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Dynamical and probabilistic approaches to the study of blackout vulnerability of the power transmission grid

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Abstract

The CASCADE probabilistic model for cascading failures gives a simple characterization of the transition from an isolated failure to a system-wide collapse as system loading increases. Using the basic ideas of this model, the parameters that lead to a similar characterization for power transmission system blackouts are identified in the OPA dynamical model of series of blackouts. The comparison between the CASCADE and OPA models yields parameters that can be computed from the OPA model that indicate a threshold for cascading failure blackouts. This is a first step towards computing similar parameters for real power transmission systems.

1. Introduction

We have developed the ORNL-PSerc-Alaska (OPA) model to study blackout dynamics in the power transmission grid [1-3]. This model incorporates self-organization processes based on the engineering response to blackouts and the long-term economic response to customer load demand. It also incorporates the apparent critical nature of the transmission system. The combination of these mechanisms leads to blackouts that range in size from single load shedding to the blackout of the entire system. This model shows a probability distribution of blackout sizes with power tails [2] similar to that observed in real blackout data from North America.

In addition to the OPA model, we have constructed CASCADE, a probabilistic model that incorporates some general features of cascading failure. A detailed description of the CASCADE model is given in Refs. [4,8]. This model shows the existence of two critical thresholds. One is associated with the minimal load needed to start a disturbance. In a power transmission system, it can be interpreted as the load increase that will cause a line (or a few independent lines) to overload and fail. The second critical threshold is associated with the minimal load transfer throughout a cascading event that can lead to a total system blackout. This type of threshold is less evident in real systems, and the parameter or parameters controlling it are not easy to identify.

Those cascading events are similar to the “domino effect.” In this case, the force needed to trip the first domino gives the first threshold. The second threshold is given by the ratio of the separation between dominos to their height; the threshold must be less than the critical value of one to cause all the dominos to fall. Of course, transmission systems are a great deal more complicated than dominos, but here we want to focus on identifying this second type of threshold.

To identify the type of threshold that causes system-wide blackouts, we compare the probabilistic model, where this threshold is easy to identify, with the dynamical model. This dynamical model incorporates the structure of a network, and a linear programming (LP) approach is used to find instantaneous solutions to the power demand. In such a model, the threshold to system-wide blackouts is not obvious, and its understanding may provide a path toward application to realistic systems.

2. Critical transitions in the CASCADE model

The CASCADE model has n identical components with random initial loads. The minimum initial load is $L_{\min}$, and the maximum initial load for each component is $L_{\max}$.. For j=1,2,...,n, component j has an initial load of L_j that is a random variable uniformly distributed in [$L_{\min}$, $L_{\max}$]. L_1, L_2, · · · , L_n are independent. Components fail when their load exceeds L_{fail}. When a component fails, a fixed amount of load P is transferred to each of the remaining components.

We assume an initial disturbance that starts the cascade by loading each component with an additional amount, D. Other components may then fail, depending on their initial loads, L_j, and the failure of any of these components will distribute an additional load, $P \geq 0$, that can cause further failures in a cascade. This model describes the cascading failure as an iterative process. In each iteration, loads fail as the transfer load, P, from other failures makes them reach the failure limit. The process stops when none of the remaining loads reaches the failure limit.

It is convenient to normalize all of the loads in the system so that they are distributed in the [0,1] interval. Thus, we normalize the initial load:

$$l_j = \frac{L_j - L_{\min}}{L_{\max} - L_{\min}}. \qquad (1)$$

Then l_j is a random variable uniformly distributed on [0, 1]. Moreover, the failure load is l_j = 1. Let

$$p = \frac{P}{L_{\max} - L_{\min}}, \quad d = \frac{D + L_{\max} - L_{fail}}{L_{\max} - L_{\min}} \quad . \qquad (2)$$

Then, p is the amount of load increase on any component when one other component fails when expressed as a fraction of the load range $L_{max} - L_{min}$. Similarly, d is the initial disturbance expressed as a fraction of the load range.

