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Dynamics, Criticality and Self-organization in a Model for Blackouts in Power Transmission Systems

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Abstract

A model has been developed to study the global complex dynamics of a series of blackouts in power transmission systems [1, 2]. This model has included a simple level of self-organization by incorporating the growth of power demand and the engineering response to system failures. Two types of blackouts have been identified with different dynamical properties. One type of blackout involves loss of load due to lines reaching their load limits but no line outages. The second type of blackout is associated with multiple line outages. The dominance of one type of blackouts versus the other depends on operational conditions and the proximity of the system to one of its two critical points. The first critical point is characterized by operation with lines close to their line limits. The second critical point is characterized by the maximum in the fluctuations of the load demand being near the generator margin capability. The identification of this second critical point is an indication that the increase of the generator capability as a response to the increase of the load demand must be included in the dynamical model to achieve a higher degree of self-organization. When this is done, the model shows a probability distribution of blackout sizes with power tails similar to that observed in real blackout data from North America.

1. Introduction

The first version of the ORNL-PSerc-Alaska (OPA) model of series of blackouts in power system transmission systems was proposed in [1, 2]. This first version of the OPA model showed how the slow opposing forces of load growth and network upgrades in response to blackouts could self organize the power system to dynamic equilibrium. Blackouts were modeled by overloads and outages of lines determined in the context of LP dispatch of a DC load flow model. This model showed complex dynamical behaviors and has a variety of transition points as a function of increasing power demand [3]. Some of these transition points have the characteristic properties of a critical transition. That is, when the power demand is close to a critical value, the probability distribution function (PDF) of the blackout size has an algebraic tail and across the critical point the system changes sharply. One such consequence of the critical transition is that at these transition points, the power served is maximum and the risk for blackouts increases sharply. This fast variation near the critical point is illustrated in Fig. 1. Therefore, it may be natural for power transmission systems to operate close to and somewhat below those points.

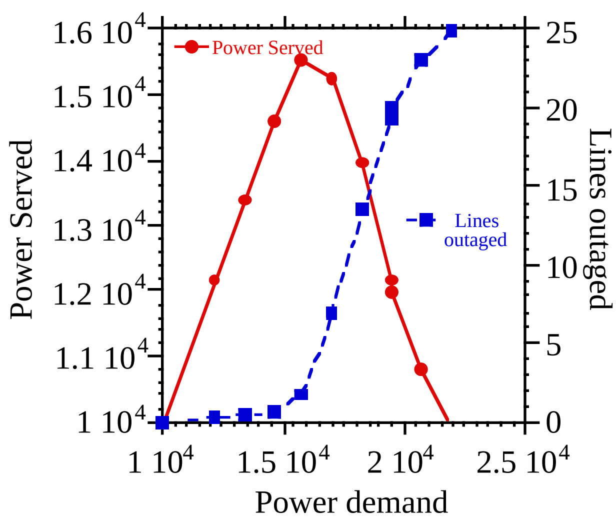

Fig. 1. Power served and number of lines outage for a tree network with 190 nodes as a function of the power demand.

The fact that, on one hand, there are critical points with maximum power served and, on the other hand, there is a self-organization process that tries to maximize efficiency and minimize risk may lead to a power transmission model governed by self-organized criticality (SOC) [4].

The operation of power transmission systems results from a complex dynamical process in which a diversity of opposing forces regulate both the maximum capabilities of the system components and the loadings at which they operate. These forces enter in a highly nonlinear manner and may cause a self-organization process to be ultimately responsible for the regulation of the system. This view of a power transmission system considers not only the engineering and physical aspects of the power system, but also the engineering, economic, regulatory and political responses to blackouts and increases in load power demand. A detailed incorporation of all these aspects of the dynamics into a single model would be extremely complicated if not intractable due to the human interactions involved. However, it is useful to consider simplified models with some approximate overall representation of the opposing forces in order to gain some understanding of the complex dynamics in such a self-organized framework and the consequences for power system planning and operation.

