-2) is that it is dynamic and that the resulting power system model (2-1) - (2-2) is (after simple algebraic operations) purely a vector differential equation with well-defined unique solutions. This circumvents a long-standing difficulty associated with the structure-preserving models which are a mixture of differential and algebraic equations and whose solution trajectories may not be well defined.

3. Bifurcations

Consider the power system model described by equations (2-1) - (2-2) in the general form

$$\dot{x} = F(x,\lambda) \qquad (3\text{-}1)$$

where x is the state vector and λ is a time-varying parameter vector. Specifically, in the power system model described in section 2, x = (δ, ω, V) and λ denotes the parameter vector that includes real and reactive power demands at each load bus. The parameters in (3-1) are subject to variation and, as a result, changes may occur in the qualitative structure of the solutions of the static equation associated with (3-1), i.e., solutions of $F(x, \lambda) = 0$ for certain values of λ. For example, a change in the number of solutions for x may occur as the parameters vary. As a result, the dynamic behavior of (3-1) may be altered.

Bifurcation theory is concerned with branchings of the static solutions of (3-1) and, in particular, it is interested in how solutions $x(\lambda)$ branch as λ varies. These changes, when they occur, are called *bifurcations* and the parameter values at which a bifurcation happens are called *bifurcation values.*

It is important in our following analysis of voltage collapse to distinguish two different periods: the period before bifurcation and the period after bifurcation. Power systems are normally operated near a stable equilibrium point. As system parameters change slowly, the stable equilibrium point changes position but remains a stable equilibrium point. This situation may be modelled with the static model $F(x,\lambda)=0$ by regarding $F(x,\lambda)=0$ as specifying the position of the stable equilibrium point x as a function of λ. (Here it would be more precise to call $F(x,\lambda)=0$ a quasistatic model since λ varies and causes corresponding variations in x). This model may also be called parametric load flow model. Exceptionally, variation in λ will cause the stable equilibrium point to bifurcate. The stable equilibrium point of (3-1) may then disappear or become unstable depending on the way in which the parameter is varied and the specific structure of the system.

After the bifurcation, the system state will evolve according to the dynamics of (3-1). (Some types of bifurcation result in the persistence of the stable equilibrium point even after the bifurcation and the static model applies just as before the bifurcation. However, we do not expect this sort of bifurcation to be typical in power systems.) To summarize, analysis of a typical bifurcation of a stable equilibrium point in a power system with slowly moving parameters has two parts:

[1] Before the bifurcation when the (quasi)static model applies.

[2] After the bifurcation when the dynamical model (3-1) applies.

The current research on voltage collapse uses the static model and only considers the system before the bifurcation. We stress that the static model is not applicable after the bifurcation.

In [24], Dobson and Chiang investigated a generic mechanism leading to disappearance of stable equilibrium points and the consequent system dynamics for one-parameter dynamical systems. A voltage collapse model was suggested based on this analysis and the results are briefly summarized in the next section.

4. A Voltage Collapse Model

Suppose that the power system model described by equations (3-1) has the specific form

$$\dot{x} = F(x, Q) \qquad (4\text{-}1)$$

where Q is a parameter such as a reactive power demand. We assume that Q varies slowly or quasi-statically with respect to the dynamics of (4-1). For example, if the system represented by equation (4-1) is initially near a stable equilibrium point $x_s(Q)$, then the dynamics will make x track $x_s(Q)$ as Q slowly varies. One typical way in which system (4-1) may lose stability is that the stable equilibrium point $x_s(Q)$ and another equilibrium point $x_1(Q)$ coalesce and disappear in a saddle-node bifurcation as parameter Q varies.

In [24], it is shown that for generic one-parameter dynamical systems the equilibrium point $x_1(Q)$ is type-one. By type-one, we mean that the corresponding Jacobian matrix has exactly one eigenvalue with a positive real part and the rest of the eigenvalues have negative real parts. Furthermore, $x_1(Q)$ lies on the stability boundary of $x_s(Q)$. The Jacobian matrix, when evaluated at $x_s(Q)$, has all of its eigenvalues with only negative real parts. However, one of eigenvalues is close to zero. At the bifurcation occurring at say, $Q = Q^*$, equilibrium points $x_s(Q)$ and $x_1(Q)$ coalesce to form an equilibrium point x^*. The Jacobian matrix evaluated at x^* has one zero eigenvalue and the real parts of other eigenvalues are negative. The eigenvector p that corresponds to the zero eigenvalue points in the direction along which the two vectors $x_s(Q)$ and $x_1(Q)$ approached each other. There is a curve made up of system trajectories which is tangent to eigenvector p at x^*. This curve is called the *center manifold* of x^* and is the union of a system trajectory W_-^c converging to x^*, the equilibrium point x^* and a system trajectory W_+^c diverging from x^*. We choose the sign of p so that it points along W_+^c. If Q increases beyond the bifurcation value Q^*, then x^* disappears and there are no other equilibrium points nearby.

Next, we consider the system dynamics described by (4-1) when Q remains fixed at bifurcation value Q^*. x^* is an unstable equilibrium point and a trajectory starting near x^* diverges from x^* approximately in the direction of p. Trajectories starting from points lying on W_+^c will move away from x^* and remain on W_+^c. Moreover, trajectories starting from points near W_+^c will move away from x^* approximately along W_+^c and in fact, the trajectories will approach W_+^c exponentially fast. The initial movement along W_+^c is slow. Recall that before the bifurcation occurs, the system state is tracking its stable equilibrium point. Therefore, at the moment the bifurcation occurs, the system state is in a neighborhood of x^*. Hence, if the system trajectory is near W_+^c at the moment that the bifurcation occurs and if Q remains fixed at its bifurcation value Q^*, then the system dynamics move near W_+^c. The system dynamics due to the bifurcation are then determined by the position of W_+^c in state space. If W_+^c is positioned so that some of the voltage magnitudes decrease along W_+^c, then we associate the movement along W_+^c with voltage collapse. This is the center manifold voltage collapse model. This model has two advantages from a computational point of view.