An analytic solution was found [4,8,9] for the probability, $f(r, d, p, n)$, of a cascade with r components failing:

$$f(r,d,p,n) = \begin{cases} \binom{n}{r} \phi(d)(rp+d)^{r-1}\left[\phi(1-rp-d)\right]^{n-r}, \\ \qquad r = 0,1,\ldots,n-1 \\ 1 - \sum_{s=0}^{n-1} f(s,d,p,n), \quad r = n \end{cases} \tag{3}$$

where p is a positive quantity and the function ϕ is

$$\phi(x) = \begin{cases} 0, & x < 0 \\ x, & 0 \leq x \leq 1 \\ 1, & x > 1 \end{cases} .$$

Equation (3) uses $0^0 \equiv 1$ and $0/0 \equiv 1$ where needed. If $d \geq 0$ and $d + np \leq 1$, then $\phi(x) = x$ and Eq. (3) reduces to the quasibinomial distribution introduced by Consul [10].

For a given system, there are two possible types of situations: (1) the system has no component failures or (2) some components in the system have failed. In the CASCADE model, there is clearly a transition from one situation to the other and the control parameter is d. The transition point is $d = 0$. The probability of failure is

$$P(d) = \begin{cases} 0, \; d < 0 \\ 1 - f(0,p,d,n) = 1 - (1-d)^n, \; d > 0 \end{cases} . \tag{4}$$

Near the critical point, the transition probability scales as nd. For large systems, it is better to introduce $\theta \equiv nd$ as the control parameter for this transition. In this way, θ remains finite for $n \to \infty$. It is also useful to consider $\lambda \equiv np$, the total load transfer from a failing component, as the second parameter in this model. The use of the λ and θ is justified in Ref. [9] by approximating CASCADE as a branching process and identifying λ and θ as parameters of the branching process. The situation with no failures is rather simple, and there is a single point in configuration space with no ambiguity in its characterization. However, the failed system has multiple possible states, each characterized by the number r of failed loads. For a given set of values for θ and λ, there is a distribution of possible states, each characterized by a probability $p_b(r,\lambda,\theta,n)$,

$$p_b(r,\lambda,\theta,n) = \frac{f(r,\lambda,\theta,n)}{1 - f(0,\lambda,\theta,n)} . \tag{5}$$

Because we are interested in system-wide collapses, an important quantity to consider is the probability of a full system cascade, $r = n$,

$$P_\infty = \frac{f(n,\lambda,\theta,n)}{1 - f(0,\lambda,\theta,n)} . \tag{6}$$

This probability has the properties of the order parameter in a critical transition. As shown in Fig. 1, this expression is such as that $P = 0$ at $\lambda < \lambda_c$, where λ_c is the critical value of λ. However, above the critical value for λ, system-wide failures are possible. In the CASCADE model, which assumes a uniform random distribution of loads, the critical point is $\lambda_c = 1$. This is the second transition point that we discussed in the introduction. It separates the localized failures of the system from system-wide cascading failures. This type of transition is the one we want to also characterize for the OPA dynamical model.

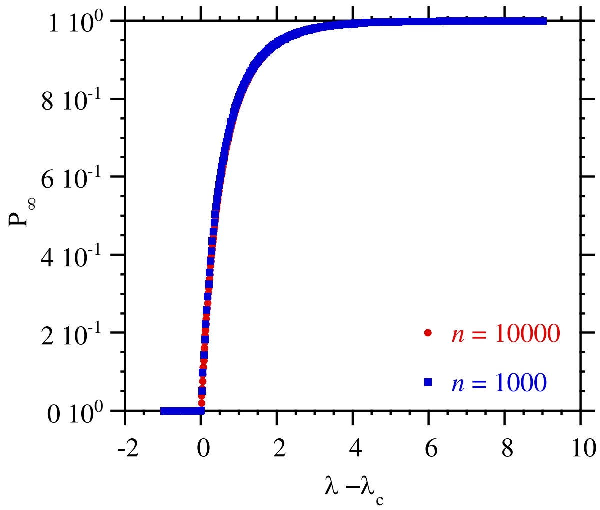

Figure 1: Probability of system-wide cascade events as a function of λ.

The parameter λ is a direct measure of the total load transferred by a failing component to the entire system. It also characterizes other properties of the system that are useful in giving a meaningful interpretation of λ for different systems. One of the approximate properties of the CASCADE model that applies when the model is not saturated due to finite size effects is that the average number of failures during the iteration k is

$$\langle r \rangle_k = \theta \lambda^k . \tag{7}$$

This is an important relationship that will be used in comparison with the dynamical model.