The OPA model is motivated by analyses of NERC data that indicate power tails in the probability distribution of the size of North American blackouts [5,7]. (Power tails decay as according to a power law and are also exhibited by complex systems near criticality.) These observations indicate the non-Gaussian character of the blackout size probability distributions and are of concern because they indicate a much larger risk of large blackouts than might be expected. Confirming and understanding this power dependence in the probability distribution tails is of course very important in doing any risk analysis of power systems.

Note that transition points are essentially determined from power systems physics and engineering constraints, however, the dynamical evolution involves aspects that are less clearly defined by simple deterministic rules. These components of the model may be developed at different levels of complexity representing different approximations to the "real" system.

The main purpose of the OPA model is to study the complex behavior of the dynamics of series of blackouts. In this paper we examine critical points of the OPA model to understand them better. This understanding allows us to extend the modeling of the self-organization of the system to represent generator upgrades as well as network upgrades. With this improvement to the OPA model, we demonstrate self-organization of the system to a critical point at which the probability distribution of blackout size resembles the probability distribution of the NERC data.

2. OPA fast dynamics blackout model

In the OPA model of [1,2], the dynamics involves two intrinsic time scales. There is a slow time scale, of the order of days to years, over which load power demand slowly increases and the network is upgraded in engineering responses to blackouts. These slow opposing forces of load increase and network upgrade self organize the system to a dynamic equilibrium. These slow dynamics are summarized in Appendix I. There is also a fast time scale, of the order of minutes to hours, over which cascading overloads or outages may lead to blackout.

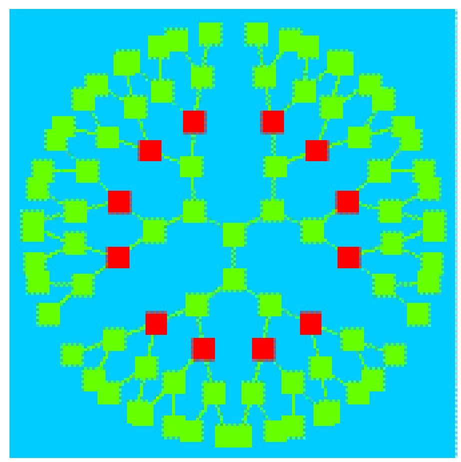

Fig. 2. A 94-node tree network with 12 generators and 82 loads.

To investigate the critical points of the OPA model in section 3, we suppress the modeling of the slow dynamics responsible for the self-organization and study only the fast dynamics of the blackouts. That is, we fix the network by suppressing the network upgrades and treat the load demands as deterministic or random parameters to be specified as inputs to the model. This section explains the fast dynamics of the OPA model.

In this paper, we investigate the blackout dynamical model applied to ideal grid networks that have a tree structure. An example of a tree network with 94 nodes is shown in Fig. 2.

In any network, the network nodes (buses) are either loads (L) (gray squares in Fig. 2), or generators (G), (black squares in Fig. 2). The power P_i injected at each node is positive for generators and negative for loads, and the maximum power injected is $P_i^{\max}$. The transmission line connecting nodes i and j has power flow F_{ij}, maximum power flow $F_{ij}^{\max}$, and the impedance of the line z_{ij}. There are $N_N = N_G + N_L$ total nodes and N_l lines, where N_G is the number of generators and N_L is the number of loads.

The blackout model is based on the standard DC power flow equation,

$$F = AP \tag{1}$$

where F is a vector whose N_L components are the power flows through the lines, F_{ij}, P is a vector whose N_N–1 components are the power of each node, P_i, with the exception of the reference generator, P_0, and A is a constant matrix. The reference generator power is not included in the vector P to avoid singularity of A as a consequence of the overall power balance.

The input power demands are either specified deterministically or as an average value plus some random fluctuation around the average value. The random fluctuation is applied to either each individual load or to "regional" groups of load nodes.