(1) Since p is tangent to W_+^c at x^*, the initial direction of W_+^c near x^* is determined by p which can be computed from the Jacobian matrix at x^*.

(2) Since W_+^c is a system trajectory, the dynamics of voltage collapse can be predicted by integrating system equations (4-1) starting on W_+^c near x^*.

The center manifold voltage collapse model can be summarized as follows: Suppose the general power system model represented by equation (4-1) has a saddle-node bifurcation at (x^*,Q^*). The dynamics at the bifurcation are described by motion along the system trajectory W_+^c starting near x^*. If some of the bus voltage magnitudes decrease along W_+^c, then the movement along W_+^c is a model for voltage collapse.

It should be stressed that not all saddle-node bifurcations of power system equations (4-1) are of the voltage collapse variety. For this reason, a thorough analysis of the system dynamics after bifurcation is essential. Furthermore, we argue that if the occurrence of a bifurcation implies that a dynamic voltage collapse occurs, then there must be a model that describes the dynamics after the bifurcation. The question is "What is the appropriate model for these dynamics?". This question is unanswered in the currently available literature. The model proposed in this section fills this gap and, perhaps more importantly, it is a computable model that allows prediction of voltage collapse.

We remark that the saddle-node bifurcation and consequent dynamics described above are typical in the sense that if a stable equilibrium point does lose stability and disappears, then it will do so in the manner described above (see the discussion of genericity in [24]). Moreover, this behavior is typical regardless of the dimension of the state space. Thus the dynamics after a saddle-node bifurcation of large scale power system models of the form (4-1) can still be modelled by movement along the one dimensional center manifold trajectory, despite the large dimension of the state space.

In this paper we investigate how the interaction between loads and generators may cause voltage collapse using the center manifold model of the dynamics after bifurcation. However, we emphasize that the center manifold model applies to any power system model of the form (4-1) after a saddle-node bifurcation. In particular, it might be applied to show how other components of power systems such as tap-changing transformers contribute to voltage collapse.

5. Computational Considerations and Physical Explanations

In this section we discuss how to compute and physically interpret the voltage collapse model proposed in last section. Suppose that the power system described by equation (4-1) is operating at the stable equilibrium point $x_s(Q_i)$, where Q_i is the reactive power demand at load bus i. Now, assume that Q_i is slowly increased while other parameters remain fixed. The equilibrium point $x_s(Q_i)$ will vary as Q_i is increased. We expect a bifurcation value Q_i^* such that

(i) $x_s(Q_i)$ is stable if $Q_i < Q_i^*$,

(ii) $x_s(Q_i)$ becomes unstable by coalescing with another type-one equilibrium point in a saddle-node bifurcation if $Q_i = Q_i^*$, and

(iii) $x_s(Q_i)$ will disappear if $Q_i > Q_i^*$, i.e. there does not exist any equilibrium point in a neighborhood of $x_s(Q_i^*)$ for $Q_i > Q_i^*$.

The bifurcation value Q_i^* can be calculated by solving the n+1 nonlinear equations

$$0 = F(x, Q_i) \qquad (5\text{-}1a)$$

$$0 = \det\left(\frac{\partial f}{\partial x}\Big|_{(x,Q_i^*)}\right) \qquad (5\text{-}1b)$$

for the bifurcation point $(x_s(Q_i^*), Q_i^*)$. These equations represent necessary conditions for a saddle-node bifurcation. Indeed, at the bifurcation, x_s is an equilibrium point (equation 5-1a) and the system Jacobian has a zero eigenvalue when evaluated at x_s (equation 5-1b). It should be noted that approaches based on repetitive load flow calculations for finding the bifurcation point often provide an interpolation point rather than an exact solution. This is because the singularity of the Jacobian matrix at the bifurcation point causes convergence problems in gradient-based algorithms such as Newton-Raphson.

The step following the calculation of the bifurcation value and the bifurcation equilibrium point is to determine the center manifold. This can be accomplished by the integration of system equation (4-1) from a point lying on the eigenvector p associated with the zero eigenvalue and sufficiently close to the bifurcation equilibrium point. However, we note that the ratio between the largest and the smallest eigenvalue of the system Jacobian when evaluated near the bifurcation equilibrium point is large. Hence the differential equations near the bifurcation equilibrium point are stiff and integration schemes designed for stiff differential equations are required.

Now we discuss the physical implications for the power system before and after the bifurcation. Before the bifurcation, the system remains at a stable equilibrium point as the reactive power demand Q_i varies. We interpret this as the system having the capability to supply the reactive power demanded by the load and identify Q_i^* as the maximum possible reactive power which the transmission system can transmit. When a system is capable of supplying a certain reactive power, we assume not only that the reactive power is supplied at some instant, but also that it is supplied in a robust way, that is, when the system state is at a stable equilibrium point. We identify the difference between Q_i^* and Q_i as the reactive power margin or reserve available. This agrees with the physical explanation in [5] that attributes voltage collapse to load reactive power supply problems.