3. The dynamical OPA model and the cascading transition

We developed the OPA model to study the dynamics of a power transmission system [1-3]. In the OPA model, the dynamics involve two intrinsic time scales.

In the OPA model, there is a slow time scale of the order of days to years, over which load power demand slowly increases and the network is upgraded in response to the increased demand. The upgrades are done in two ways. Transmission lines are upgraded as engineering responses to blackouts and maximum generator power is increased in response to increasing demand. The transmission line upgrade is implemented as an increase in maximum power flow, $F_{ij}^{\max}$, for the lines that have overloaded during a blackout. That is, $F_{ij}^{\max}(t) = \mu F_{ij}^{\max}(t-1)$ if line ij overloads during a blackout. We take μ to be a constant. These slow, opposing forces of load increase and network upgrade self-organize the system into a dynamic equilibrium. As discussed elsewhere [3], this dynamical equilibrium is close to the critical points of the system [5, 6].

In the OPA model, there is also a fast time scale, of the order of minutes to hours, over which cascading overloads or outages may lead to a blackout. Cascading blackouts are modeled by overloads and outages of lines determined in the context of LP dispatch of a DC load flow model. Random line outages are triggered with a probability p_0. They simulate the consequence of intentional or accidental events. A cascading overload may also start if one or more lines are overloaded in the solution of the LP problem. In this situation, we assume that there is a probability, p_1, that an overloaded line will become an outage. When a solution is found, the overloaded lines of the solution are tested for possible outages. If there are one or more line outages, we reduce the maximum power flow allowed through this line by several orders of magnitude. In this way, there is practically no power flow through this line. Once the power flow through the lines is reduced, a new solution is then calculated. This process can lead to multiple iterations, and the process continues until a solution with no more line outages is found. The overall effect of the process is to generate a possible cascade of line outages that is consistent with the network constraints and the LP dispatch optimization.

The OPA model allows us to study the dynamics of blackouts in a power transmission system. This model shows dynamical behaviors characteristic of complex systems. It has a variety of transition points as power demand is increased [5, 6]. These transition points are related to a limitation in the generator power and/or single line overloads. These transition points correspond to single failures of the system and are the first type of transition discussed above. However, in contrast to the CASCADE model, there are multiple sources of single failure in this model.

Here, we study the critical point from the perspective of triggering system-wide blackouts as described in the previous section. The first thing to consider is the possible separation between regimes of single failures and regimes with cascading failures. For this model, calculation of the probability of a system collapse event is not possible. It would be necessary to carry out calculations for a very long time to obtain the necessary statistics. In particular, close to the transition, the required computational time is beyond our present capabilities. We need another approach.

In the OPA model, we find the separation between the two regimes as a function of two parameters, Γ and μ. Here, Γ is the ratio of minimal generator power margin, $(\Delta P/P)_c \equiv (P_G - P_0)/P_0$, to the root mean square of the fluctuation of the load demand $g \equiv \left\langle \left[(P_D - P_0)/P_0 \right]^2 \right\rangle^{1/2}$.

$$\Gamma = (\Delta P/P)_c / g \quad . \qquad (8)$$

P_G is the minimal generator power available, $P_0 = \hat{P}_0 e^{\hat{\lambda} t}$ is the mean load demand that increases at a constant rate $\hat{\lambda}$, and P_D is the actual load demand that fluctuates around the mean value.

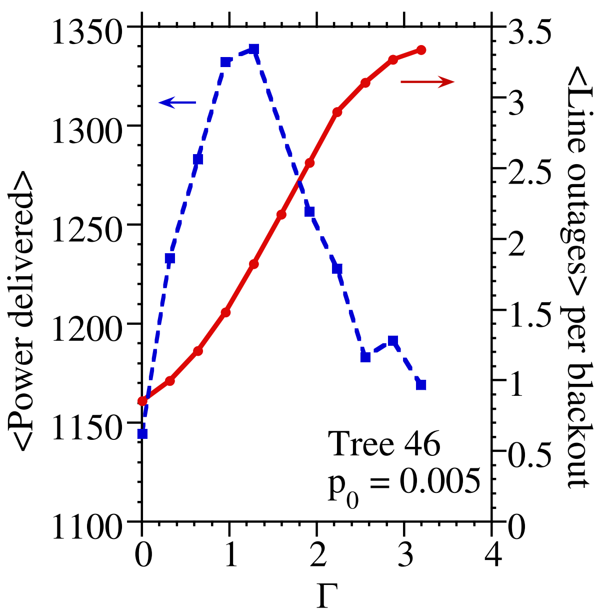

Figure 2: Averaged power delivered and number of line outages per blackout as a function of Γ.