The generator power dispatch is solved using standard LP methods. Using the input power demand, we solve the power flow equations, Eq. (1), with the condition of minimizing the following cost function:

$$\text{Cost} = \sum_{i\in G} P_i(t) - W \sum_{j\in L} P_j(t) \tag{2}$$

We assume that all generators run at the same cost and all loads have the same priority to be served. However, we set up a high price for load shed by setting $W = 100$. This minimization is done with the following constraints:

1) Generator power $\quad 0 \le P_i \le P_i^{\max} \quad i \in G$
2) Load power $\quad P_j \le 0 \quad j \in L$
3) Power flows $\quad |F_{ij}| \le F_{ij}^{\max}$
4) Power balance $\quad \sum_{i\in G\cup L} P_i = 0$

This linear programming problem is numerically solved using the simplex method as implemented in [6]. The assumption of uniform cost and load priority can of course be relaxed but changes to the underlying dynamics are not likely from this.

In solving the power dispatch problem for low load power demands, the initial conditions are chosen in such a way that a feasible solution of the linear programming problem exists. That is, the initial conditions yield a solution without line overloads and without power shed. Increases in the average load powers and random load fluctuations can cause a solution of the linear programming with line overloads or requires load power to be shed. At this point, a cascading event may be triggered.

A cascading overload may start if one or more lines are overloaded in the solution of the linear programming problem. We consider a line overloaded if the power flow through this line is within 1% of $F_{ij}^{\max}$. At this point, we assume that there is a probability p_2 that an overloaded line will outage. If an overloaded line outages, we reduce its corresponding $F_{ij}^{\max}$ by large amount (making it effectively zero) to simulate the outage, and a new solution is calculated. This process can require multiple iterations and continues until a solution is found with no more outages.

This fast dynamics model does not attempt to capture the intricate details of particular blackouts, which may have a large variety of complicated interacting processes also involving, for example, protection systems, dynamics and human factors. However, the fast dynamics model does represent cascading overloads and outages that are consistent with some basic network and operational constraints.

Calculations with the fast dynamics model are carried out by slowly increasing the average load demands over 10^5 iterations. If one regards each model run as occurring at successive peak daily loads (when blackouts are most likely), then this corresponds to an average load demand increasing slowly in a power network under fixed conditions over a period of 10^5 days. This is not because we try to simulate a real power transmission network, as this time scale is too long for the power system remaining under the same rules and conditions. Rather it is done in order to accumulate the necessary statistics to calculate the PDFs and other statistical measures needed to understand the system and ultimately do risk analysis. This emphasizes the problem of working with real data [5,7], which has only been available for a period of time more than an order of magnitude shorter than the time used in these calculations.

3. Critical points of the OPA fast dynamics blackout model

This section studies the behavior of blackouts in the fast dynamics model of section 2 as the average load demand is increased. As the power demand increases, we found several transition points. Some of these transition points have the characteristic properties of a critical transition. That is, when the load power demand is close to a critical value, the probability distribution function (PDF) of the blackout size has an algebraic tail and at the critical loading the risk for blackouts increases sharply. In particular, there is a sudden change in the rate of change of load shed as a function of the power demand. These transitions are caused by limits in the power system and they can be grouped in two types of limiting conditions:

1. Limits set by the available power generation.
2. Limits set by the transmission capacity of the grid.

An example with two of these limits is shown in Fig. 3. For a tree network with 382 nodes (12 generators and 370 loads), we increase power demand by increasing all loads at the same rate. The load demand in this example is deterministic and there is no random fluctuation in the load demands. As we reach a power demand of 31480, the total generator capacity, load power shedding starts. As the demand continues to increase, all power above 31480 is shed. When the demand reaches 45725, the power flow in some lines reaches the line power flow limit and some line outages are produced. This causes a further increase in the load power shed.