At the bifurcation the system state moves along the center manifold trajectory and the system behavior depends on where the trajectory goes in state space. Two cases may occur. In the first case, the system trajectory diverges and tends to infinity. This implies that the system will not settle into steady-state at some finite value in the state space. We note that the model breaks down and no longer applies when the trajectory leaves some bounded region. For example, in a voltage collapse, if the voltage falls sufficiently, protection devices will change the power system structure so that the assumed power system model no longer applies. In the second case, the system trajectory tends to another stable equilibrium point. If the system model remained applicable as the system state moved to the new stable equilibrium point, then the center manifold model predicts that the system trajectory will converge to a different, stable configuration after the bifurcation.

6. A Numerical Example

In order to physically illustrate the center manifold voltage collapse model, we consider the power system model shown in Figure 1, which is taken from [23]. This system consists of a load bus and two generator buses. One of the generator busses is treated as a slack bus. The load is modeled by a simplified induction motor in parallel with a constant P-Q load and constant impedance as described previously in equation (2-2). For easy reference, the equations are given by

$$P_d = P_0 + P_1 + K_{p\omega}\dot{\delta} + K_{pv}(V + T\dot{V})$$

$$Q_d = Q_0 + Q_1 + K_{q\omega}\dot{\delta} + K_{qv}V + K_{qv2}V^2$$

where P_0, Q_0 are the constant real and reactive powers of the motor and P_1, Q_1 are the P-Q load.

The dynamics of the non-slack-bus generator is described by the swing equation

$$M\ddot{\delta}_m + D\dot{\delta}_m = P_m + V_mVY_m\sin(\delta - \delta_m - \theta_m) + V_m^2Y_m\sin\theta_m$$

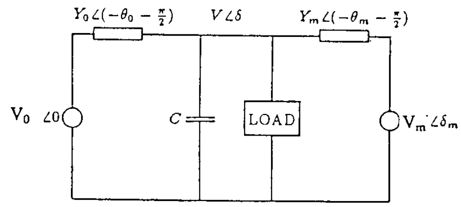

Fig. 1. A simple power system.

where M, D and P_m are the generator moment inertia, damping coefficient and mechanical power, respectively. The load bus includes a capacitor as part of its constant impedance representation in order to maintain the voltage magnitude at a nominal and reasonable value. It is convenient to derive the Thevenin equivalent circuit with the capacitor. The adjusted values are [23]

$$V_0' = \frac{V_0}{(1 + C^2Y_0^{-2} - 2CY_0^{-1}\cos\theta_0)^{1/2}}$$

$$Y_0' = Y_0(1 + C^2Y_0^{-2} - 2CY_0^{-1}\cos\theta_0)^{1/2}$$

$$\theta_0' = \theta_0 + \tan^{-1}\{\frac{CY_0^{-1}\sin\theta_0}{1 - CY_0^{-1}\cos\theta_0}\}$$

The real and reactive powers supplied by the network to the load are given by

$$P = -V_0'VY_0'\sin(\delta + \theta_0') - V_mVY_m\sin(\delta - \delta_m + \theta_m) + (Y_0'\sin\theta_0' + Y_m\sin\theta_m)V^2 \quad (6\text{-}1a)$$

$$Q = V_0'VY_0'\cos(\delta + \theta_0') + V_mVY_m\cos(\delta - \delta_m + \theta_m) - (Y_0'\cos\theta_0' + Y_m\cos\theta_m)V^2 \quad (6\text{-}1b)$$

Thus, the resulting system equations are

$$\dot{\delta}_m = \omega_m \quad (6\text{-}2a)$$

$$M\dot{\omega}_m = -D\omega_m + P_m + V_mVY_m\sin(\delta - \delta_m - \theta_m) + V_m^2Y_m\sin\theta_m \quad (6\text{-}2b)$$

$$K_{q\omega}\dot{\delta} = -K_{qv}V - K_{qv2}V^2 + Q - Q_0 - Q_1 \quad (6\text{-}2c)$$

$$TK_{q\omega}K_{pv}\dot{V} = K_{p\omega}K_{qv2}V^2 + (K_{p\omega}K_{qv} - K_{q\omega}K_{pv})V + K_{p\omega}(Q_0 + Q_1 - Q) - K_{q\omega}(P_0 + P_1 - P) \quad (6\text{-}2d)$$

where P and Q are from equations (6-1a) and (6-1b). The load parameter values used in the simulation are: $K_{p\omega} = 0.4$, $K_{pv} = 0.3$, $K_{q\omega} = -0.03$, $K_{qv} = -2.8$, $K_{qv2} = 2.1$, $T = 8.5$, $P_0 = 0.6$, $Q_0 = 1.3$, $P_1 = Q_1 = 0.0$ and the network and generator parameter values were $Y_0 = 20.0$, $\theta_0 = -5.0$, $V_0 = 1.0$, $C = 12.0$, $Y_0' = 8.0$, $\theta_0' = -12.0$, $V_0' = 2.5$, $Y_m = 5.0$, $\theta_m = -5.0$, $V_m = 1.0$, $P_m = 1.0$, $M = 0.3$, $D = 0.05$. All parameter values are in per unit except for angles, which are in degrees.