Varying Γ and/or μ is not necessarily a realistic way of modeling the transmission system, but it allows us to understand its dynamics. For a 46-node tree network, we have done a sequence of calculations for different values of the minimal generator power margin $(\Delta P/P)_c$ at a constant g and μ. We have changed this margin from 0 to 100%. For each value of this parameter, we have carried out the calculations for more than 100,000 days in a steady-state regime. This number of days gives us reasonable statistics for the evaluations. One way of looking at the change of characteristic properties of the blackouts with Γ is by plotting the power delivered and the averaged number of line outages per blackout. These plots are shown in Fig. 2. We can see that at low and high values of Γ the power served is low. In the first case, because of limited generator power, the system cannot deliver enough power when there is a relatively large fluctuation in load demand. At high Γ, the power served is low because the number of line outages per blackout is large.

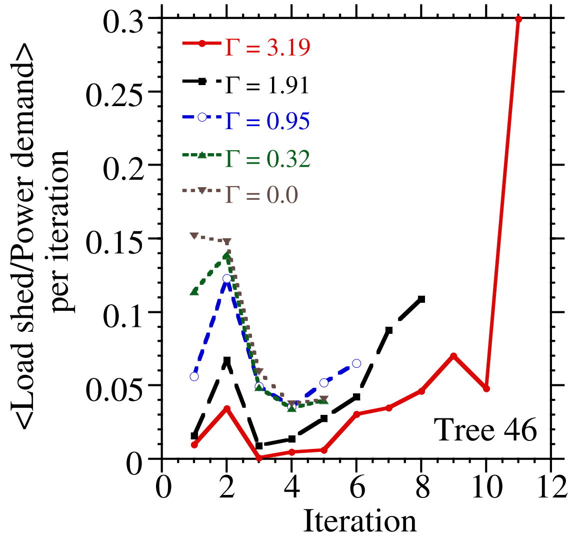

Figure 3: Averaged load shed per blackout normalized to the power demand as a function of iteration number for different values of Γ.

Looking at averaged quantities is not a good way of identifying the demarcation between single (or a few independent) failures and cascading events. To have a better sense of this demarcation, we have calculated the load shed per iteration, normalized to the total power demand, for all blackout events. In Fig. 3, we have plotted the averaged value over all the blackout events for five different values of Γ. We can see that at very low Γ the averaged event is limited to less than five iterations; most of the load is shed during the first couple of iterations. This is typical of isolated failures in a system. However, for large values of Γ, sufficient power is available in the first few iterations with very low load shed. The number of iterations of the cascade events increases and the load shed increases with the iteration number. These are the characteristic properties of large cascading events. At about Γ = 1.0, where the power served has a maximum (Fig. 2), there is the transition from one type of event to the other.

A similar study can be done keeping the parameter Γ fixed and varying the upgrading rate μ. In Fig. 4 and for the 94-node tree network, we show the distribution of the number of line outages for the worst blackouts in a year for different values of μ. We see that for a high upgrade rate, the number of line outages is rather small. However, as μ decreases, the worst blackouts involve a large part of the network.

The Γ and μ parameters have no direct connection to the parameter used in the probabilistic model to characterize the transition from a single failure to a cascading failure. Using the guidance of the CASCADE model, we will try to identify a parameter analogous to λ in the OPA model. To do so, we need to find a way of comparing both models.