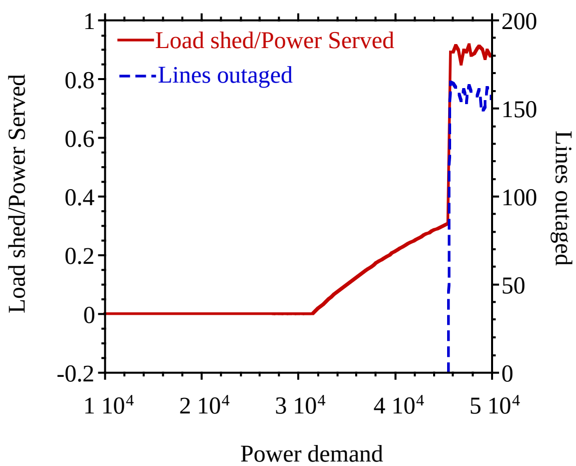

Fig. 3. Normalized power shed and number of outaged lines for a tree network with 382 nodes as a function of power demand.

Why is there a second transition after the total power served is kept constant that is therefore independent of the level of demand? The reason is that the individual loads increase and the power shed is not uniform over all loads. Therefore, even if the total power served is constant, the power delivered to some of the loads is increased as the total demand increases and that leads to overloading lines and possible line outages. The second transition point occurs at the same value of the power demand even in the absence of the first critical point, because it depends on the power of individual loads and the maximum power flow that the lines connecting them can carry. These results come from studying a sequence of cases under the same conditions but without random load fluctuations. The important point is that the first transition point is a function of the total power demand, while the second depends on the local value of the loads near the lines that are closer to overload.

Some of these transition points have the characteristic properties of a critical transition. For the calculation shown in Fig. 3, we have used the power demand as control parameter and we have done a scan starting with all load nodes having the same power loads and not allowing for fluctuations. Clearly the power generation limit (the first inflection point in the load-shed curve in Fig. 3) behaves as a second order transition point, characterized by a continuous function with discontinuous derivative. The critical point in this case is given by the generator power margin reaching zero, that is $\Delta P \equiv \sum_{i \in G} P_i - P_{Demand} = 0$. The load shed is a continuous function of the load power demand, but its derivative with respect to the load power demand is discontinuous at the transition point.

In the proximity of the generator critical point, the PDF of the normalized load shed has an algebraic tail. To calculate the PDF, we have to introduce noise into the system and this is done by introducing random fluctuations of the load power demands. The load fluctuations are controlled by the parameter γ described in Appendix I. For a given value of γ, the average fluctuation induced in the total power demand is $(\gamma - 1)/\left(2\sqrt{N_L}\right)$. Therefore, we can also use the parameter γ as a control parameter to scan over the critical point.

For the sequence of results in Fig. 4, the first critical point is reached with γ = 1.35. This corresponds to an averaged fluctuation in the power demand of 10%. As γ increases, more of the fluctuations in power demand reach the critical point, and the PDF of the normalized load shed develops an algebraic tail with decay index close to –1. Above the critical point, the PDF changes to an exponential tail. This is shown in Fig. 4 for a 94-node tree network. We have chosen network conditions with the generator power limit well below the limits set by the transmission lines in order to avoid interacting with the line limits critical point. In this case, we have given an averaged value to the generation margin capability of 5% and varied the maximum daily oscillation of the loads. Naturally, in this situation there are no outages in the system and power shedding is simply due to a supply shortage.

The second transition point in Fig. 3 is associated with line limits and is more difficult to characterize. To identify this transition point, it is useful to define the fraction of overload for a given line as