The reactive power demand Q_1 is chosen as the system parameter in (4-1). In order to compute bifurcation value Q_1 and the associated bifurcation equilibrium point, the following approximate formulas [23] are useful. The approximate bifurcation value is

$$Q_1^* = \frac{(-K_{qv} + V_0'Y_0' + V_mY_m)^2}{4(K_{qv2} + Y_0' + Y_m)} - Q_0 \quad (6\text{-}3)$$

and the approximate voltage magnitude at the bifurcation equilibrium point is

$$V^* = \frac{(-K_{qv} + V_0'Y_0' + V_mY_m)}{2(K_{qv2} + Y_0' + Y_m)} \qquad (6\text{-}4)$$

Formulas (6-3) and (6-4) are derived from the approximate static model [23]

$$Q_0 + Q_1 - (-K_{qv} + V_0'Y_0' + V_mY_m)V + (K_{qv2} + Y_0' + Y_m)V^2 = 0 \qquad (6\text{-}5)$$

These approximations show the relationship between the bifurcation point and certain load, transmission network and generator parameters. The two values generated by these approximations were used as an initial values for finding the saddle-node bifurcation point of system (6-2) with Q_1 being the parameter. The bifurcation equilibrium point is $x^* = (\delta_m^*, \omega^*, \delta^*, V^*) = (0.348, 0.0, 0.138, 0.925)$ and the bifurcation value is $Q_1^* = 11.41$. The eigenvector associated with the zero eigenvalue at the bifurcation equilibrium point is $p = (0.23, 0.0, 0.099, -0.97)$. The relatively large negative component of the eigenvector associated with voltage indicates that at the bifurcation point, the initial movement of the system dynamics will be in a direction such that voltage magnitude decreases while the other state variables remain fairly constant.

In this example, the system dynamics at the bifurcation are described by the center manifold W_+^c which can be obtained by numerical integration methods designed for stiff equations. Figure 2 show the system dynamics, starting from the point $x_0 = x^* + 0.01$ p, in the voltage and angle space respectively. Figure 2 demonstrates the dynamics of a voltage collapse phenomenon after a saddle-node bifurcation. Note that during the entire process the bus angles remain fairly constant.

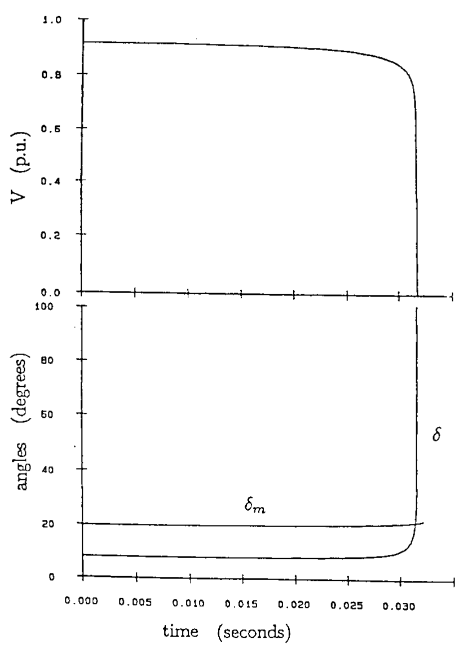

Fig. 2. Voltage magnitude and angle at load bus when bifurcation occurs.

Case 1: slowly varying reactive power demand

The previous example demonstrated the center manifold model for the dynamics of voltage collapse after a saddle node bifurcation. Now we simulate the behavior of the example both before and after the bifurcation to illustrate the entire process (we use the same set of parameters as that in the above except $Y_m = 1.0$). We assume that the reactive power demand Q_1 varies slowly, linearly increasing starting from below the bifurcation value Q_1^*. In particular, $Q_1 = 10.912 + 0.02t$. The initial system state is found by solving (6-2a) with $Q_1 = 10.917$; i.e., performing a load flow. The differential equations were then numerically integrated using a stiff differential equation solver. The result is shown in Figure 3. As expected, a voltage collapse occurs at the time when Q_1 passes Q_1^*. Before the bifurcation, the system state tracks the stable equilibrium point as it varies slowly with Q_1 and the static model is a good approximation to the system behavior. The voltage decrease before the bifurcation is slow because the variation of Q_1 is slow. The system can supply sufficient reactive power to the load while $Q_1 < Q_1^*$. After the bifurcation, the behavior is similar to that predicted by the dynamical center manifold model presented above. (The difference between the two is that in this example, Q_1 continues to slowly increase after the bifurcation, while the center manifold model assumes that Q_1 is fixed at Q_1^*.)

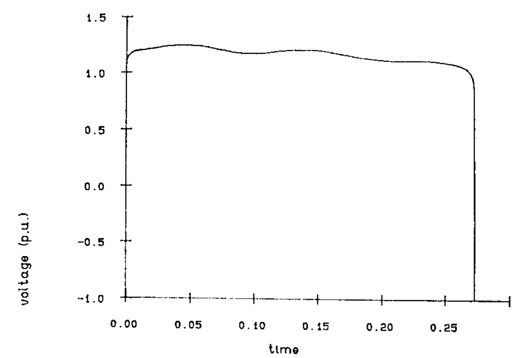

Fig. 3. Voltage magnitude at load bus when the reactive power demand is slowly varied.

Case 2: system parameters versus bifurcation values

During normal operation, it is important to understand the effects of varying different system parameters on the system's capacity to supply reactive power. In other words, we need to determine how varying system parameters affects the position of bifurcation point. During normal operation, there is a stable equilibrium point and the static model (6-2) with left hand side zero is applicable. We argue that enhancement of the transmission capability (say, by increasing the transmission parameter Y_m or Y_0) would increase the capacity of the transmission network to supply reactive power. This is illustrated in Figure 4 showing the relationship between the transmission line parameter and the bifurcation value. The figure indicates that a larger transmission capacity ensures a larger bifurcation value.