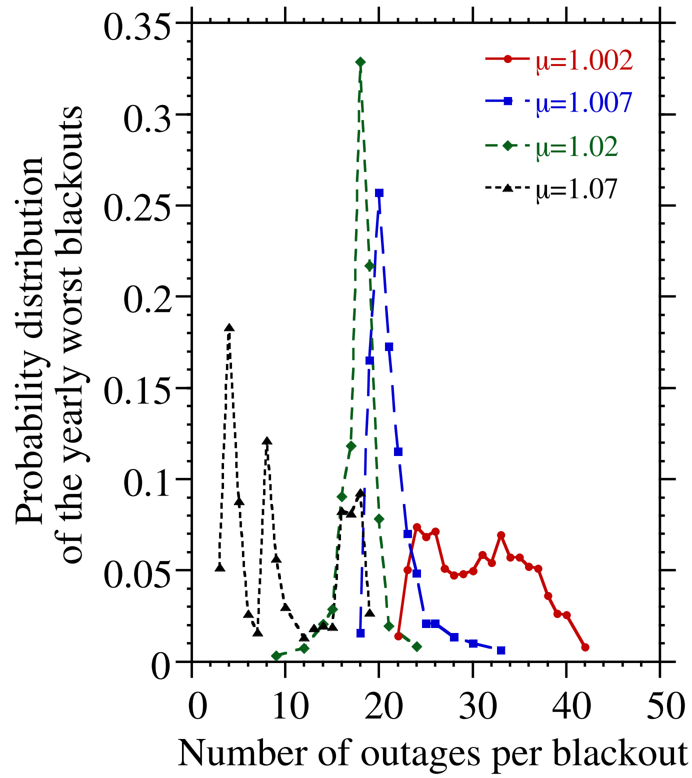

Figure 4: Probability distribution of the number of outages per blackout for the worst yearly blackouts. The calculation is for the 94-node tree network and Γ = 0.96.

4. Averaged number of line outages per iteration

In relating the OPA model to CASCADE, we will interpret the component failures in CASCADE as line outages in OPA. We can then associate the normalized loads, $l_i \in [0,1]$, in CASCADE to the fractional line overloads, M_i, in OPA. The fractional line overload for line i is defined as

$$M_i = \frac{F_i}{F_i^{\max}} \ , \qquad (9)$$

where F_i is the power flow through line i and $F_i^{\max}$ is the maximum possible power flow through this line. For each network considered, the fraction of overloads M_i is also distributed in [0,1], but the distribution is not necessarily random. The average value of the M_i's as the average value of the l_i's in the CASCADE model gives no information on the criticality of the system. It only provides some information on the distribution of loads.

There are several ways of interpreting the parameter λ within the OPA model, and, of course, these different methods do not necessarily lead to the same value for λ. One way is to calculate the averaged number of line outages, $\langle N_{out}(j) \rangle$, per step j in cascading failures, and in analogy with Eq. (7) define

$$\lambda_{eff}(j) \equiv \langle N_{out}(j) \rangle^{1/j} \ . \qquad (10)$$

A priori, there is no reason for λ_{eff} to be independent of j or to have any value similar to the critical value found in the CASCADE model.

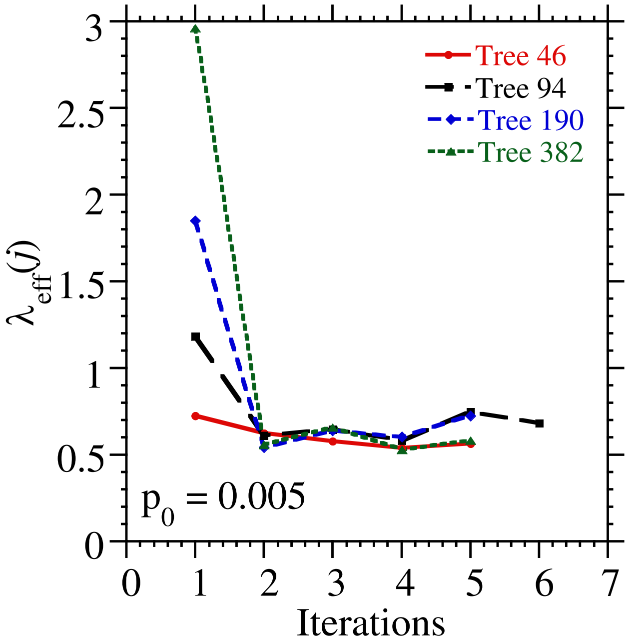

Figure 5: $\lambda_{eff}(j)$ as a function of iteration number for different tree networks.

In Fig. 5, we have plotted $\lambda_{eff}(j)$ as a function of j for four network configurations. These $\lambda_{eff}(j)$networks have a tree-like structure with three line connections per node. These types of networks were discussed in Ref. [2]. The four networks considered here have 46, 94, 190, and 382 nodes.