$$M_{ij} = \frac{F_{ij}}{F_{ij}^{\max}} \quad (3)$$

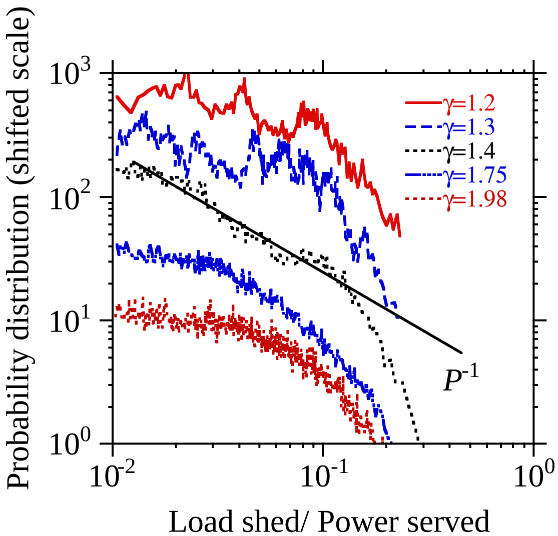

Fig. 4. PDF of the normalized load shed for a tree 94-node tree network for different levels of the load fluctuations.

We can then calculate $M_{\max} \equiv \max_{ij} M_{ij}$. The second transition point in Fig. 3 is given by $M_{\max} = 1$. The properties of this type of transition point depend on the value of the parameter p_2, the probability that an overloaded line outages. Of course, if $p_2 = 0$, there are no line outages and this transition point has similar properties as the generator limit, and looks like a second order transition. However, for $p_2 = 1$, all overload lines outage. This is the value of p_2 used in the calculation shown in Fig. 3 and the transition point has some of the features of a first order transition characterized by a discontinuous jump in the function. In Fig. 5, we show examples of transitions for these different values of the relevant parameters. For values of p_2 between 0 and 1, we have intermediate situations that are more difficult to characterize. Only for $p_2 = 0$ does the PDF of the load shed near the critical point have a clear algebraic tail.

The full classification of the properties of these transition points for all values of the parameters is beyond the scope of this paper.

In trying to describe the realistic dynamics of power transmission systems, it is found that the best choice of parameters is when both critical points are close to each other. In this case, we can combine the existence of power tail in the PDF of the load shed with the presence of outages.

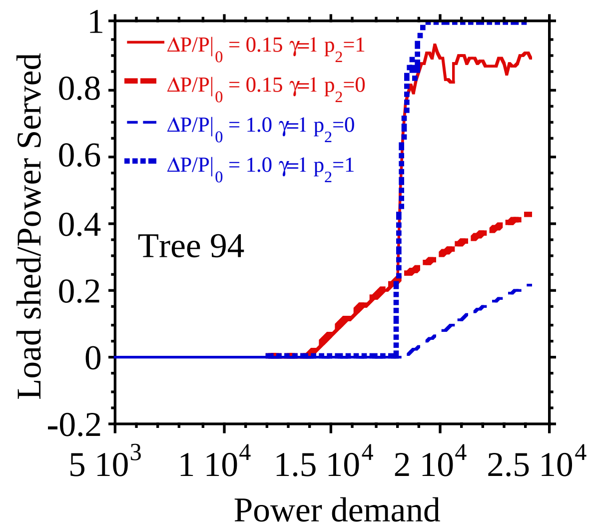

Fig. 5. Normalized power shed for a tree network with 94 nodes as a function of load power demand. Four different scenarios are included.

4. Self-organization dynamics

To transform the model described in Section 2 into a self-organized dynamical system we must include some of the opposing forces that act on the power transmission system. One example of these opposing forces is the growth of the demand and the system response through upgrading the system. These opposing mechanisms were incorporated in the model in Refs. [1, 2] and a short description of this model is given in the Appendix I. In this model, all loads are multiplied by a fixed constant $\gamma > 1$ at the start of the day. This causes an exponential increase in the average load demand. The generator capabilities are incremented in the same way; therefore, the generator limits are never reached in this model. The response mechanism was triggered by the blackout outages. When there is a blackout, the overload lines have their limits incremented by multiplying them by $\mu > 1$. The combined effect of these slow dynamics is that the system self-organizes close to the critical point of the outages (second point in the example of Fig. 3).