One efficient way to increase the capacity of the transmission network to supply reactive power to a load bus is to install (or increase) capacitors at that bus. This is common utility system practice for transmission systems as well as distribution systems. The relationship between the bifurcation value of system (6-2) and the amount of capacity installed at the load bus is shown in Figure 5 and supports this viewpoint. This also indicates that static var compensation at the receiving end of a long distance transmission line can improve system conditions relative to a possible voltage collapse.

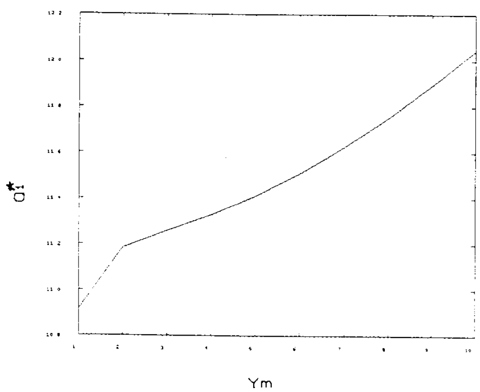

Fig. 4. Relationship between the transmission line parameter and the bifurcation value.

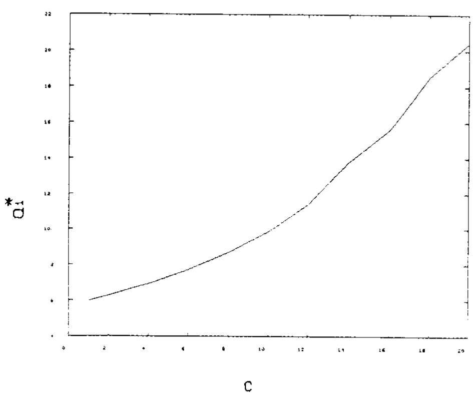

Fig. 5. Relationship between the installed capacity at the load bus and the bifurcation value.

A reasonable way to protect the system from voltage collapse is to move the bifurcation equilibrium point outside the permissible region of operating points. For example, this could be done by keeping the voltage magnitude in the bifurcation equilibrium point below 0.95 p.u. The relationship between the voltage magnitude at the bifurcation equilibrium point of system (6-2) and the amount of capacity installed at the load bus as well as the transmission line parameter are shown in Figures 6 & 7 respectively.

We have compared the results in Fig. 4-7 for the (exact) static model ((6-2) with left hand side zero) to those from (6-3) and (6-4), which were derived from the approximate static model (6-5). The exact and approximate static models are in close agreement, with maximum relative errors of 4%, 2%, 4%, 1% for Fig. 4-7 respectively.

Although reactive power demand seems to be a relevant parameter for voltage collapse [23], we note that the description of dynamics after saddle-node bifurcation of a stable equilibrium point may apply to other types of instabilities in power systems and other parameters such as real power demand may prove to be relevant for more general instabilities. The central point is to determine which parameters play a key role in the process of interest. This requires comprehensive knowledge of the characteristic properties of each power system and engineering judgement.

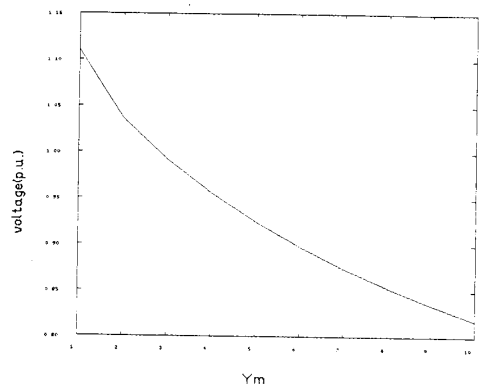

Fig. 6. Relationship between the transmission line parameter and the voltage magnitude at the bifurcation point.

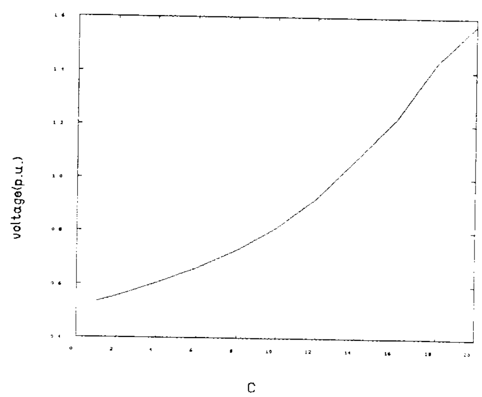

Fig. 7. Relationship between the installed capacity at the load bus and the voltage magnitude at the bifurcation point.

7. Conclusions

Several voltage collapses have had a period of slowly decreasing voltage followed by an accelerating collapse in voltage. In this paper we clarify the use of static and dynamic models to explain this type of voltage collapse where the static model is used before a saddle-node bifurcation and the dynamic model is employed after the bifurcation.

Before the bifurcation, a static model may be used to explain the slow voltage decrease. The closeness of the system to bifurcation may be interpreted physically in terms of the ability of transmission systems to transmit reactive power to load buses. Simulation results show how this ability varies with system parameters. We suggest that voltage collapse could be avoided by manipulating system parameters so that the bifurcation point is outside the normal operating region.

After the bifurcation, the system dynamics is modelled by the center manifold voltage collapse model [23,24]. The essence of this model is that the system dynamics after bifurcation are captured by the center manifold trajectory. The behavior predicted by the model is found simply by numerically integrating the system differential equations to obtain this trajectory.