The numerical results in Fig. 5 show that $\lambda_{eff}(j)$ is weakly varying with j for $j > 1$. For large values of j, the statistics are rather poor and the evaluation of λ_{eff} may have significant error bars. For the first iteration, we found strong variations of $\lambda_{eff}(1)$ with the size and conditions of the network. These variations are understandable because the calculations in Fig. 3 are done for a fixed probability, p_0, of the event being initiated by a line outage. As the number of lines increases, we can have more than one event simultaneously triggered by these random events. Changing the value of p_0 significantly changes $\lambda_{eff}(1)$. However, the change of p_0 has only a weak effect on $\lambda_{eff}(j)$ for $j > 1$. In Fig. 6, we show the effect of changing p_0 on $\lambda_{eff}(j)$.

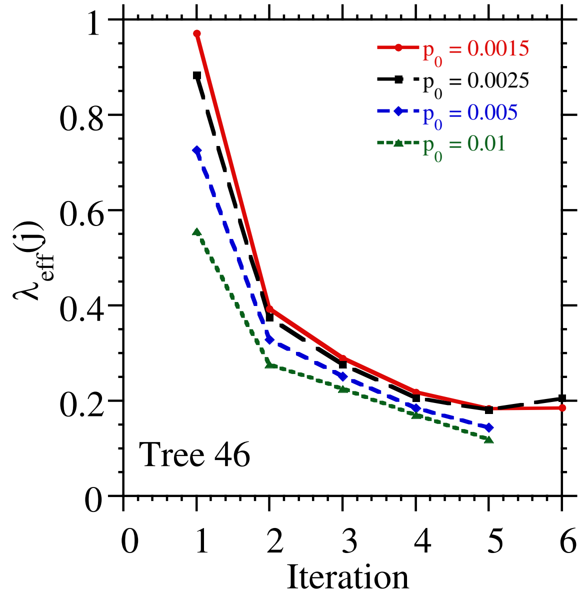

Figure 6: $\lambda_{eff}(j)$ as a function of iteration number for different values of the probability of an accidentally triggered line outage.

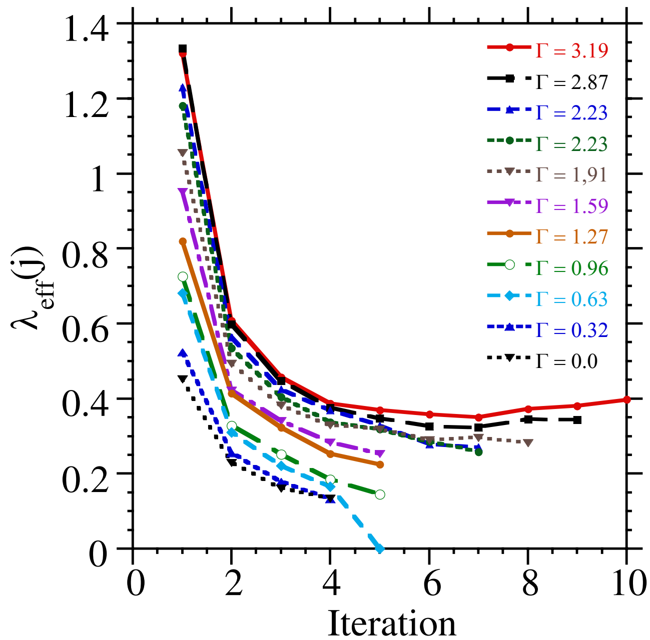

Figure 7: $\lambda_{eff}(j)$ as a function of iteration number for different values of Γ for the 46-node tree network.

Let us now consider the sequence of calculations in which Γ is varied for the 46-node tree network. We have seen that by varying Γ we can change the blackout events from a single failure to cascading events (Fig. 3). In Fig. 7, we have plotted $\lambda_{eff}(j)$ versus j for these different values of Γ. We can see that $\lambda_{eff}(j)$ increases uniformly with Γ. Also, the dependence on the iteration number, $j > 1$, becomes weaker. This may reflect the change in the dynamics going from blackouts dominated by generation limitations to blackouts that are dominated by line outages. The comparison with the

CASCADE model is relevant in the latter regime. The existence of a single λ describing the cascade process is one of the more significant results of these comparisons.