In order to have a self-organization mechanism that includes the growth of maximum generator power, we have used tested several algorithms. One of the simplest forms incorporating such a mechanism is based on the increase of maximum generator power as a response to the load demand. We have limited the model to increases in maximum generator power at the same nodes that initially had generators. In doing so, we have implemented the following rules:

a. The increase in power is quantized. This may reflect the upgrade of a power plant or adding generators. We have tried two possibilities. The increase is taken to be either a fixed quantity or a fixed ratio to the total power. The second approach seems to work better, in the sense of convergence to a steady state. Therefore, we introduce the quantity

$$\Delta P_a \equiv \kappa\left(P_T / N_G\right) \qquad (4)$$

Here, P_T is the total power demand, N_G is the number of generator nodes, and κ is a parameter that we have taken to be a few percent.

b. To be able to increase the maximum power in node j, the sum of the power flow limits of the lines connected to j should be 20% larger than the existing generating power plus the addition at node j. The 20% value is an arbitrary quantity that provides a safety margin so that the line ratings are coordinated with the generator capabilities.

c. A second condition to be verified before any maximum generator power increase is that the mean generator power margin has reached a threshold value. That is, we define the mean generator power margin at a time *t* as:

$$\frac{\Delta P}{P} = \frac{\sum_{j \in G} P_j - P_0 e^{\lambda t}}{P_0 e^{\lambda t}} \qquad (5)$$

where P_0 is the initial power load demand.

d. Once condition c) is verified, we choose a node at random to test condition b). If the chosen node verifies condition b), we increase its power by the amount given by Eq. (4). If condition b is not verified, we choose another node at random and iterate. After power has been added to a node, we recalculate the mean generator power margin, Eq. (5), and continue the process till $\Delta P/P$ is above the prescribed quantity. This is motivated by the fact that utilities are, in general, likely to build a power plant where the transmission capacity already exists.

This algorithm seems to provide the self-organization process that we were looking for in the following sense. The maximum generator power stays close to and below the critical point, which results in PDFs with power law tails. The new measure of criticality is the mean generator power margin and this quantity converges to a steady state in a reasonable amount of time. Its mean values is approximately

$$\left\langle \frac{\Delta P}{P} \right\rangle = \left.\frac{\Delta P}{P}\right|_{threshold} - \frac{\kappa}{2N_G}. \qquad (6)$$

This result is reasonable because only one node gets an increase at a given time.

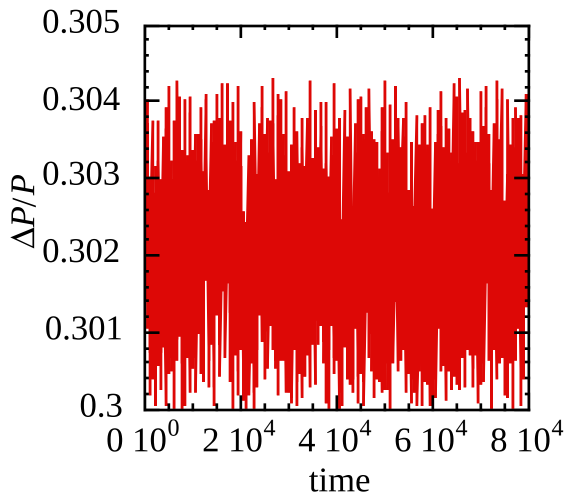

Fig. 6. Time evolution of $\Delta P/P$ for a case with κ = 0.04, threshold $\Delta P/P$ = 0.3.

It is also possible to introduce a time delay between the detection of a limit in the generation margin and the increase of the maximum generator power. This delay would represent the construction time. However, the result is the same as increasing the value of κ in Eq. (4), which can also give an alternative interpretation for κ.

In Fig. 6, we have plotted an example of time evolution of the generator margin $\Delta P/P$ for a case with κ = 0.04, threshold $\Delta P/P = 0.3$, and 10 generator nodes. Increasing κ gives larger oscillations. The results, characterized by the PDF of the normalized power shed, do not seem to depend on the particular value of κ. They do depend on the value taken for the threshold of $\Delta P/P$. If the latter is not the critical value, we do not get critical behavior, as expected. Note that the critical value for $\Delta P/P$ is a function of the maximum oscillation of the power load as described in Section 3.