The simulation result shows that a slowly increasing reactive power demand might cause a voltage collapse. This contrasts with the conjecture that it is the protection system that causes the abrupt change in trajectory. In our simple power system example we chose to investigate how a dynamic load and generator might cause voltage collapse. The modelling in this example is probably oversimple; nevertheless we regard it as an important first step towards the goal of demonstrating voltage collapse in a realistic power system model.

We comment on future research directions. The voltage collapse model is very general and may be applied to any system of differential equations with a slowly varying parameter. Therefore we are confident that it can be applied to more realistic power system models. (Indeed, since the theory is a general account of typical system behavior, useful applications in other areas may well emerge.) It is not yet clear what power system models are adequate for analyzing real voltage collapses either in system size or the components modelled. More research in devising suitable dynamic models of system components is needed. For example, the generator control loops or limitations might need to be modelled. Improved dynamic load models are needed and further experience in applying our theory to realistic power system models is needed in order to judge to what extent our approximation of slowly varying parameters is appropriate.

Most of the previous analysis of voltage collapse has considered only the period before bifurcation. In this paper we show by an example that voltage collapse may be studied before bifurcation with a static model and after bifurcation with a dynamic model. We are encouraged that the simulation of our example captures qualitative features of a real voltage collapse. We hope to extend our modeling to larger or more detailed power system models.

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DISCUSSION

HANS GLAVITSCH, Swiss Federal Institute of Technology, Zürich, Switzerland. In this paper the authors address a very important aspect of the voltage collapse problem and present a number of interesting results for which they are to be commended.

The problem is the inclusion of the dynamic behavior of the system under a collapse situation. In the paper there is a numerical example whereby its dynamics is due to derivatives of the voltage at the load bus. From the discusser's point of view there are two questions to be answered:

1) Is the dynamic behavior of the system near a collapse situation adequately represented ?

2) Is the load model representative ?

As to the first question it seems that voltage stability in the sense of steady state stability can be represented this way. The system could be extended by adding the voltage regulator or other features of the system. However, it is to be realized which effects are the dominant ones which contribute to voltage instability, e.g. tap changers, domestic loads, etc. Thereby the time evolution of the instability effect is important. If there is a substantial portion of induction motor load the interaction with the supply system is quite different from a system having a regulated domestic load. Thus, the models of the supply system and of the load as far as their time behavior is concerned have to be adequately chosen. The important aspect is that an assessment can be done for small deviations only.

The second question has wider implications. From the knowledge of power systems it is to be expected that it is not easy to model load over a wide range of voltage magnitudes. When the voltage drops below 0.85 p.u. a model which is representative for the normal voltage level is probably not accurate anymore. This comment is already implied in the first question. However, there is another point which may be explained by the following.

If an ac source is taken having a reactance attached which supplies a reactive load only and the load is constant (constant Q) there is a voltage limit. Increasing the load beyond the limit does not yield a solution for the voltage. If according to the paper an extension to the load model (proportional to V, V^2, $\dot{V}$) is made there is a mathematical solution, i.e. the voltage follows the manifold beyond the bifurcation point. The main observation thereby is that the reactive load is not constant anymore but varies with the voltage.

The question behind this new model is if the basic insufficiency of the socalled static approach (constant Q) having no solution beyond the bifurcation point has been resolved. What the new model does is essentially a reduction of the reactive load. In the moment a reduction of the load is permissible the static approach will also offer a solution which has a different character but anyway there is one.

It seems that by assuming a different load model the character of the problem has been changed substantially. The question of supplying a constant reactive load beyond the limiting voltage magnitude remains as it is. Technically there are several ways out. Either it is agreed that the reactive load is allowed to change or it is maintained that the load is really constant. In the latter case there is no solution to the load case. In the first case there is a variety of solutions of which there is one as shown in the paper. What remains, however, is the voltage behavior beyond the bifurcation point. The model as assumed here is just an outlet for the voltage to be continued. As mentioned earlier it can hardly be assumed that the load model under normal conditions is also valid for lower voltage levels. Hence, caution has to be exercised when interpreting the voltage beyond the bifurcation point. With other words, one has to be aware of the correctness of the load model.

The authors are invited to give their opinion on these aspects of the problem.

M.M. BEGOVIC, A.G. PHADKE, Virginia Polytechnic Institute and State University, Blacksburg, VA 24060: The authors are to be commended for their effort to explain the dynamics of voltage collapse in power systems using the static bifurcation theory. The parameter dependent state space model

$$\dot{x} = F(x,\lambda)$$

is subjected to geometric contortions caused by the slow change of λ which moves the system to a static bifurcation and causes the disappearance of a stable equilibrium point for a certain critical value of parameter λ^0. The conditions (5–1a) and (5–1b) in the paper define the dependence of a saddle–node bifurcation value on the changes of one parameter. The subsequent system trajectory along the center manifold W^c is characterized by the initial slow dynamics, which complies well with the observations of the actual collapse cases. The discussers agree that proper load modeling is instrumental in assessing the correct bifurcation value and have two questions:

1) The use of the composite dynamic load model ((2–2a), (2–2b) in the paper) is an obvious convenience, because it allows to analyze the system in a state space without a vector field (flow) defined on it, which would be the case if loads were modeled without dynamics. In reality, however, it is reasonable to expect that at least some of the loads would have to be modeled as nonlinear without dynamics, implying the change of the system model into

$$\begin{aligned} \dot{y} &= F(y,z,\lambda) \\ 0 &= G(y,z,\lambda) \end{aligned}$$

where $y \in \mathbb{R}^n$, $z \in \mathbb{R}^m$, $\lambda \in \mathbb{R}^k$, $F:\mathbb{R}^{n+m+k} \to \mathbb{P}$, $\mathbb{P} \subset \mathbb{R}^n$, $G:\mathbb{R}^{n+m+k} \to \mathbb{Q}$, $\mathbb{Q} \subset \mathbb{R}^m$. Would any of the authors' results be applicable to the analysis of the bifurcation reached for $\lambda = \lambda^0$

$$\begin{aligned} F(y,z,\lambda^0) &= 0 \\ G(y,z,\lambda^0) &= 0 \\ \det[DF(y,z,\lambda^0)\ DG(y,z,\lambda^0)]^t &= 0 \end{aligned}$$

which can also produce voltage collapse?