The dependence of λ_{eff} on j is not just a peculiarity of the structure of the ideal tree networks. In Fig. 8, we show the calculated $\lambda_{eff}(j)$ for the IEEE 118 bus network [7]. We can see that $\lambda_{eff}(j)$ is also weakly dependent on j for $j > 1$.

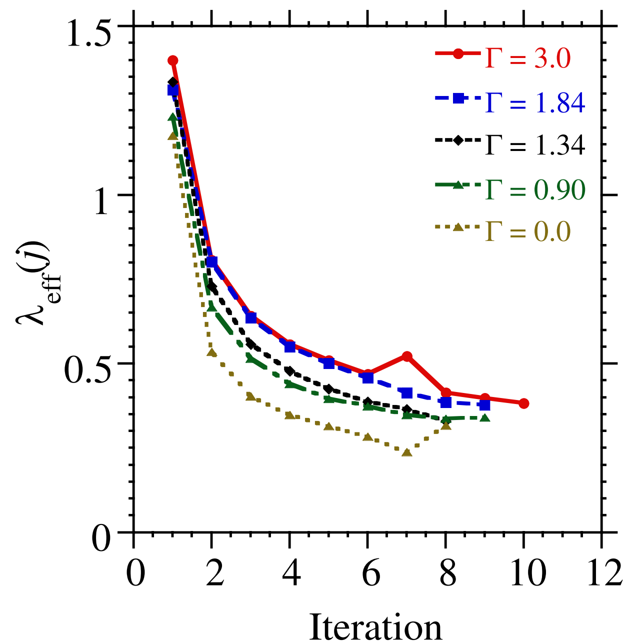

Figure 8: $\lambda_{eff}(j)$ as a function of iteration number for six values of Γ for the IEEE 118 bus network.

It is not surprising that $\lambda_{eff}(j)$ is larger for the first iteration than for the following ones. In the OPA model, unlike in the CASCADE model, there is power shed during each iteration. This power shed reduces the stress over the system and accordingly reduces the probability of line outages at high iterations. Therefore, we believe that the value of $\lambda_{eff}(j)$ for $j = 1$ is the most significant one to be compared with the parameters of the CASCADE model.

We can summarize the stability properties to cascading events of these networks by plotting in the Γ-μ plane the line $\lambda_{eff}(1) = 1$. This line gives the demarcation between the region with $\lambda_{eff}(1) > 1$, where cascading events are possible, and $\lambda_{eff}(1) < 1$, where the cascading events are suppressed. Such a plot is shown in Fig. 9 for the 46-node and 94-node tree networks and for the IEEE 118 bus network. The position of the line $\lambda_{eff}(1) = 1$ in the Γ-μ plane changes with the network configuration, but the three networks show a very similar structure.

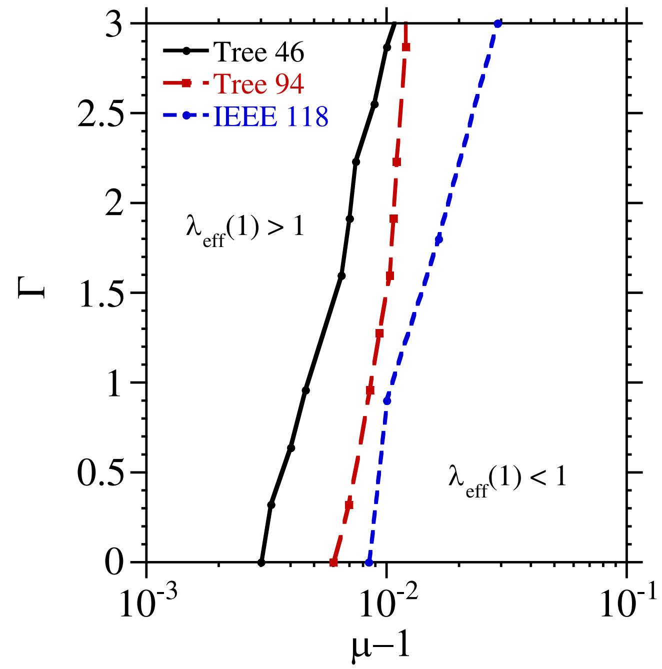

Figure 9: $\lambda_{eff}(1) = 1$ in the Γ−μ plane for the 46-node and 94-node tree networks and for the IEEE 118 bus network.