Once we have determined from the load scans of Section 3 what the critical points are, we can explore the dynamics of self-organization. In combining the two dynamical loops, the real self-organized critical point that the system is operating near is the point where the two types of critical points are close to each other. Therefore, both the line improvement and the generator upgrade in the dynamics of the system are needed in the dynamical evolution. Once we have both of them together, the PDFs of the power shed have well-developed power tails. This is shown in Fig. 7, where the PDF of the load shed normalized to the total power demand for three different tree configurations has been plotted.

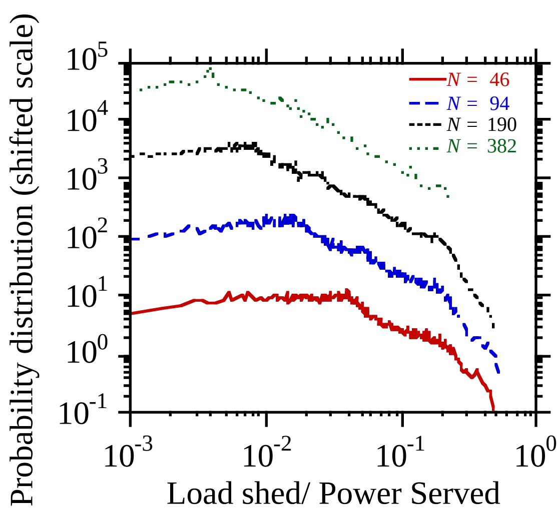

Fig. 7. PDF of the load shed normalized to the total power demand for four different tree networks.

The power-law-scaling region increases with the number of nodes in the network. The power decay index is practically the same for the four networks and close to –1.0. The particular values of the decay index for each network are given in Table I.
The range of the power tail region is defined as the ratio of the maximum and minimum load shed described by the power law. From the values obtained for the four networks listed in Table I, we can see that the range scales with the network size.

Table I

Number of nodes	PDF decay index	Range of power tail
46	-1.13	7
94	-1.16	17
190	-1.16	36
382	-1.21	62

The PDFs plotted in Fig. 7 are very similar to the PDFs of the normalized load shed obtained by direct power demand scan near the critical points with an important difference that they are now self organized.

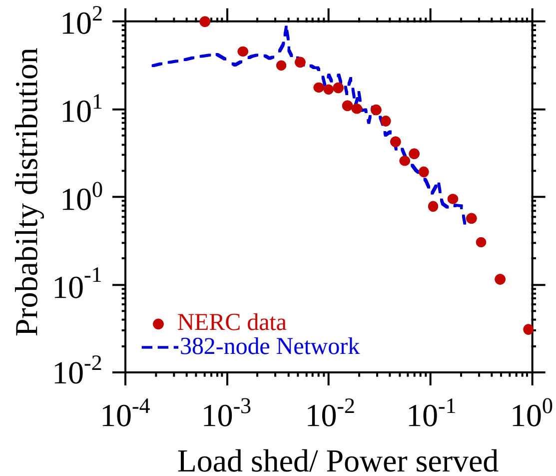

Fig. 8. PDF of the normalized load shed for the 382-node tree network and for the North American blackouts in 15 years of NERC data normalized to the largest size blackout

The form of these PDFs, or at least their power tail, seems to have a universal character. Therefore, we can try to compare the PDF of the normalized load shed obtained for the largest network with the PDF of the blackouts obtained in the analysis of the data from the last 15 years in the U.S. We have plotted the latter normalizing the events to the largest value over this period of time.

The level of agreement between the power tails of these two PDFs is remarkable. This seems to indicate that the dynamical model for the blackout has captured some of the main features of the data.