2) The generator model used in the paper applies swing dynamics equations to the constant voltage source. What do the authors think would be an appropriately simple model relevant for voltage collapse analysis on a system of larger size?

C. O. Nwankpa and **S. M. Shahidehpour**, (*Department of Electrical and Computer Engineering, Illinois Institute of Technology, Chicago, Illinois*): The authors should be commended for their interesting paper on the modeling of power system dynamics after bifurcation. We agree with the authors "...that not all saddle-node bifurcations of power system equations (4-1) are of the voltage collapse variety". The authors underline the use of a "(quasi)static model" before bifurcation and the dynamical model (3-1) after bifurcation.

Would the authors explain how they would handle the situation where the bifurcation does not take place at the operating point (load flow solution) but along some trajectories of the system. This would occur due to the general assumption used in studying the problem that the rotor angles would be static (constant). In this regard, we would suggest a time scale separation of differential equations involved in (3-1) arising from singularly perturbed equations accounting for slowly varying generator angles' effects on the bifurcation values as shown in [A,B,C].

We strongly agree with the authors comments on the relationship between a slowly varying parameter within the power system and two periods of slow and fast variations in voltage. The question that should be asked here is whether a one parameter dynamical model is a good approximation of this phenomenon? As mentioned above, in voltage collapse studies, rotor angles may be viewed as slowly varying parameters along with the possibility of varying Q_i values. In this type of multi-parameter dynamical system the equilibrium point $x_1(\mathbf{Q})$ will not be type-one, so how will the eigenvector p be computed from the Jacobian matrix at x^*? How will the integration of the system equation (4–1) be performed? How seriously will this affect the computational effort?

We are interested in the interpretation of the center manifold trajectory describing where the trajectory goes in state space after bifurcation. It is a known fact from bifurcation theory of multi-dimensional nonlinear systems that the assumption (iii) of Section 5 indicating that there does not exist any equilibrium point in the neighborhood of $x_s(Q_i^*)$ for $Q_i > Q_i^*$ is a rather weak one. From this theory, the system trajectory may diverge and instead of tending to infinity or another s.e.p., it may tend to an u.e.p. in which the system model would simulate the persistence of the former s.e.p. [D].

Author's comments on these points will be appreciated.

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H. D. Chiang, I. Dobson, R. J. Thomas, J. S. Thorp and **L. Fekih Ahmed**: We would like to thank the discussors for their interest in the paper and valuable comments. A point-to-point response to each discussor is presented below.

In response to Mr. Nwankpa and Dr. Shahidehpour:

[1] When employing bifurcation theory to analyze voltage collapse, it is important to distinguish between two different spaces: the state space and the parameter space. In the power system model described in section 2, the state vector consists of all bus angles, generator bus frequencies as well as load bus voltage magnitudes while the parameter vector includes real and reactive power demand at each bus. Variations in the parameters may cause the system to encounter a bifurcation which could lead to voltage collapse. Moreover, variations in the parameter vector will cause a change in the state vector. Hence, it is not appropriate to view rotor angles as slowly varying parameters; they are state variables.

[2] It is shown in [A1] that one typical way in which a stable equilibrium point of a one-parameter dynamical system disappears is through a saddle-node bifurcation. In power system applications, a one-parameter dynamical system is a system together with one of the following conditions:

- the reactive (or real) power demand at one load bus varies while the others remain fixed,
- both the real and reactive power demand at a load bus varies and their variations can be parameterized. Again the others remain fixed,
- the real and/or reactive power demand at some collection of load buses varies and their variations can be parameterized while the others are fixed.

It would be desirable to develop a voltage collapse model so that saddle-node bifurcations would still arise generically when several power system parameters are allowed to freely varying. However, we note that the one-parameter theory is sufficient to illustrate voltage collapse for certain power system models. For example, Tamura et al. [A2] and Begovic & Phadke [A3] provide examples of saddle-node bifurcations associated with voltage collapse due to variation of the reactive power demand of a single load bus and due to slow increases in both the real and reactive power demand of all load buses, respectively.

[3] Before we respond to the question of how to handle the situation where "the bifurcation does not take place at the operating point (i.e., the load-flow solution) but along some trajectories of the system," it might be helpful to review the concepts associated with bifurcation. The term bifurcation can be broadly used to describe qualitative changes in the trajectory structure of a dynamical system as the parameters of the system are varied. In this paper, the focus is on the so-called local bifurcation theory which is concerned with the bifurcation of equilibrium points or with situations where the problem can be cast into this form such as in the study of bifurcations of closed orbits via a local Poincare map. The goal of the local bifurcation theory is to investigate the nature of the static solutions as parameters vary. In this case, local bifurcation theory assumes a static model. It is clear that the popular P-V and Q-V curves used in many utilities to analyze voltage collapse fall in the category of static models and therefore local bifurcation theory applies. On the other band, global bifurcation theory is concerned with qualitative changes in the phase portrait of an extended state space. Global bifurcations are often characterized by an absence of a transversality condition between the stable and unstable manifolds of equilibrium points and closed orbits. Typical examples of global bifurcations are homoclinic and heteroclinic bifurcations. In terms of the question raised it is not clear to us how to define a bifurcation that occurs along some trajectory of the system. We presume the question is how to relate global bifurcation theory to voltage collapse. This interesting question needs further investigation because the global bifurcation theory is far from complete mainly due to the fact that techniques for global analysis of trajectory structure are just under development.