5. Load transfer during a cascading event

Another interpretation of the parameter λ in the CASCADE model is the total load transfer associated with a failing line. To calculate this transfer load, we use a tree network and we cause a single line outage at a time. We operate at very low power to prevent any of the M_i s from reaching 1 after the chosen line outage because that can cause a reorganization of the power that leads to a different solution. For each line outage, we calculate the effective λ_{0j} in the following way:

$$\lambda_{0j} = \frac{1}{M_j^0} \sum_{i=1}^{\bar{N}_L} \left(M_i^1 - M_i^0 \right) \quad , \qquad (11)$$

where, $\bar{N}_L$ is the number of lines minus 1 because there is only one line outage. The superscript of the M_i's indicates step zero, the value of the M_i before the line outage, or step 1, after the line outage. The transfer load is normalized to M_j because we need the value of the transferred load when $M_j = 1$. This calculation is more elaborate than calculation of a standard line-outage power-distribution factor because the generation redispatches after the line outage.

We calculate λ_{0j} for each line j of the network and repeat the calculation n times for different random values of the loads. Then, we average λ_{0j} over the lines and over the calculated n samples. This gives us another determination of the effective λ, $\langle \lambda_0 \rangle$. We have done the calculation of $\langle \lambda_0 \rangle$ for the tree 46 configuration and several values of Γ. In Fig. 10, we compare these results to the $\lambda_{eff}(1)$ calculated in the previous section. We can see that the values are quite similar. This result is interesting because this method for determining $\langle \lambda_0 \rangle$ can be applied to a real power transmission network and this parameter can be used as an alternative way of determining how close a system is to the cascading threshold.

6. Conclusions

The CASCADE model gives a simple characterization for the transition from an isolated failure to a system-wide collapse. The characterization of this transition is very important, not only for power systems but for any large, man-made, networked system. The control parameter for this transition is directly related to the load transfer during cascading events. In real systems, perhaps more than one parameter can characterize this transition. Here, we have looked for ways of determining this control parameter for power transmission systems to quantify the way in which cascading failures propagate.

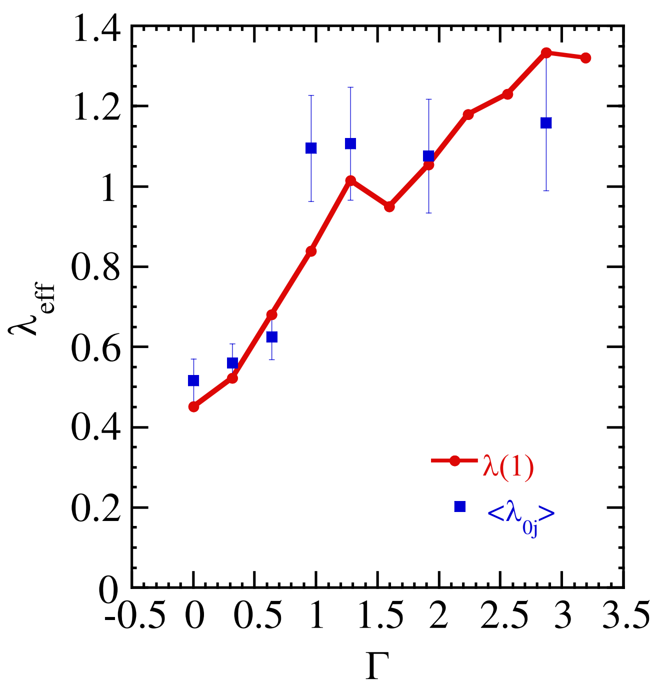

Figure 10: Comparison of the calculated $\lambda_{eff}(1)$ using the two methods discussed in this paper.

The OPA model gives a test bed to apply some of the concepts developed in the simpler probabilistic models. Using an analogy between the two types of models, we have been able to identify a similar transition from an isolated failure to a system-wide collapse in OPA. Furthermore, in defining the transition between these two operational regimes, we have been able to correlate the two parameters Γ and μ, which are related to the operation of the system, to $\langle \lambda_0 \rangle$, which can be determined for a real power transmission system. The relationship between those parameters and the threshold for cascading failure may lead to some practical criteria that will be applicable to the design and operation of power transmission systems.

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