5. Discussion and conclusions

The opposing forces organizing the system, which in this paper are crudely represented by load demand increases and upgrades to network and generation capacity, may also be seen as the outcomes of design and operational procedures that trade off system security (risk of blackout) and maximizing the energy or peak power delivered. It is not clear how the self-organization we model is divided between design and planning procedures and operational procedures. In power system design and planning, design rules can incorporate previous experience of blackouts or designs may be tested by extensive simulations. In operations, real blackouts do of course occur and have significant impacts on operation, upgrades and repair, but there are also engineering responses to simulated events. A key question addressed but not fully answered in this paper, assumes from the power tails observed in the NERC data that North American power systems have been operated near a critical point and asks why or how this arises. Operational security criteria such as the n-1 criterion do influence the power system loading and planning, as well as the probability of cascading outages, and it would be interesting to determine the extent to which application of these criteria would lead to operation of the system near critical loading at peak load. That is, do specific security criteria contribute to self-organization to criticality? A fundamental understanding of the relation between security criteria and the risk or distribution of blackouts is particularly needed as the tested practices of the past are changed to accommodate deregulated power systems.

To understand the complex dynamics of series of blackouts in power systems, we proposed the OPA model. The OPA model incorporated a simple level of self-organization by including the growth of the load power demand and the engineering response to system failures. This model shows a variety of possible transition points, some linked to line outages in the system and others linked to the limits in the generation capacity. We have found that critical behavior emerges when these transition points correspond to similar power demand levels. Close to this critical point, the system reaches a maximum capacity for power transmission and the PDF of the blackout sizes have a power tail.

For the system to self-organize near to this critical point, we have to model the dynamics of the upgrading of the generator capacity as a response to the increased load demand. This leads to a double response loop in the model: 1) line upgrades and 2) generator upgrades. Under these conditions, the model has the characteristic properties of a system governed by self-organized criticality. The PDF of the blackouts size has the same power dependence that have been found from the analysis of NERC data for the North American power grid over a period of 15 years.

In these studies, we have limited the application of the model to idealized power grid systems with tree structure. Applications to more realistic power networks are under way.

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Appendix I: Slow dynamics of load increase and network upgrade in OPA model.

The slow dynamics proposed in [1, 2] model the growth of the demand and response to the blackout by upgrades in the grid transmission capability. The slow dynamics is carried out by a simple set of rules. At the beginning of the day *t*, we apply the following rules:

1. Growth of the power demand. All loads are multiplied by a fixed parameter λ that represents the daily rate of increase in electricity demand. On the bases of the past electricity consumption, we had estimated that $\lambda = 1.00005$. This value corresponds to a yearly rate of 1.8%.

$$P_i(t) = \lambda P_i(t-1) \quad \text{for } i \ni L \qquad \text{(AI-1)}$$

Equally, the maximum generator power is increased at the same rate

$$P_i^{\max}(t) = \lambda P_i^{\max}(t-1) \quad \text{for } i \ni G \qquad \text{(AI-2)}$$

2. Power transmission grid improvement. We assume a gradual improvement in the transmission capacity of the grid in response to the outages and blackouts. This improvement is implemented through an increase of $F_{ij}^{\max}$ for the lines that have overload during a blackout. That is.

$$F_{ij}^{\max}(t) = \mu F_{ij}^{\max}(t-1) \qquad \text{(AI-3)}$$

if the line *ij* overloads during a blackout. We take μ to be a constant and this parameter is the main control parameter in this system

3. Daily power fluctuations. To represent the daily local fluctuations on power demand, all power loads are multiplied by a random number *r*, such that $1/\gamma \le r \le \gamma$. We generally choose γ in the range 1 to 1.4. We also assign a probability for a random outage of a line. We represent a line outage by multiplying its impedance by a large number κ_1 and dividing its corresponding $F_{ij}^{\max}$ by another large

number κ_2. In the present calculations, these numbers are of the order of 1000.

After applying these three rules to the network parameters, we look for a solution of the power flow problem using linear programming as described in section 2.

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