[4] Suppose that the power system described by equation (4-1) is operating at the stable equilibrium point $x_s(Q_i)$, where Q_i is the reactive power demand at load bus i. If Q_i is slowly increased while other parameters remain fixed the equilibrium point $x_s(Q_i)$ will vary as Q_i is increased. It can be shown [A1] (not by assumption!) that a bifurcation value Q_i^* exists such that

(i) $x_s(Q_i)$ is stable if $Q_i < Q_i^*$,
(ii) $x_s(Q_i)$ becomes unstable by coalescing with another type-one equilibrium point $x_1(Q_i)$ in a saddle-node bifurcation if $Q_i = Q_i^*$. Just before the bifurcation, $x_1(Q_i)$ is on the stability boundary of $x_s(Q_i)$ and $x_1(Q_i)$ is the closest unstable equilibrium point to $x_s(Q_i)$. and
(iii) $x_s(Q_i)$ will disappear if $Q_i > Q_i^*$, i.e. there does not exist any equilibrium point in a neighborhood of $x_s(Q_i^*)$ for $Q_i > Q_i^*$.

[5] At a saddle-node bifurcation point, the system trajectory moves along the center manifold. One of two cases may occur. In the first case, the system trajectory diverges and tends to infinity. In the second case, the system trajectory tends to another stable equilibrium point. We do not expect that the system trajectory will tend to an unstable equilibrium point. This is explained as follows: Just before the bifucation occurs a part of the unstable manifold of the type-one equilibrium point $x_1(Q_i)$ converges to the stable equilibrium point $x_s(Q_i)$ [A1] and another part of the unstable manifold, under a generic condition termed a transversality condition, either converges to another stable equilibrium point or tends to infinity [A4, A5]. Hence, the unstable manifold of $x_1(Q_i)$ cannot tend to an unstable equilibrium point. And we expect the existence of a one-to-one map between the center manifold and the part of the unstable manifold that either converges to another stable equilibrium point or tends to infinity.

In response to Drs. Begovic and Phadke:

1. The use of the dynamic load model (2-2a) & (2-2b) is not for any purpose of convenience. The dynamic load model, derived by Walve based on field tests, is used in order to better capture the behavior of the load during periods of dynamic swing than static load models, such as constant P-Q, do. Moreover, the resulting power system model is purely a vector differential equation with a well-defined vector field as well as unique solutions. On the other hand, if loads are modelled as nonlinear functions without dynamics, then the resulting power system model is a mixture of differential equations and algebraic equations. Algebraic equations can arise from load models that are idealizations of some unmodelled dynamics

which normally tend to act so that the algebraic equations are satisfield. This kind of system may not be well posed globally; namely, some system trajectories may not be defined for positive time. One common approach to resolving this difficulty is to use singular perturbation ideas to develop dynamics for the voltage magnitudes and angles of load buses which somehow generalize the algebraic equations as follows:

$$\dot{y}=F(y, z, \lambda)$$

$$\epsilon\dot{z}=G(y, z, \lambda)$$

where ϵ is a small number. This does indeed yield a power system model of the form (4-1). However, it is unclear how to choose the value ϵ and how to physically justify such choices.

2. The voltage collapse model presented in the paper is applicable to any generator model described by a differential equation. Thus, the generator model could include the effects of flux decay as well as controllers of the generator such as exciters and governors. However, at this point, it is not clear to what degree of complexity the generator should be modelled for the purpose of voltage collapse analysis.

In response to Professor Glavitsch:

We share the viewpoint of Professor Glavitsch relative to the importance of load model in the analysis of voltage collapse problems. We believe that load characteristics have a significant effect on system dynamics and hence on the quality of stability analysis. We also believe that load behavior during significant system dynamics cannot be adequately described by static load models such as constant P-Q and should be modelled as dynamic load models. We admit that a universal dynamic load model for voltage collapse analysis is still not available and may remain so in the near future. We also agree that load models for a different mixture of industrial loads, commercial loads and residential loads should be different. But the problem is in what sense load models should be different? (e.g., totally different in form or just different in the coefficients of the same form?).

It should be stressed that the proposed voltage collapse model is not tailored to any particular load model such as (2-2a) and (2-2b). This voltage collapse model is applicable to loads that can be described by differential equations. In other words, different dynamic load models indeed lead to different system dynamics. However, the system dynamics after bifurcation are still captured by the center manifold trajectory. Of course, different dynamic load models result in different center manifold trajectories.

The load model suggested in this paper is based on load dynamics due to Walve [25]. It is clear from [25] that the load model is valid for nominal voltage with small deviations only, as was noted by Professor Glavitsch. When the voltage magnitude of a load bus is outside its nominal range, the load model is probably not adequate anymore and needs to be modified. In such situations one plausible way is to describe the load model as a set of differential equations with each differential equation representing the load model for a certain operating condition. In this case the proposed voltage collapse model is still applicable.

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